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DIOPTRICAE

PARS SECVNDA,

CONTINENS

LIBRVM SECVNDVM,

DE

CONSTRVCTIONE TELESCOPIORVM DIOPTRICORVM

CVM

APPENDICE

DE

CONSTRVCTIONE TELESCOPIORVM CATOPTRICO-DIOPTRICORVM.

AVCTORE
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ACAD. PETROP. PARISIN. ET LOND.

PETROPOLI
Impenſis Academiae Imperialis Scientiarum
1770.

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DE

CONSTRVCTIONE TELESCOPIORVM

SECTIO PRIMA.

DE

TELESCOPIIS PRIMI GENERIS,

QVAE

LENTE OCVLARI CONCAVA INSTRVCTA OBIECTA SITV ERECTO REPRAESENTANT.

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CAPVT I.

DE
TELESCOPIIS IN GENERE.

Definitio I.

1.

Teleſcopium eſt inſtrumentum dioptricum obiectis valde remotis ſpectandis inſeruiens.

Coroll. I.

2. Cum ergo diſtantia obiecti ſit valde magna, in calculo quantitatem a, qua diſtantia obiecti a lente obiectiua deſignatur, tanquam infinitam ſpectare licet, ideoque α denotabit diſtantiam focalem lentis obiectiuae, neglecta ſcilicet eius craſſitie.

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Coroll. 2.

3. Cum poſuerimus $a = \mathrm{A}a$, ob $a = \infty$ erit numerus A euaneſcens ideoque et $\mathrm{A} = 0$ et $\mathfrak{A} = \frac{\mathrm{A}}{\mathrm{A}+1} = 0$. Hinc ergo in formulis ſupra traditis litterae A et $\mathfrak{A}$ ita ex calculo eliminabuntur, vt loco $\mathrm{A}a$ et $\mathfrak{A}a$ ſcribatur α.

Definitio. 2.

4. In teleſcopiis campus apparens non ex ipſa obiecti conſpicui quantitate aeſtimatur, ſed ex angulo, ſub quo haec pars conſpicua nudo oculo cerneretur.

Corollarium.

5. Littera ergo Φ, quam ſupra in noſtras formulas introduximus, denotabit ſemidiametrum campi adparentis vel potius eius tangentem; quia autem hic angulus plerumque eſt valde paruus, is ipſe loco tangentis ſine errore, praecipue ſi multiplicatio ſit notabilis, uſurpatur.

Definitio 3.

6. Multiplicatio in teleſcopiis ex ratione quantitatis per inſtrumentum viſae, ad quantitatem, qua idem obiectum in eadem diſtantia remotum nudo oculo cerneretur, aeſtimari ſolet.

Co-

Coroll. 1.

7. Quia ergo ſupra in genere multiplicationem ad diſtantiam b retulimus; obiecti vero diſtantia poſita eſt $=a$, erit quoque $b=a$.

Coroll. 2.

8. Exponens ergo multiplicationis $=m$ hoc caſu indicat, quoties angulus, ſub quo diametrum cuiuspiam obiecti per teleſcopium cernimus, maior ſit angulo, ſub quo idem obiectum nudis oculis cerneretur.

Scholion 1.

9. Hoc ſcilicet intelligendum eſt, quamdiu de angulis ſatis paruis eſt ſermo; quando autem anguli ſunt maiores, exponens multiplicationis m declarabit, non quoties ipſe angulus, ſub quo obiectum quodpiam per teleſcopium cernitur, ſed quoties eius tangens maior ſit tangente eius anguli, ſub quo idem obiectum nudis oculis eſſet apparitur um, ita ut etiamſi multiplicatio m foret infinita, tamen angulus viſionis non ultra 90° excreſcere poſſet, dum ſcilicet quantitas obiecti ab axe teleſcopii aeſtimatur.

Scholion 2.

10. His igitur obſeruatis formulae ſupra erutae facile ad teleſcopia accomodantur eoque non nihil ſim-

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pliciores euadunt. Praeterea vero etſi pro varia oculi conſtitutione diſtantia iuſta littera l deſignata ſit maxime diuerſa, tamen hic iſta diuerſitas ſeponi ſolet, quia teleſcopium ad unam oculi ſpeciem accomodatum in praxi facile ad quosuis alios oculos accommodatur, et quia plerumque diſtantia iuſta l ſatis eſt magna prae oculi diſtantia ab ultima lente, eaque adeo pro multis oculis in infinitum excreſcit, commode ſtatuemus $l=\infty$. Hinc ſi ultimae lentis diſtantiae determinatrices ſint f et ζ poſt eamque locus oculi $=O$, ob $O=\zeta+l$, diſtantia ζ debebit eſſe infinita, ſcilicet $\zeta=O-l$, ita ut ſit $\frac{\zeta}{l}=-1$; ſiue $\frac{l}{\zeta}=-1$ atque ob $\zeta=\infty$ euidens eſt, ultimae lentis diſtantiam focalem fore $=f$.

Problema I.

11. Ex quotcunque lentibus teleſcopium fuerit compoſitum, elementa exponere, quibus ſupra vſi ſumus, ad eius conſtructionem determinandam, ſimulque relationem eorum diuerſorum elementorum inter ſe repraeſentare.

Solutio.

Pro qualibet lente 1° conſiderauimus eius rationem refractionis pro radiis mediae naturae. 2° Eius binas diſtantias determinatrices cum numero arbitrario λ. 3° Nunc etiam cuiusque lentis diſtantiam focalem in

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in calculum introducemus. 4° Etiam introduximus rationes aperturarum pro ſingulis lentibus littera π indicatas. Quae elementa pro ſingulis lentibus ſequenti modo ob oculos ponamus:

Lentes	Ratio refr.	Diſt. det	Num. arb.	Diſt. foc.	Rat. apert.
I^{ma}	$n : 1.$	$a, \alpha.$	λ	$p.$	o
II^{da}	$n' : 1$	$b, \beta.$	λ'	$q.$	π
III^{tia}	$n'' : 1.$	$c, \gamma.$	λ''	$r.$	π'
IV^{ta}	$n''' : 1.$	$d, \delta.$	λ'''	$s.$	π''
V^{ta}	$n'''' : 1.$	$e, \varepsilon.$	λ''''	$t.$	π'''
VI^{ta}	$n''''' : 1.$	$f, \zeta.$	λ^{V}	$u.$	π''''

etc.

deinde etiam poſuimus $A = \frac{\alpha}{a}$; $B = \frac{\beta}{b}$; $C = \frac{\gamma}{c}$; $D = \frac{\delta}{d}$; $E = \frac{\varepsilon}{e}$; $F = \frac{\zeta}{f}$ etc. tum vero etiam $\mathfrak{A} = \frac{A}{A+1}$; $\mathfrak{B} = \frac{B}{B+1}$; $\mathfrak{C} = \frac{C}{C+1}$; $\mathfrak{D} = \frac{D}{D+1}$; $\mathfrak{E} = \frac{E}{E+1}$; $\mathfrak{F} = \frac{F}{F+1}$ etc.

His expoſitis modo ante vidimus ob $a = \infty$, fore $A = o$ et $\mathfrak{A} = o$, quibus valoribus ita eſt vtendum, vt ſit $Aa = \alpha$ et $\mathfrak{A}a = p$ atque $p = \alpha$; pro ſequentibus autem lentibus habebimus

$$q = \mathfrak{B}b;\ r = \mathfrak{C}c;\ s = \mathfrak{D}d;\ t = \mathfrak{E}e;\ u = \mathfrak{F}f.$$

etc.

unde viciſſim per diſtantias focales erit

$$b = \frac{q}{\mathfrak{B}} = \frac{B+1}{B}q. \text{ et } \beta = (B+1).\ q$$

$$c = \frac{r}{\mathfrak{C}} = \frac{C+1}{C}r. \text{ et } \gamma = (C+1).\ r$$

$$d = \frac{s}{\mathfrak{D}} = \frac{D+1}{D}s. \text{ et } \delta = (D+1).\ s$$

etc.

deinde

deinde pro rationibus aperturarum habuimus ſupra ſequentes relationes: poſita ſcilicet ſemidiametro campi apparentis $=\Phi$

$$1^\circ.\quad \frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{A\alpha}{b}=\frac{\alpha}{b}\ \text{ſeu}$$

$$\frac{\mathfrak{B}\pi}{\Phi}=\frac{\alpha+b}{b}\ \text{vel}\ \frac{\pi}{\Phi}=\frac{\alpha+b}{q}$$

$$2^\circ.\quad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{AB.\alpha}{c}=\frac{B\alpha}{c}$$

$$3^\circ.\quad \frac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\frac{ABC.\alpha}{d}=\frac{BC.\alpha}{d}$$

$$4^\circ.\quad \frac{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}{\Phi}=\frac{ABCD\alpha}{e}=\frac{BCD\alpha}{e}$$

etc.

atque hinc viciſſim valores ſequentes elicuimus:

$$a=\infty \qquad \alpha=p$$

$$b=\frac{p\Phi}{\mathfrak{B}\pi-\Phi} \qquad \beta=\frac{B.p\Phi}{\mathfrak{B}\pi-\Phi}$$

$$c=\frac{B.p\Phi}{\mathfrak{C}\pi'-\pi+\Phi} \qquad \gamma=\frac{BC.p\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

$$d=\frac{BC.p\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi} \qquad \delta=\frac{BCD.p\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

$$e=\frac{BCD.p\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi} \qquad \varepsilon=\frac{BCDE.p\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$

etc.

Coroll. I.

12. In ſuperioribus iam ſatis oſtenſum eſt, quomodo ex binis diſtantiis determinatricibus ſingulas lentes conſtrui oporteat; quem in finem valores litterarum ϱ, σ, τ, quibus etiam adiungimus ν et μ, recordari neceſſe eſt, qui ſunt

$\varrho +$

$$\varrho+\sigma=\frac{1}{n-1};\ \sigma-\varrho=\frac{2n+2}{n+2}$$

$$\tau=\frac{n\sqrt{(4n-1)}}{4(n-1)(n+2)};\ \mu=\frac{n(4n-1)}{8(n-1)^2(n+2)}$$

$$\text{et } \nu=\frac{4(n-1)^2}{4n-1};\ \mu\nu=\frac{n}{2(n+2)}$$

Coroll. 2.

13. His valoribus pro quavis ratione refractionis cognitis pro diſtantiis determinatricibus a, α; cum numero arbitrario λ facies lentis ſequenti modo definientur:

Radius faciei

$$\text{anter:} = \frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$\text{poſter:} = \frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

ad quod exemplum omnes reliquae lentes ſunt conſtruendae.

Coroll. 3.

14. Cum confuſio ex tali lente oriunda fiat minima, ſumto $\lambda=1$; operae pretium erit inveſtigare, quantum numerum pro λ accipi oporteat, ut ambae lentis facies fiant inter ſe aequales; reperitur hunc in finem

$$\sqrt{(\lambda-1)}=\frac{a-\alpha}{a+\alpha}\cdot\frac{2(n+2)(n-1)}{n\sqrt{(4n-1)}}$$

hincque

$$\lambda=1+\frac{(a-\alpha)^2}{(a+\alpha)^2}\cdot\frac{4(nn-1)^2}{n^2(4n-1)}$$

cum iam ſit $\frac{\overline{a-\alpha}^2}{\overline{a+\alpha}^2} = 1 - \frac{4a\alpha}{(a+\alpha)^2}$; erit

$$\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} - \frac{16a\alpha(nn-1)^2}{(a+\alpha)^2 n^2 (4n-1)}$$

quare ſi fuerit vel $a = \infty$ vel $\alpha = \infty$ erit

$$\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)}$$

Scholion.

15. Quo noſtra inveſtigatio latius pateat; ſingulis lentibus peculiares refractionis rationes tribuamus, quoniam nunc quidem compertum eſt, diuerſas uitri ſpecies ratione refractionis inter ſe diſcrepare, ita tamen, vt ualor numeri n intra limites 1, 50 et 1, 60 contineatur, quamobrem pro praxi conſultum erit, pro ſingulis ualoribus intra hos limites contentis ualores litterarum ϱ, σ, τ, μ, ν et $\mu\ \nu$ hic exhibere; quem in finem ſequentem tabulam hic ſubiungemus. Quod uero ad differentialia numerorum n attinet, de iis nihil definio, ſi quidem experimenta Dollondi ueritati ſunt conſentanea, praeterquam quod ſi $n = 1,53$ pro uitro coronario, $n' = 1,58$ pro chryſtallino, ſit per experimenta

$$dn : dn' = 2 : 3;\ \frac{dn}{n-1} : \frac{dn'}{n'-1} = 7 : 10.$$

n.	ϱ.	σ.	τ.	μ.	ν.	$\mu\nu$.
1.50.	0.2858.	1.7143.	0.9583.	1.0714.	0.2000.	0.2143.
1.51.	0.2653.	1.6956.	0.9468.	1.0420.	0.2065.	0.2151.
1.52	0.2456.	1.6776.	0.9358.	1.0140.	0.2129.	0.2159.
1.53	0.2267.	1.6601.	0.9252.	0.9875.	0.2196.	0.2168.
1.54.	0.2083.	1.6434.	0.9149.	0.9622.	0.2260.	0.2176.
1.55.	0.1907.	1.6274.	0.9051.	0.9381.	0.2326.	0.2182.
1.56.	0.1737.	1.6119.	0.8956.	0.9151.	0.2393.	0.2192.
1.57.	0.1573.	1.5970.	0.8864.	0.8932.	0.2461.	0.2199.
1.58.	0.1414.	1 5827.	0.8775.	0.8724.	0.2529.	0.2206.
1.59.	0.1259.	1.5689.	0.8689.	0.8525.	0.2597.	0.2214.
1.60.	0.1111.	1.5555.	0.8607	0.8333.	0.2666.	0.2221.

Problema 2.

16. Ex quotcunque lentibus teleſcopium fuerit compoſitum, definire conditiones, vt ſingularum lentium interualla fiant poſitiua.

Solutio.

Quomodocunque diſtantiae determinatrices lentium ratione ſignorum + et − ſint adfectae, ſemper neceſſe eſt, vt quantitates $\alpha + b$; $\beta + c$; $\gamma + d$; $\delta + e$ etc., quibus diſtantiae lentium exprimuntur, fiant poſitiuae; quodſi ergo loco harum litterarum ualores ante exhibiti ſubſtituantur, ſequentibus conditionibus ſatisfieri oportet:

$$a+b = \frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi - \mathfrak{P}} > 0$$

$$\beta + c = \frac{B.\mathfrak{P}p.(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \mathfrak{P})(\mathfrak{C}\pi' - \pi + \mathfrak{P})} > 0$$

$$\gamma + d = \frac{BC.\mathfrak{P}p.(\mathfrak{D}\pi'' - (1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \mathfrak{P})(\mathfrak{D}\pi'' - \pi' + \pi - \mathfrak{P})} > 0$$

$$\delta + e = \frac{BCD.\mathfrak{P}p.(\mathfrak{E}\pi''' - (1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \mathfrak{P})(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \mathfrak{P})} > 0$$

etc.

circa quas diſtantias obſeruari conuenit, quasdam earum etiam fieri poſſe $= 0$, quando ſcilicet duae pluresue lentes ſibi inuicem immediate iunguntur, quemadmodum in lentibus obiectiuis euenire poſſe ſupra uidimus, nunquam autem vlla harum diſtantiarum fieri debet negatiua.

Coroll. 1.

17. Hinc manifeſtum eſt, ſi fuerit $\pi = 0$, tum diſtantiam inter lentem primam et ſecundam euaneſcere; ac ſi praeterea ſit $\pi' = 0$, etiam tertia lens praecedentibus immediate iungetur, et quarta lens inſuper iis adiungetur, ſi quoque fuerit $\pi'' = 0$, quod quidem euenit in lentibus obiectiuis compoſitis ſeu multiplicatis, vti ſupra iam eſt oſtenſum.

Coroll. 2.

18. Diſtantia autem inter lentem primam et ſecundam fiet maior nihilo, uel tribuendo ipſi π ualorem poſitiuum, quoties ſcilicet fuerit $\frac{\mathfrak{B}p}{\mathfrak{B}\pi - \mathfrak{P}}$ quantitas poſi-

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poſitiua uel tribuendo ipſi π ualorem negatiuum, quoties $\frac{\mathfrak{B}p}{\mathfrak{B}\pi-\Phi}$ fuerit quantitas negatiua.

Coroll. 3.

19. Quoniam $\alpha = p$ eſt quantitas poſitiua, caſus notari merentur;

1°. $b = -p$; 2°. $b = 0$; 3°. $b > 0$.

Primo caſu interuallum primum euaneſcit, ideoque erit uel $\pi = 0$ uel $\mathfrak{B} = 0$, quod autem fieri nequit, quia foret $B = 0$, ideoque $\frac{\beta}{b} = 0$, ac propterea $\beta = 0$, cuiusmodi autem lens non datur, niſi etiam ſit $b = 0$; vnde in hoc primo caſu neceſſario habebitur $\pi = 0$. Secundo caſu, quo $b = 0$, lens ſecunda cadet in ipſam imaginem a prima lente proiectam fietque $\mathfrak{B}\pi - \Phi = \infty$, quia neque p neque Φ eſſe poteſt $= 0$, vnde prodibit pro hoc caſu $\mathfrak{B} = \infty$ et $B = -1$. hoc eſt $\beta = -b = 0$, vnde patet, hoc caſu ambas diſtantias determinatrices ſecundae lentis euaneſcere; nihilo uero minus eius diſtantiam focalem q ualorem quemcunque retinere poſſe, cum ſit $q = \mathfrak{B}b$, ob $\mathfrak{B} = \infty$ et $b = 0$ Caſu denique tertio, quo $b > 0$, fieri debet $\mathfrak{B}\pi - \Phi > 0$ ſeu $\mathfrak{B} > \frac{\Phi}{\pi}$.

Coroll. 4.

20. Quod hic de caſu ſecundo notauimus, ualet quoque de qualibet alia lente, quae in locum imaginis a lente praecedente formatae conſtituitur; tum

enim eius diſtantiarum determinatricium anterior euaneſcit, vnde et poſterior neceſſario euaneſcere debet; eueniat enim hoc in lente quarta, cuius diſtantiae determinatrices ſunt d et δ, et diſtantia focalis s, et quia eſt $\frac{1}{s} = \frac{1}{d} + \frac{1}{\delta}$ ſi ergo ſit $d = 0$, neceſſario quoque fiet $\delta = 0$, cum enim ſit $\delta = \frac{sd}{d-s}$, poſito $d = 0$, fiet utique $\delta = 0$ tum uero hinc etiam cognoſcimus, fore $\frac{\delta}{d} = -1 = D$; ita, vt hoc quoque caſu ſit $D = -1$ et $\mathfrak{D} = \infty$.

Problema 3.

21. Si teleſcopium ex quotcunque lentibus fuerit compoſitum, definire aperturas ſingularum lentium, vt omnes radii ab obiecto per lentem obiectiuam ingreſſi ſimul per omnes lentes ſequentes transmittantur.

Solutio.

Hic non obiectum quodcunque eſt intelligendum, ſed tantum quod per teleſcopium conſpici poteſt totum, ita, vt eius ſemidiameter adparens conueniat cum ſemidiametro campi apparentis, quam ſtatuimus $= \Phi$. Quodſi iam lentis obiectiuae ponatur ſemidiameter aperturae $= x$, ſupra oſtendimus, ſemidiametros aperturae ſingularum lentium ſequentium ſequenti modo determinari:

Semid.

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Semid. apert.

Lentis II$^{dae.}$ $\frac{\mathfrak{B}\pi p \pm x}{\mathfrak{B}\pi - \mathfrak{P}} \cdot \Phi$

. . . III$^{tiae.}$ $\frac{P.\mathfrak{C}\pi' p \pm x}{\mathfrak{C}\pi' - \pi + \mathfrak{P}} \cdot \Phi$

. . . IV$^{tae.}$ $\frac{BC.\mathfrak{D}\pi''.p \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \mathfrak{P}} \cdot \Phi$

. . . V$^{tae.}$ $\frac{BC\Gamma.\mathfrak{E}\pi'''.p \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \mathfrak{P}} \cdot \Phi$

etc.

ſingulae hac expreſſiones conſtant duabus partibus, et ſignum ambiguum $\pm$ indicat, ambas partes capi debere poſitiuas, etiamſi forte ambae uel ſaltim altervtra fuerit negatiua. Nihil autem impedit, quominus hae aperturae capiantur maiores, etiamſi haec amplificatio omni vſu deſtituatur. Quin etiam ſufficit, has ſemidiametros maiori tantum parti, quae plerumque eſt prior, aequales ſumſiſſe, quia hinc nullum aliud incommodum eſt metuendum, niſi quod extremitates campi adparentis al quarto obſcurius repraeſententur; atque vt lentes tantae aperturae ſint capaces, pro litteris π, π', π'', π''' etc. tam exiguas fractiones ſumi oportet, vti ſupra eſt expoſitum, ueluti $\frac{1}{4}$, $\frac{1}{5}$ uel adhuc minores.

Coroll. 1.

22. Priores partes harum formularum multo concinnius exprimi poſſunt, ſi diſtantias focales in calculum-

culum introducamus; tum enim eae ſequenti modo exprimentur:

$$\pi\, q;\ \pi'\, r;\ \pi''\, s;\ \pi'''\, t \text{ etc.}$$

quae expreſſiones immediate ex natura litterarum π, π', π'' etc. ſupra expoſita ſequuntur.

Coroll. 2.

23. Hinc etiam alterae partes illarum formularum concinnius exprimi poterunt, cum ſit $\frac{\Phi}{\mathfrak{B}\pi - \Phi} = \frac{b}{p} = \frac{q}{\mathfrak{B}p}$; et $\frac{\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = \frac{r}{B\mathfrak{C}.p}$ et $\frac{\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \frac{s}{BC.\mathfrak{D}.p}$; vnde ſuperiores formulae ita repraeſentari poſſunt.

Semid. apert.
Lentis

$$\text{I}^{mae.} = x$$

$$\text{II}^{dae.} = \pi\, q \pm \frac{q\,x}{\mathfrak{B}.p}$$

$$\text{III}^{tiae.} = \pi'\, r \pm \frac{r\,x}{B\mathfrak{C}.p}$$

$$\text{IV}^{tae.} = \pi''\, s \pm \frac{s\,x}{BC.\mathfrak{D}.p}$$

$$\text{V}^{tae.} = \pi'''\, t \pm \frac{t\,x}{BCD.\mathfrak{E}.p}$$

Coroll. 3.

24. Quodſi ergo eueniat, vt litterarum π, π', π'', π''' etc. quaepiam euaneſcat; tum pro lente re-

reſpondente ſemidiameter aperturae ſoli ſecundae parti aequalis ſumi debet. Aliis uero caſibus, quibus pars prima maior eſt ſecunda, ſufficit aperturam ex ſola prima parte definiri.

Scholion.

25. Caſus iſte, quo litterarum π, π', π'' etc. quaepiam fit $= 0$, tum habet locum, quando lens reſpondens in eiusmodi loco collocatur, quem ſupra pro idoneo loco oculi aſſignauimus, in quo ſcilicet radii ab extremitate obiecti per centrum lentis primae transmiſſi iterum uspiam cum axe concurrunt. In hoc enim loco lens conſtituta nulla alia apertura indigebit, niſi ea, quae ob aperturam lentis obiectiuae requiritur. Quare probe notandum eſt, quoties quaepiam lens in tali loco collocatur, pro ea ualorem ipſius π reſpondentis fore $= 0$. et uiciſſim. Quoniam igitur plerumque pars aperturae ab x pendens fit ualde parua, huiusmodi lentes commodiſſime loco diaphragmatum, quae uulgo in teleſcopiis adplicari ſolent, vſurpari poterunt, vt earum tam exigua apertura radii peregrini excludantur.

Problema 4.

26. Ex quotcunque lentibus teleſcopium fuerit compoſitum, definire rationem multiplicationis m, qua obiecta per id uiſa aucta conſpicientur.

Solutio.

Ex formulis, quas iam ſupra pro multiplicatione inuenimus, obtinebimus pro ſingulis lentium numeris ſequentes formulas.

Pro num. lent.	Ratio Multiplicationis.
I.	$m = + 1$ ob $\frac{\alpha}{l} = - 1$.
II.	$m = - \frac{\alpha}{b}$ ob $\frac{\beta}{l} = - 1$.
III.	$m = + \frac{\alpha\beta}{bc}$ ob $\frac{\gamma}{l} = - 1$.
IV.	$m = - \frac{\alpha\beta\gamma}{bcd}$ ob $\frac{\delta}{l} = - 1$.
V.	$m = + \frac{\alpha\beta\gamma\delta}{bcae}$ ob $\frac{\varepsilon}{l} = - 1$.
etc.	

hic ſcilicet notandum eſt, ſi pro m prodeat ualor poſitiuus, obiectum ſitu erecto, ſin autem negatiuus, ſitu inuerſo repraeſentatum iri. Viciſſim igitur ſi uelimus, vt teleſcopium v. gr. centies multiplicet, duo caſus ſunt euoluendi, alter, quo repraeſentatio requiritur erecta, alter, quo inuerſa, ac priori caſu ſtatuimus $m = + 100$; poſteriori uero $m = - 100$, ita, vt tunc ſatis ſit perſpicuum, quomodo pro quouis lentium numero ualores litterarum α, b; β, c; γ, d. etc. eſſe debeant comparati.

Co-

Coroll. 1.

27. Si litteras latinas maiuſculas introducere uelimus, erit pro duabus lentibus $m = -\frac{\alpha}{b}$; pro tribus $m = +\frac{\alpha}{c}$. B. pro quatuor $m = -\frac{\alpha}{d}$ B C. pro quinque $m = +\frac{\alpha}{e}$ B C D. etc.

Coroll. 2.

28. Cum porro ſit $\alpha = p =$ diſtantiae focali lentis obiectiuae, et littera latina minuſcula in his formulis denotet diſtantiam focalem lentis vltimae, formulae iſtae pro multiplicatione concinnius hoc modo repraeſentantur:

I. $m = +1$.
II. $m = -\frac{p}{q}$.
III. $m = +\frac{p}{r}$. B.
IV. $m = -\frac{p}{s}$ BC.
V. $m = +\frac{p}{t}$ BCD.

etc.

Scholion.

29. In hoc problemate pro caſu vnius lentis inuenimus $m = +1$, quo indicatur, obiecta per vnicam lentem non aucta, ſed naturali quantitate ſpectari; id quod per ſe eſt manifeſtum, quoniam diſtantiam oculi iuſtam l infinitam aſſumimus; tum enim

 erit

erit etiam $\alpha = p = \infty$ ideoque haec lens habebit ſuas facies inter ſe parallelas, per quam obiecta perinde cernuntur, ac nudis oculis; deinde pro caſu duarum lentium inuenimus $m = \frac{-p}{q}$; quare cum p ſit poſitiuum, ſi q fuerit negatiuum, teleſcopium referet obiecta ſitu erecto et aucta in ratione $p : q$, ſeu quoties diſtantia focalis lentis obiectiuae maior fuerit, quam diſtantia focalis lentis ocularis concauae; ſin autem lens ocularis quoque fuerit conuexa ſeu q poſitiuum, obiecta cernentur ſitu inuerſo ac toties aucta, quoties q continebitur in p. Tum uero hinc etiam liquet, ob $\alpha = p$ et $b = q$ diſtantiam inter has duas lentes $\alpha + b$ ſeu longitudinem teleſcopii fore aequalem quantitati $p + q$, vti ſatis conſtat. At ſi plures lentes adhibeantur, ratio multiplicationis non amplius per ſolas diſtantias focales lentium obiectiuae et ocularis determinatur, ſed inſuper ratio eſt habenda numerorum B, C, D etc. ſeu lentium intermediarum.

Problema 5.

30. Ex quotcunque lentibus teleſcopium fuerit compoſitum, definire locum oculi ſeu eius diſtantiam poſt vltimam lentem ocularem.

Solutio.

Hanc diſtantiam ſupra littera O indicauimus ſtatimque uidimus pro caſu vnicae lentis fore $O = o$.

Pro-

Pro caſu autem duarum lentium inuenimus $O = \frac{\mathfrak{B} b \pi}{\pi - \Phi}$, quae ob $\beta = \infty$ hincque $B = \infty$ et $\mathfrak{B} = 1$ abit in hanc $O = \frac{b\pi}{\pi - \Phi}$. Cum autem porro ſit $b = \frac{p\Phi}{\pi - \Phi}$, ideoque $\pi - \Phi = \frac{p\Phi}{b}$; habebitur $O = \frac{b^2 \pi}{p\Phi} = \frac{q^2 \pi}{p\Phi}$ et ob $m = -\frac{p}{q}$ ſeu $p = -m q$ erit $O = \frac{-\pi}{m.\Phi}$ et ob $\frac{\pi}{\Phi} = \frac{p+q}{q}$ habebitur etiam $O = -\frac{(p+q)}{m} = +\frac{m-1}{m} \cdot q$

Pro caſu trium lentium ob $\gamma = \infty$ ideoque $C = \infty$ et $\mathfrak{C} = 1$ habebimus $O = \frac{c\pi'}{\pi' - \pi + \Phi}$; eſt uero $c = r = \frac{B p \Phi}{\pi' - \pi + \Phi}$ et $m = \frac{p}{r} . B$ atque hinc $p B = m r$ adeoque $c = \frac{m r \Phi}{\pi' - \pi + \Phi}$; vnde erit $O = \frac{\pi'}{m \Phi} . r$

Pro caſu quatuor lentium ob $\delta = \infty$ ideoque $D = \infty$ et $\mathfrak{D} = 1$ inuenimus $O = \frac{d\pi''}{\pi'' - \pi' + \pi - \Phi}$; at eſt $d = s = \frac{BCp\Phi}{\pi'' - \pi' + \pi - \Phi} = \frac{-m s \Phi}{\pi'' - \pi' + \pi - \Phi}$ hincque $\pi'' - \pi' + \pi - \Phi = -m\Phi$ adeoque $O = -\frac{\pi''}{m\Phi} . s$

Quo haec ad plures lentes accommodari queant, tabulam ſequentem ſubiungam.

Num. lent.	Locus oculi
I.	$O = 0$
II.	$O = \frac{b\pi}{\pi - \mathfrak{P}} = -\frac{\pi}{m\Phi} \cdot q$
III.	$O = \frac{c\pi'}{\pi' - \pi + \Phi} = \frac{\pi'}{m\Phi} \cdot r$
IV.	$O = \frac{d\pi''}{\pi'' - \pi' + \pi - \mathfrak{P}} = -\frac{\pi''}{m\Phi} \cdot s$
V.	$O = \frac{e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \mathfrak{P}} = \frac{\pi'''}{m\Phi} \cdot t$
	etc.

Coroll. 1.

31. Ex ſuperioribus hic repeti conueniet, ſi ualor ipſius O prodeat poſitiuus, tum pro oculo locum idoneum inueniri, ex quo totus campus adparens conſpici queat; ſin autem pro O prodeat ualor negatiuus, tum oculum lenti ultimae immediate adplicari debere hocque caſu campum apparentem per aperturam pupillae determinari.

Coroll. 2.

32. Caſu duarum lentium diſtantiam O concinnius exhibere licuit, cum eſſet $O = \frac{m-1}{m} \cdot q = (1 - \frac{1}{m})q$; vnde ſtatim patet pro repraeſentatione erecta, vbi q eſt quantitas negatiua, diſtantiam O pariter fore negatiuam ideoque oculum lenti oculari immediate adplicari debere. At ſi lens ocularis fuerit conuexa et repraeſentatio inuerſa, tum oculum in certa diſtantia poſt lentem ocularem collocari debere.

Pro-

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Problema 6.

33. Ex quotcunque lentibus teleſcopium fuerit compoſitum, ſi diſtantia oculi poſt lentem ocularem prodierit poſitiua, definire campum apparentem ſeu eius ſemidiametrum Φ, quem conſpicere licebit.

Solutio.

Cum ſit $b = a$, ex formulis generalibus ſupra inuentis pro quouis lentium numero habebimus ſequentes campi apparentis determinationes:

Num. lent.	Semid. campi apparentis
I.	$\Phi =$ indetermin.
II.	$\Phi = \frac{-\pi}{m-1}$
III.	$\Phi = \frac{-\pi+\pi'}{m-1}$
IV.	$\Phi = \frac{-\pi+\pi'-\pi''}{m-1}$
V.	$\Phi = \frac{-\pi+\pi'-\pi''+\pi'''}{m-1}$

etc.

Coroll. 1.

34. Si m denotat numerum poſitiuum, eo ſemper indicatur, repraeſentationem obiectorum eſſe erectam; ſin autem teleſcopium in ſitu inuerſo repraeſentet, tum ſemper m numero negatiuo eſt exprimendum, uti iam ſupra eſt monitum.

Co-

Coroll. 2.

35. Ex his formulis etiam patet, quo maior fuerit multiplicatio m, eo minorem fore, ceteris paribus, campum adparentem et cum litterae π, π', π'', π''' etc. denotare poſſint fractiones non maiores, quam $\frac{1}{4}$ uel $\frac{1}{5}$ ſiue poſitiuas, ſiue negatiuas, euidens eſt, augendo numerum lentium campum apparentem continuo magis augeri poſſe.

Scholion.

36. Hoc modo ſemidiameter campi apparentis per fractionem quandam reperietur expreſſus, quae tanquam pars radii ſeu ſinus totius eſt ſpectanda. Quare cum arcus circuli radio aequalis contineat circiter $57^\circ\ 17'$ ſeu $3437'$, fractio pro Φ inuenta in minuta prima conuertetur, ſi ea multiplicetur per numerum 3437, hocque modo ſpatium in coelo, quod per teleſcopium quodcunque conſpicitur, facillime in gradibus et minutis definietur, vbi inſuper notari conuenit, angulum hunc Φ hic ſemper vt poſitiuum ſpectari; ſi enim prodeat negatiuus, id ſemper eſt indicio, rationem multiplicationis m quoque negatiue eſſe capiendam ſeu repraeſentationem eſſe inuerſam.

Problema 7.

37. Ex quotcunque lentibus teleſcopium fuerit compoſitum, ſi diſtantia oculi poſt lentem vltimam prodierit negatiua, definire campum apparentem ſeu eius ſemidiametrum Φ, quem conſpicere licet.

So-

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Solutio.

Si prodeat diſtantia O negatiua, ideoque oculus in hoc loco collocari nequeat, iam ſupra vidimus, tum oculum lenti vltimae immediate adplicari debere, quaſi eſſet $O = 0$; hocque caſu campum apparentem non amplius per aperturas lentium definiri, ſed potiſſimum ab apertura pupillae pendere, cuius ſemidiametrum littera ω deſignauimus, quae ob inſignem oculi variationem a parte vigeſima digiti usque ad $\frac{1}{15}$ dig. augeri poteſt, quod euenire ſolet, ſi oculus in loco valde obſcuro verſetur. Pro hoc igitur caſu ex ſupra traditis determinatio campi apparentis ſequenti modo ſe habebit:

Pro caſu duarum lentium ob $\mathfrak{B} = 1$ et $b = q$, erit primo $\pi q = \omega$; deinde $\Phi = \frac{\pi\omega}{\pi p + \omega}$, quae expreſſio ob $\pi = \frac{\omega}{q}$ abit in hanc, $\Phi = \frac{\omega}{p+q} = \frac{-\omega}{(m-1)q}$; $= \frac{-\pi}{m-1}$ ob $p = -mq$, quae expreſſio, quia hoc caſu q negatiuum valorem obtinet, per ſe fit poſitiua.

Pro caſu trium lentium primo ob $\mathfrak{C} = 1$ et $c = r$ erit $\pi' r = \omega$; tum vero fit $\frac{\mathfrak{B}p\Phi\pi'}{\pi' - \pi + \Phi} = \omega$ ſeu $\frac{mr\Phi\pi'}{\pi' - \pi + \Phi} = \omega$; vnde inuenitur $\Phi = \frac{(\pi' - \pi)\omega}{mr\pi' - \omega} = \frac{(\pi' - \pi)\omega}{(m-1)\pi' r} = \frac{\pi' - \pi}{m-1}$.

Pro caſu quatuor lentium ob $\mathfrak{D} = 1$ et $d = s$ erit primo $\pi'' s = \omega$, tum vero $\frac{\mathfrak{BC}p\pi''\Phi}{\pi'' - \pi' + \pi - \Phi} = \omega$ $= \frac{-ms\pi''\Phi}{\pi'' - \pi' + \pi - \Phi}$ vnde inuenitur $\Phi = \frac{(\pi'' - \pi' + \pi)\omega}{\omega - ms\pi''} = \frac{-(\pi'' - \pi' + \pi)\omega}{(m-1)\pi'' s}$ $= -\frac{(\pi'' - \pi' + \pi)}{m-1}$.

Quas determinationes in ſequenti tabula repraeſentemus:

Pro num. lent. erit		et pro campo apparente.
II.	$\pi = \frac{\omega}{q}$	$\Phi = \frac{-\omega}{(m-1)q} = \frac{-\pi}{m-1}$.
III.	$\pi' = \frac{\omega}{r}$	$\Phi = \frac{-(\pi-\pi')\omega}{(m-1)\pi' r} = \frac{-\pi+\pi'}{m-1}$.
IV.	$\pi'' = \frac{\omega}{s}$	$\Phi = \frac{-(\pi-\pi'+\pi'')\omega}{(m-1)\pi'' s} = \frac{-\pi+\pi'-\pi''}{m-1}$.
V.	$\pi''' = \frac{\omega}{t}$	$\Phi = \frac{-(\pi-\pi'+\pi''-\pi''')\omega}{(m-1)\pi''' t} = \frac{-\pi+\pi'-\pi''+\pi'''}{m-1}$.

etc.

Coroll. 1.

38. Hinc patet, formulas pro ſemidiametro campi apparentis Φ non diſcrepare *a* caſu praecedente; verum autem diſcrimen in hoc conſiſtit, quod caſu praecedente vltima litterarum π, π', π'' etc. ab arbitrio noſtro pendebat, dummodo intra limitem praeſcriptum $\frac{1}{4}$ vel $\frac{1}{3}$ contineretur, hic autem ea a conſtitutione pupillae determinari debeat.

Coroll. 2.

39. Eatenus ergo hoc caſu campus apparens minor fit, quam caſu praecedente, quatenus litterarum π, π', π'' etc. vltimae minor valor tribui debet, id quod fit, ſi fractiones $\frac{\omega}{q}$, $\frac{\omega}{r}$, $\frac{\omega}{s}$ etc. minores fuerint, quam limes ille $\frac{1}{4}$ uel $\frac{1}{3}$. Sin autem huic limiti prodierint

dierint aequales, vtroque caſu idem habebitur campus apparens.

Coroll. 3.

40. Hinc autem concludere non licet, ſi iſtae fractiones $\frac{\omega}{q}$, $\frac{\omega}{r}$, $\frac{\omega}{s}$ etc. maiores fiant limite praeſcripto, tum hoc poſteriori caſu campum adeo maiorem viſum iri, propterea quod ipſa lentis poſtremae natura non permittit maiorem valorem litterae reſpondentis π. Atque ob hanc cauſſam nequidem conuenit tam exiguas lentes oculares admittere, vt valor vltimae litterae π limitem $\frac{1}{4}$ uel $\frac{1}{3}$ ſuperans prodeat, quia tum ipſa huius lentis apertura minor capi deberet, quam pupilla.

Scholion.

41. Nihil autem obſtat, quominus lenti oculari apertura maior tribuatur, quam pupillae, quandoquidem inde nullum aliud incommodum eſſet metuendum, niſi quod non omnes radii per hanc lentem transmiſſi in oculum ingrederentur; quod autem tantum abeſt, vt ſit incommodum, vt potius inſigne lucrum inde obtineri poſſit; tum enim pupilla ſucceſſiue per totam lentis aperturam vagari poterit, quo id commodi conſequemur, vt ſucceſſiue alias atque alias obiecti partes conſpiciamus. Id quod in teleſcopiis ad praecedentem caſum pertinentibus locum habere nequit. Determinatio igitur vltimae litterarum π, π', π'' etc. in proble-

blemate exhibita ei tantum fini inſeruit, vt inde magnitudo campi vno obtutu viſi rite definiatur, cum adeo inſigne lucrum exſpectari queat, ſi lenti oculari multo maior apertura tribui queat; ex quo iam ratio multo clarius perſpicitur, cur lentes oculares nimis paruas euitari conueniat.

Problema 8.

42. Si teleſcopium ex quotcunque lentibus fuerit compoſitum atque adeo ſingulae lentes ex diuerſis vitri ſpeciebus ſint formatae, definire ſemidiametrum confuſionis, qua repraeſentatio obiectorum erit inquinata.

Solutio.

Iam in limine huius capitis cuilibet lenti peculiarem refractionis rationem tribuimus, huncque in finem litteras n, n', n'', n''' etc. in calculum introduximus. Quare tantum opus eſt, vt formulas in additamento poſtremo I^{mae} partis inuentas ad caſum teleſcopiorum, quo fit $a = \infty$; $b = a$; hincque $A = 0$ et $A\,a = \alpha = p$, transferamus; ad quod efficiendum ex denominatoribus ſingulorum membrorum factor A^3 cum factore communi coniungatur, vt fiat in eius denominatore $A^3 . a\,a\,b = \alpha^3 = p^3$. Quo facto pro quolibet lentium numero ſemidiameter confuſionis ſequenti formula exprimetur:

$$\frac{m x^3}{4 p^3}\left\{\begin{array}{l}\mu\ \lambda + \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)\Phi}{B^3(\mathfrak{B}\pi-\Phi)} \\ \quad + \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)\Phi}{B^3C^3(\mathfrak{C}\pi'-\pi+\Phi)} \\ \quad + \frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)\Phi}{B^3C^3D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} \end{array}\right.$$

etc.

quae, ſi ſingula membra in duas partes diſcerpantur, commodius exprimi poterit ob valores $\frac{B}{B+1} = \mathfrak{B}$; $\frac{C}{C+1} = \mathfrak{C}$ etc. Erit ſcilicet haec expreſſio:

$$\frac{m x^3}{4 p^3}\left\{\begin{array}{l}\mu\ \lambda + \frac{\mu'\Phi}{\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right) \\ \quad + \frac{\mu''\Phi}{B^3\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right) \\ \quad + \frac{\mu'''\Phi}{B^3C^3\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right) \end{array}\right.$$

etc.

quae porro, formulis §. 23 in ſubſidium vocatis ad hanc formam redigitur:

$$\frac{m x^3}{4 p^3}\left\{\begin{array}{l}\mu\ \lambda + \frac{\mu' q}{\mathfrak{B}^2 \cdot p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right) \\ \quad + \frac{\mu'' \cdot r}{B^4\mathfrak{C}^2 \cdot p}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu''}{C}\right) \\ \quad + \frac{\mu''' \cdot s}{B^4C^4\mathfrak{D}^2 \cdot p}\left(\frac{\lambda'''}{\mathfrak{D}^2}+\frac{\nu'''}{D}\right) \end{array}\right.$$

etc.

atque hinc pro quouis lentium numero ſemidiameter confuſionis ſequenti modo exprimetur:

Pro duabus lentibus ob $B = \infty$; $b = q$; $\mathfrak{B} = 1$. erit ſemidiameter confuſionis $= \frac{mx^3}{4p^3} (\mu\lambda + \frac{\mu'\lambda' . q}{p})$ quae forma ob $p = -mq$ reducitur ad hanc:

$$\frac{mx^3}{4p^3} (\mu\lambda - \frac{\mu'\lambda'}{m})$$

Pro tribus lentibus ob $C = \infty$ et $\mathfrak{C} = 1$ et $B . p = mr$ erit ſemidiameter confuſionis $=$

$$\frac{mx^3}{4p^3} \begin{cases} \mu\lambda + \frac{\mu' q}{\mathfrak{B} . p} (\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}) \\ \qquad + \frac{\mu'' . \lambda''}{B^3 . m .} \end{cases}$$

Pro quatuor lentibus ob $D = \infty$ et $\mathfrak{D} = 1$ et $B\, C\, p = -ms$ erit ſemidiameter confuſionis $=$

$$\frac{mx^3}{4p^3} \begin{cases} \mu\lambda + \frac{\mu' q}{\mathfrak{B} . p} [\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}] \\ \qquad + \frac{\mu'' . r}{B^4 . \mathfrak{C}^2 . p} [\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}] \\ \qquad - \frac{\mu''' \lambda''' .}{B^3 C^3 . m} \end{cases}$$

Pro quinque lentibus ob $E = \infty$ et $\mathfrak{E} = 1$ et $B\, C\, D\, p = mt$ erit ſemidiameter confuſionis $=$

$$\frac{mx^3}{4p^3} \begin{cases} \mu\lambda + \frac{\mu' q}{\mathfrak{B}^2 p} [\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}] \\ \qquad + \frac{\mu'' . r}{B^4 \mathfrak{C} . p} [\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}] \\ \qquad + \frac{\mu''' s}{B^4 C\, \mathfrak{D}^2 . p} [\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu'''}{D}] \\ \qquad + \frac{\mu'''' . \lambda''''}{B^3 C^3 D^3 . m} \end{cases}$$

etc.

Co-

Coroll. 1.

43. His igitur formulis ſemidiameter confuſionis per numerum ſeu fractionem quandam numericam expreſſa reperitur, quae fractio in gradus, minuta et ſecunda conuerſa indicabit, ſub quanto angulo ſingula obiectorum puncta per teleſcopium conſpiciantur, quippe in quo effectus confuſionis exiſtit.

Coroll. 2.

44. Ne igitur haec confuſio fiat intolerabilis, neceſſe eſt, vt ſemidiameter confuſionis infra certum limitem ſubſiſtat; pro quo limite ſupra hanc formulam conſtituimus $\frac{1}{4k^3}$ exiſtente $k = 40$ uel $k = 30$ circiter.

Coroll. 3.

45. Quodſi ergo in genere numeros in clauſulis contentos ponamus $= \mathrm{N}$, efficiendum eſt, vt $\frac{m x^3}{4 p^3}. \mathrm{N}$ non excedat limitem $\frac{1}{4k^3}$; ex quo ſtatui debebit $\frac{m x.^3}{p^3}. \mathrm{N} = \frac{1}{k^3}$, vnde quantitas p ſeu diſtantia focalis lentis obiectiuae determinatur fiet ſcilicet $p = k\ x. \sqrt[3]{m.N}$.

Coroll. 4.

46. Si porro gradus claritatis littera y indicetur, vt ſupra fecimus, vbi vidimus, capi debere $x = m\,y$ et vulgo ſtatui $y = \frac{1}{30}$ dig., vnde ſatis notabilis

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tabilis gradus claritatis oritur; aequatio modo inuenta erit $p = m\,k\,y\,\sqrt[3]{(m.\ N)}$; vnde patet, caeteris paribus, diſtantiam focalem lentis obiectiuae p ſequi rationem ſesquitriplicatam multiplicationis m, vbi notandum quia $y = \frac{1}{30}$ dig. et $k = 40$. fore propemodum $k\,y = \frac{4}{3}$ dig. ſeu quaſi 1 dig. plus uel minus ſecundum circumſtantias.

Scholion 1.

47. Ne quem offendat, quod ex hac aequatione valorem ipſius p defininimus, cum tamen haec quantitas iam inſit in numero N; notandum eſt, hic non tam ipſam quantitatem p, quam eius rationem ad reliquas diſtantias focales q, r, s etc. in numerum N ingredi quae rationes cum aliunde vt iam cognitae ſpectari poſſint, noſtra aequatio vtique eſt idonea, ex qua valor abſolutus ipſius p determinetur, id quod fit ex quantitate x, quae in digitis expreſſa habetur, cum ſit $x = m\,y$ et y in partibus digiti detur, ſeu capiatur $y = \frac{1}{30}$ dig. ſiue maior ſiue minor, prout maior uel minor claritatis gradus deſideratur.

Scholion 2.

48. Cum maxime ſit optandum, vt haec confuſio penitus ad nihilum redigatur fiatque $N = 0$, ſi hoc ſucceſſerit, oſtendendum adhuc eſt, quomodo hinc diſtantia focalis lentis obiectiuae p definiri debeat, ſiquidem pro caſu $N = 0$ noſtra aequatio daret $p = 0$;

quod

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quod cum fieri nequeat, ad eius aperturam ſeu quantitatem x eſt reſpiciendum, quae quia ex gradu claritatis y cum multiplicatione m coniuncto eſt data, huic lenti neceſſario tantam diſtantiam focalem p tribui oportet, vt lens tantae aperturae fiat capax: ad minimum ſcilicet debet eſſe $p > 5\,x$ atque interdum adhuc maius, prout lentis facies magis prodeunt incuruatae. In genere autem obſeruandum eſt, nihil impedire, quo minus maior ſtatuatur quantitas p dummodo non fiat minor.

Problema 9.

49. Si teleſcopium quotcunque lentibus conſtet oculique diſtantia poſt vltimam lentem inuenta fuerit poſitiua, definire lentium diſpoſitionem, vt obiecta ſine margine colorato conſpiciantur.

Solutio.

Quoniam huic conditioni iam ſupra generatim ſatisfecimus, aequatio ibi inuenta ad caſum praeſentem teleſcopiorum accommodemus ac videbimus, ſcopum obtineri, ſi huic aequationi ſatisfieri poſſit

$$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{p\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{Bq\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{BCp\Phi}$$

etc.

quam ad ſingulos lentium numeros applicemus.

Pro duabus lentibus ob $b = q$ erit

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi q}{\Phi p} = -\frac{dn'}{n'-1} \cdot \frac{\pi}{m\Phi}.$$

Pro tribus lentibus ob $c = r$ erit

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi b}{\Phi p} + \frac{dn''}{n''-1} \cdot \frac{\pi' r}{B\Phi p}$$

ſiue

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi b}{\Phi p} + \frac{dn''}{n''-1} \cdot \frac{\pi'}{m\Phi}$$

Pro quatuor lentibus ob $d = s$ et $BCp = -ms$ erit

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi b}{\Phi p} + \frac{dn''}{n''-1} \cdot \frac{\pi' c}{B\Phi p} - \frac{dn'''}{n'''-1} \cdot \frac{\pi''}{m\Phi}.$$

Pro quinque lentibus ob $e = t$ et $BCDp = mt$ erit

$$0 = \frac{dn'}{n'-1} \cdot \frac{\pi b}{\Phi p} + \frac{dn''}{n''-1} \cdot \frac{\pi' c}{B\Phi p}$$
$$+ \frac{dn'''}{n'''-1} \cdot \frac{\pi'' d}{BC\Phi p} + \frac{dn''''}{n''''-1} \cdot \frac{\pi'''}{m\Phi}.$$

Coroll. 1.

50. Cum pro caſu duarum lentium ſit $\frac{-\pi}{\Phi} = m - 1$, habebitur hacc aequatio $0 = \frac{dn'}{n'-1} \cdot \frac{m-1}{m}$ quod cum fieri non poſſit, manifeſtum eſt teleſcopia ex duabus lentibus compoſita a vitio marginis colorati liberari non poſſe.

Co-

Coroll. 2.

51. Si omnes lentes ex eadem vitri ſpecie ſint factae, aequationes noſtras per factores differentiales diuidere licebit indeque eaedem formulae reperiuntur, quae pro hoc caſu ſupra ſunt datae.

Problema 10.

52. Si teleſcopium quotcunque conſtet lentibus oculique diſtantia poſt vltimam lentem inuenta fuerit negatiua, definire lentium diſpoſitionem, vt obiecta ſine margine colorato conſpiciantur.

Solutio.

Ex ſuperioribus pro quouis lentium numero ſequentibus aequationibus erit ſatisfaciendum.

Pro duabus lentibus ſi ſuperior aequatio per A multiplicetur, habebitur

$$0 = \frac{dn}{n-1} \cdot B \pi p,$$ quod ob $B = \infty$ fieri nequit.

Pro tribus lentibus multiplicando per A habebitur ob $C = \infty$

$$0 = \frac{dn}{n-1} \cdot B \pi' p + \frac{dn'}{n'-1} \cdot b ((B+1) \pi' - \pi)$$

Pro quatuor lentibus ob $D = \infty$ habebitur

$$0 = \frac{dn}{n-1} \cdot BC \pi'' p + \frac{dn'}{n'-1} \cdot b ([B+1] C \pi'' - \pi)$$
$$+ \frac{dn''}{n''-1} \cdot c \left(\frac{[C+1] \pi'' - \pi'}{B}\right)$$

Pro

Pro quinque lentibus ob $E = \infty$

$$0 = \frac{dn}{n-1} . BCD\pi''' p + \frac{dn'}{n'-1} b ([B+1]CD\pi''' - \pi)$$
$$+ \frac{dn''}{n''-1} . c \left(\frac{[C+1]D\pi''' - \pi'}{B}\right)$$
$$+ \frac{dn'''}{n'''-1} . d \left(\frac{[D+1]\pi''' - \pi''}{BC}\right)$$

Problema II.

53. Si teleſcopium ex quotcunque lentibus ſit compoſitum, eam definire lentium diſpoſitionem, vt omnis confuſio a diuerſa radiorum refrangibilitate oriunda penitus tollatur.

Solutio.

Ex ſupra traditis pro omni lentium numero aequationem exhibere poſſumus, qua ſcopo propoſito ſatisfiet, multiplicando enim per A^2 habebitur

$$0 = \frac{dn}{n-1} . \alpha + \frac{dn'}{n'-1} . \frac{b}{\mathfrak{B}} + \frac{dn''}{n''-1} . \frac{c}{\mathfrak{C} . B^2}$$
$$+ \frac{dn'''}{n'''-1} . \frac{d}{\mathfrak{D} . B^2 C^2} + \frac{dn''''}{n''''-1} . \frac{e}{\mathfrak{E} . B^2 C^2 D^2}$$

etc.

quae ob $\alpha = p$; $b = \frac{q}{\mathfrak{B}}$; $c = \frac{r}{\mathfrak{C}}$; $d = \frac{s}{\mathfrak{D}}$ etc. abit in hanc

$$0 = \frac{dn}{n-1} . p + \frac{dn'}{n'-1} . \frac{q}{\mathfrak{B}^2} + \frac{dn''}{n''-1} . \frac{r}{B^2 \mathfrak{C}^2}$$
$$+ \frac{dn'''}{n'''-1} . \frac{s}{B^2 C^2 . \mathfrak{D}^2} + \frac{dn''''}{n''''-1} . \frac{t}{B^2 C^2 D^2 . \mathfrak{E}^2}$$

etc.

Hinc

Hinc ergo pro ſingulis lentium numeris nanciſcimur ſequentes aequationes adimplendas.

Pro duabus lentibus ob $\mathfrak{B} = 1$

$$0 = \frac{dn}{n-1} \cdot p + \frac{dn'}{n'-1} \cdot q$$

Pro tribus lentibus ob $\mathfrak{C} = 1$

$$0 = \frac{dn}{n-1} \cdot p + \frac{dn'}{n'-1} \cdot \frac{q}{\mathfrak{B}^2} + \frac{dn''}{n''-1} \cdot \frac{r}{B^2}$$

Pro quatuor lentibus ob $\mathfrak{D} = 1$

$$0 = \frac{dn}{n-1} \cdot p + \frac{dn'}{n'-1} \cdot \frac{q}{\mathfrak{B}^2} + \frac{dn''}{n''-1} \cdot \frac{r}{B \cdot \mathfrak{C}^2}$$
$$+ \frac{dn'''}{n'''-1} \cdot \frac{s}{B \cdot C^2}$$

Pro quinque lentibus ob $\mathfrak{E} = 1$

$$0 = \frac{dn}{n-1} \cdot p + \frac{dn'}{n'-1} \cdot \frac{q}{\mathfrak{B}^2} + \frac{dn''}{n''-1} \cdot \frac{r}{B \cdot \mathfrak{C}^2}$$
$$+ \frac{dn'''}{n'''-1} \cdot \frac{s}{B^2 C \cdot \mathfrak{D}^2} + \frac{dn''''}{n''''-1} \cdot \frac{t}{B \cdot C^2 D^2}.$$

Coroll. I.

54. Cum ſit $\mathfrak{B} = \frac{q}{b}$; $\mathfrak{C} = \frac{r}{c}$; $\mathfrak{D} = \frac{s}{d}$ etc. tum vero $B = \frac{\beta}{b}$; $C = \frac{\gamma}{c}$; $D = \frac{\delta}{d}$ etc. aequatio generalis per pp ſeu $\alpha\alpha$ diuiſa abibit in ſequentem formam:

$$0 = \frac{dn}{n-1} \cdot \frac{1}{p} + \frac{dn'}{n'-1} \cdot \frac{bb}{\alpha\alpha} \cdot \frac{1}{q} + \frac{dn''}{n''-1} \cdot \frac{b^2.cc}{\alpha\alpha.\beta\beta} \cdot \frac{1}{r}$$
$$+ \frac{dn'''}{n'''-1} \cdot \frac{bb.cc.dd}{\alpha\alpha.\beta\beta.\gamma\gamma} \cdot \frac{1}{s} + \frac{dn''''}{n''''-1} \cdot \frac{b^2.c^2.d^2.e^2}{\alpha^2.\beta^2.\gamma^2.\delta^2} \cdot \frac{1}{t}$$

etc.

quae aequatio commodior videtur praecedente.

E 3 Co-

Coroll. 2.

55. Quod ad numerum horum terminorum attinet, perſpicuum eſt, eum eſſe numero lentium aequalem neque igitur opus eſt, vt hanc formulam ſeorſim ad quemlibet lentium numerum accommodemus.

Coroll. 3.

56. Si omnes lentes ex eadem vitri ſpecie eſſent confectae, tum haec aequatio per coëfficientes differentiales diuidi poſſet prodiretque

$$0 = \frac{1}{p} + \frac{b^2}{\alpha^2} \cdot \frac{1}{q} + \frac{b^2 c^2}{\alpha^2 \beta^2} \cdot \frac{1}{r} + \frac{b^2 c^2 d^2}{\alpha^2 \beta^2 \gamma^2} \cdot \frac{1}{s}$$

etc.

cui autem nullo modo ſatisfieri poteſt.

Scholion 1.

57. Quod haec aequatio, quando omnes lentes ex eadem vitri ſpecie ſunt paratae, nullo modo ſubſiſtere queat; ſequenti modo oſtendi poteſt. Cum ſit $\frac{1}{p} = \frac{1}{\alpha}$; $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$; $\frac{1}{r} = \frac{1}{c} + \frac{1}{\gamma}$; $\frac{1}{s} = \frac{1}{d} + \frac{1}{\delta}$; ſi hi valores ſubſtituantur ſingulaque membra poſt primum in duas partes diſcerpantur, aequatio induet hanc formam:

$$0 = \frac{1}{\alpha} + \frac{b}{\alpha^2} + \frac{b^2 c}{\alpha^2 \beta^2} + \frac{b^2 c^2 d}{\alpha^2 \beta^2 \gamma^2}$$
$$+ \frac{b^2}{\alpha^2 \beta} + \frac{b^2 c^2}{\alpha^2 \beta^2 \gamma} + \frac{b^2 c^2 d^2}{\alpha^2 \beta^2 \gamma^2 \delta}$$

etc.

hic

hic iam iungantur iterum bini termini et aequatio prodiens ita erit comparata:

$$0 = \frac{\alpha + b}{\alpha^2} + \frac{b^2(\beta + c)}{\alpha^2 \beta^2} + \frac{b^2 c^2 (\gamma + d)}{\alpha^2 \beta^2 \gamma^2} + \frac{b^2 c^2 d^2}{\alpha^2 \beta^2 \gamma^2 . \delta}$$

quia nunc $\alpha + b$; $\beta + c$; $\gamma + d$ vt lentium diſtantiae neceſſario ſunt poſitiuae, omnes plane termini usque ad vltimum neceſſario poſitiui ſunt; ultimus autem terminus $\frac{b^2 c^2 d^2}{\alpha^2 \beta^2 \gamma^2 \delta}$ ob $\delta = \infty$ per ſe euaneſcit, ſcilicet pro caſu quatuor lentium, quem hic conſiderauimus.

Scholion 2.

58. His igitur praeparatis iam poſſemus ad diuerſa genera teleſcopiorum conſtituenda progredi, ſingularumque ſpecierum conſtructionem docere. Sed quoniam ea, quae ſupra de lentibus multiplicatis ſunt tradita, maximum vſum in perficiendis teleſcopiis habere poſſunt, dum ſcilicet loco lentium ſimplicium multiplicatae adhibentur, quae multo minorem confuſionem pariant, conſultum videtur, ea hic repetere et ad teleſcopia accommodare. Inprimis autem ex formula pro ſemidiametro confuſionis inuenta patet, lentem obiectiuam in ea praecipuas partes tenere; ſiquidem pro ea fuerit $\lambda = 1$, quare ſi eius loco lens multiplicata ſubſtituatur, pro qua valor numeri λ vehementer ſit minor vel adeo euaneſcat; ſtatim maximum inde commodum adipiſcimur, dum tota confuſio ad valde exiguum vel fortaſſe ad nihilum redigitur.

Quo-

Quocirca in capite ſequente praecipuas lentes compoſitas, quas in locum lentis obiectiuae ſubſtituere licebit, enumerabimus, et pro ſingulis valorem ipſius λ indicabimus, vt deinceps pro circumſtantiis hinc depromi poſſint.

CAPVT

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CAPVT II.

DE
LENTIBUS OBIECTIUIS
COMPOSITIS ATQUE PERFECTIS.

Problema I.

59.

Conſtructionem lentis obiectiuae ſimplicis, quae minimam confuſionem pariat, deſcribere.

Solutio.

Cum lens ſimplex minorem confuſionem parere nequeat, quam ſi fuerit $\lambda = 1$. ſtatuamus ſtatim $\lambda = 1$ et cum ſit $a = \infty$, ex iis, quae ſupra ſunt tradita, facile intelligitur, hanc lentem ita conſtrui debere, vt ſit

$$\text{radius faciei} \begin{cases} \text{anterioris} = \frac{a}{\sigma} \\ \text{poſterioris} = \frac{\alpha}{\varrho} \end{cases}$$

vbi numeri σ et ϱ ex ratione refractionis ſunt ſumendi ſecundum tabulam §. 15. exhibitam. Pro variis igitur vitri ſpeciebus haec conſtructio ita ſe habebit; ſcilicet cum ſit $\alpha = p$, erit radius faciei

pro *n*.	anterioris F.	poſterioris G.
1. 50.	0. 58333. *p*.	3. 4989. *p*.
1. 51.	0. 58976. *p*.	3. 7693. *p*.
1. 52.	0. 59609. *p*.	4. 0717. *p*.
1. 53.	0. 60234. *p*.	4. 4111. *p*.
1. 54.	0. 60849. *p*.	4. 8008. *p*.
1. 55.	0. 61448. *p*.	5. 2439. *p*.
1. 56.	0. 62039. *p*.	5. 7571. *p*.
1. 57.	0. 62617. *p*.	6. 3573. *p*.
1. 58.	0. 63183. *p*.	7. 0722. *p*.
1. 59.	0. 63739. *p*.	7. 9428. *p*.
1. 60.	0. 64288. *p*.	9. 0009. *p*.

Coroll. 1.

60. Cum in expreſſione pro ſemidiametro confuſionis λ multiplicetur per μ, ex §. 15. intelligitur, confuſionem, ceteris paribus, eo fieri minorem, quo maior fuerit ratio refractionis *n*, ita, vt hoc reſpectu ea vitri ſpecies, quae maximam refractionem habet, reliquis ſit anteferenda.

Coroll. 2.

61. Vulgo lentes obiectiuae vtrinque aequaliter conuexae confici ſolent, pro quo caſu operae pretium erit, inueſtigare, quanto numerus λ vnitatem ſit ſuperaturus; quia autem eſt $F = \frac{\alpha}{\sigma - \tau \cdot \sqrt{\lambda - 1}}$ et $G = \frac{\alpha}{\rho + \tau \cdot \sqrt{\lambda - 1}}$ poſito $F = G$ erit $\sqrt{\lambda - 1} = \frac{\sigma - \rho}{2\tau} = \frac{2(nn-1)}{n\sqrt{(4n-1)}}$; tum vero habe-

habebitur $\frac{1}{F} + \frac{1}{G} = \frac{\sigma+\rho}{\alpha} = \frac{1}{(n-1)\alpha} = \frac{2}{p}$, ſeu $F = G = 2[n-1]\alpha = 2[n-1]p$. Quod autem ad λ attinet, pro caſu $n = 1,55$ erit $\sqrt{\lambda - 1} =$ [illegible] $= 0.79367$, hincque $\lambda = 1.62991$; vnde patet, quanto maiorem confuſionem talis lens obiectiua pariat.

Coroll. 3.

62. Si lentem obiectiuam conuexo-planum facere velimus, vt eius facies poſterior fiat plana ſeu $G = \infty$, erit $\sqrt{\lambda - 1} = \frac{-\rho}{\tau}$ et $F = \frac{\alpha}{\sigma+\rho} = [n-1]\alpha$; et pro caſu, quo $n = 1.55$, $\lambda = 1.0443$, vnde confuſio non niſi perparum ſuperat illam, quae oritur ex caſu $\lambda = 1$.

Coroll. 4.

63. Sin autem eadem lens plano-conuexa inuertatur, vt ſit $F = \infty$, ideoque $\sqrt{[\lambda - 1]} = \frac{\sigma}{\tau}$ et $G = \frac{\alpha}{\rho+\sigma} = [n-1]\alpha$, erit pro caſu $n = 1,55$, $\lambda = 4,2329$, ita, vt talis lens plus quam quadruplo maiorem pariat confuſionem, quam noſtra lens commendata.

Coroll. 5.

64. Patet ergo, ſi lens adhibeatur plano conuexa, quantum interſit, vtrum facies eius conuexa an plana verſus obiectum dirigatur, cum poſteriori caſu confuſio circiter quater maior fiat, quam priore.

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Problema 2.

65. Conſtructionem lentis obiectiuae duplicatae, ſiquidem ambae lentes ex eadem vitri ſpecie ſint confectae, deſcribere, quae minimam confuſionem pariat.

Solutio.

Ex §. 113 libri ſup., cum hic ſit $a = \infty$ et $\beta = p$, colligimus ſequentem conſtructionem:

Pro lente priori

Radius faciei $\begin{cases} \text{anterioris} = \frac{2p}{\sigma} \\ \text{poſterioris} = \frac{2p}{\varrho} \end{cases}$

Pro lente poſteriori

Radius faciei $\begin{cases} \text{anterioris} = \frac{2p}{2\sigma - \varrho} \\ \text{poſterioris} = \frac{2p}{2\varrho - \sigma} \end{cases}$

ac ſi haec lens duplicata loco lentis obiectiuae adhibeatur, pro ea erit $\lambda = \frac{1-\nu}{4}$ quos valores pro praecipuis tantum vitri ſpeciebus determinemus:

Contemplemur igitur primo vitrum coronarium, pro quo $n = 1,53$ et cum ſit $\varrho = 0,2266$, $\sigma = 1,6602$, erit $2\sigma - \varrho = 3.0938$ et $2\varrho - \sigma = -1.2070$; tum vero ob $\nu = 0.2194$ prodit $\lambda = 0.1951$, atque habetur ſequens conſtructio

Pro

Pro vitro coronario $n = 1, 53$.

Pro lente priori

radius faciei $\begin{cases} \text{anterioris} = 1, 2047.\ p. \\ \text{poſterioris} = 8, 8262.\ p. \end{cases}$

Pro lente poſteriore

radius faciei $\begin{cases} \text{anterioris} = 0, 6464.\ p. \\ \text{poſterioris} = -1, 6570.\ p. \end{cases}$

et $\lambda = 0.\ 1951$.

Ponamus nunc $n = 1, 55$ pro vitro ordinario, eritque $\varrho = 0.\ 1907$. $\sigma = 1.6274$ et $2\sigma - \varrho = 3.0641$. $2\varrho - \sigma = -1.\ 2460$. $\nu = 0.\ 2326$; hinc $\lambda = 0.1918$, vnde elicitur ſequens conſtructio

Pro vitro communi $n = 1, 55$.

Pro lente priore

radius faciei $\begin{cases} \text{anterioris} = 1, 2289.\ p. \\ \text{poſterioris} = 10, 4876.\ p. \end{cases}$

Pro lente poſteriore

radius faciei $\begin{cases} \text{anterioris} = 0, 6527.\ p. \\ \text{poſterioris} = -1, 6053.\ p. \end{cases}$

et $\lambda = 0.\ 1918$.

Ponamus porro $n = 1, 58$ pro vitro chryſtallino, eritque $\varrho = 0.\ 1413$, $\sigma = 1.\ 5827$, $2\sigma - \varrho = 3.\ 0241$;

$2\rho - \sigma = -1.3001$, $\nu = 0.2529$; hincque $\lambda = 0.1868$. vnde habetur ſequens conſtructio:

Pro vitro chryſtallino $n = 1,58$

Pro lente priori

radius faciei $\begin{cases} \text{anter.} = +1.26366.\ \text{p.} \\ \text{poſter.} = +14.15421.\ \text{p.} \end{cases}$

Pro lente poſteriori

radius faciei $\begin{cases} \text{anter.} = +0.66135.\ \text{p.} \\ \text{poſter.} = -1.53834.\ \text{p.} \end{cases}$

et $\lambda = 0.1868$.

Problema 3.

66. Conſtructionem lentis triplicatae, ſiquidem omnes tres lentes ex eadem vitri ſpecie ſint confectae, deſcribere, quae minimam confuſionem pariat.

Solutio.

Ex §. 135. libri ſup., cum hic ſit $a = \infty$ et $\gamma = p$ colligimus hanc conſtructionem:

Pro lente

prima, radius faciei $\begin{cases} \text{anterioris} = \frac{3p}{\sigma} \\ \text{poſterioris} = \frac{3p}{\rho} \end{cases}$

ſecunda, radius faciei $\begin{cases} \text{anterioris} = \frac{3p}{2\sigma-\rho} \\ \text{poſterioris} = \frac{3p}{2\rho-\sigma} \end{cases}$

tertia, radius faciei $\begin{cases} \text{anterioris} = \frac{3p}{3\sigma-\rho} \\ \text{poſterioris} = \frac{3p}{3\rho-2\sigma} \end{cases}$

pro qua lente triplicata valor ipſius λ eſt $\lambda = \frac{7-8\nu}{4\cdot 9}$

Quare

Quare pro praecipuis vitri ſpeciebus valores horum radiorum euoluamus.

Cum igitur ſit pro vitro coronario $n = 1, 53$, $\varrho = 0, 2266$, $\sigma = 1, 6602$ $2\sigma - \varrho = 3. 0938$; $2\varrho - \sigma = -1. 2070$ $3\sigma - 2\varrho = 4. 5274$; $3\varrho - 2\sigma = -2. 6406$. atque ob $\nu = 0. 2194$ reperitur $\lambda = 0. 0461$. atque ſequens habetur conſtructio:

Pro vitro coronario, $n = 1, 53$.

Pro lente prima

radius faciei $\begin{cases} \text{anterioris} = 1. 8070. \text{ p.} \\ \text{poſterioris} = 13. 2393. \text{ p.} \end{cases}$

Pro lente ſecunda

radius faciei $\begin{cases} \text{anterioris} = 0. 9696. \text{ p.} \\ \text{poſterioris} = -2. 4855. \text{ p.} \end{cases}$

Pro lente tertia

radius faciei $\begin{cases} \text{anterioris} = 0. 6626. \text{ p.} \\ \text{poſterioris} = -1. 1361. \text{ p.} \end{cases}$

tum vero pro hac lente triplicata erit $\lambda = 0. 0461$.

Pro vitro communi, $n = 1, 55$.
cum ſit $\varrho = 0. 1907$; $\sigma = 1. 6274$ $2\sigma - \varrho = 3. 0641$; $2\varrho - \sigma = -1, 2460$; $3\sigma - 2\varrho = 4. 5008$, $3\varrho - 2\sigma = -2. 6827$, erit:

Pro lente prima

radius faciei $\begin{cases} \text{anterioris} = +1. 8433. \text{ p.} \\ \text{poſterioris} = +15. 7315. \text{ p.} \end{cases}$

Pro

Pro lente ſecunda

radius faciei $\begin{cases} \text{anterioris} = +0.9790.\ p. \\ \text{poſterioris} = -2.4079.\ p. \end{cases}$

Pro lente tertia

radius faciei $\begin{cases} \text{anterioris} = +0.6665.\ p. \\ \text{poſterioris} = -1.1182.\ p. \end{cases}$

atque ob $\nu = 0.2326$ erit $\lambda = 0.0422$.

Pro vitro chryſtallino, $n = 1,58$.

$\varrho = 0,1413$; $\sigma = 1.5827$. $2\sigma - \varrho = 3.0241$; $2\varrho - \sigma = -1.3001$. $3\sigma - 2\varrho = 4.4655$; $3\varrho - 2\sigma = -2.7415$.

Pro lente prima

radius faciei $\begin{cases} \text{anterioris} = +1.8954.\ p. \\ \text{poſterioris} = +21.2313.\ p. \end{cases}$

Pro lente ſecunda

radius faciei $\begin{cases} \text{anterioris} = +0.9920.\ p. \\ \text{poſterioris} = -2.3075.\ p. \end{cases}$

Pro lente tertia

radius faciei $\begin{cases} \text{anterioris} = +0.6718.\ p. \\ \text{poſterioris} = -1.0942.\ p. \end{cases}$

et quia eſt $\nu = 0.2529$, erit $\lambda = 0.0362$.

Co-

Coroll. 1.

67. Si ergo huiusmodi lens ſiue duplicata ſiue triplicata loco lentis obiectiuae adhibeatur, ſummus eius vſus in hoc conſiſtit, vt ſemidiameter confuſionis ob imminutum valorem ipſius λ multo minor reddatur, hincque diſtantia focalis lentis obiectiuae haud mediocriter minor ſumi poſſit.

Coroll. 2.

68. Deinde etiam hinc patet, quo maior fuerit refractio ſeu numerus *n*, pro huiusmodi lente obiectiua, eo maius lucrum in conſtructionem teleſcopiorum redundare, quia tum non ſolum numerus λ prodit minor, ſed etiam numerus μ, per quem λ multiplicari oportet.

Scholion.

69. Huiusmodi autem lentes duplicatae et triplicatae in obiectiuae lentis locum ſubſtituendae nihil plane conferunt ad alterum confuſionis genus, quod ex diuerſa radiorum refrangibilitate naſcitur, diminuendum, ſed aequationes in capite Imo datae pro hoc genere confuſionis tollendo prorſus manent eaedem ac ſi lens obiectiua eſſet ſimplex; verum reliquae lentes duplicatae et triplicatae, quas ſupra in additamento commendauimus, primum etiam terminum in aequatione pro diſperſione ante inuenta ad nihilum redi-

 gunt,

gunt, in quo praecipua pars huius confuſionis continetur. Quocirca in hoc capite illas lentium tam duplicatarum, quam triplicatarum, ſpecies repeti conueniet.

Definitio 4.

70. Lens obiectiua perfecta eſt, quae non ſolum nullam parit confuſionem ab apertura oriundam, ſed etiam nullam plane radiorum diſperſionem gignit.

Coroll. 1.

71. Si igitur talis lens adhibeatur, numerus λ penitus euaneſcet, vnde ſemidiameter confuſionis multo fit minor, quam pro lentibus obiectiuis compoſitis hactenus explicatis.

Coroll. 2.

72. Ex ſuperioribus etiam ſatis intelligitur, ad huiusmodi lentes perfectas conſtruendas duas ad minimum diuerſas vitri ſpecies requiri et quia experimenta circa alias vitri ſpecies adhuc deſiderantur, alias ſpecies adhibere non licet, praeter vitrum coronarium et chryſtallinum, quibus Clariſſimus Dollondus eſt vſus.

Problema 4.

73. Lentem obiectiuam duplicatam, partim ex vitro coronario $n = 1, 53$, partim ex chryſtallino $n = 1, 58$ compoſitam conſtruere.

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Solutio.

In additamento ad calcem capitis III. partis praecedentis annexo duas huiusmodi lentes perfectas dedimus, quarum alterius lens prior ex vitro coronario, poſterior vero ex vitro chryſtallino erat confecta; alterius vero contra lens prior ex vitro chryſtallino, poſterior vero ex coronario; has duas lentium perfectarum ſpecies hic referamus.

I. Lens obiectiua perfecta duplicata

Pro lente priori, ex vitro coronario $n = 1,53$ parata

radius faciei $\begin{cases} \text{anterioris} = + 0.1807.\ p. \\ \text{poſterioris} = + 1.3239.\ p. \end{cases}$ Crown Glaſſ.

Pro lente poſteriori ex vitro chryſtallino $n = 1,58$ parata

radius faciei $\begin{cases} \text{anterioris} = - 0.4770.\ p. \\ \text{poſterioris} = - 0.5191.\ p. \end{cases}$ Flint Glaſſ.

quae capax eſt aperturae, cuius ſemidiameter eſt $x = 0.0452.\ p.$

II. Lens obiectiua perfecta duplicata.

Pro lente priori, ex vitro chryſtallino $n = 1,58$ parata

radius faciei $\begin{cases} \text{anterioris} = - 2.0545.\ p. \\ \text{poſterioris} = - 0.2828.\ p. \end{cases}$ Flint Glaſſ.

 Pro

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Pro lente poſteriori ex vitro coronario $n = 1,53$ parata

radius faciei $\begin{cases} \text{anterioris} = + 0.4568.\ p. \\ \text{poſterioris} = + 0.2438.\ p. \end{cases}$ Crown Glaſſ.

eritque ſemidiam. aperturae $x = 0.0609.\ p.$

vbi notandum eſt, p deſignare diſtantiam focalem ipſius lentis duplicatae.

Coroll. 1.

74. Cum igitur harum lentium poſterior maiorem admittat aperturam, quam prior, haec illi ſine dubio eſt anteferenda, quoniam, vt infra patebit, omnis teleſcopiorum perfectio eo redit, vt lens obiectiua quam maximam aperturam admittat.

Coroll. 2.

75. Obſeruandum hic eſt, vtroque caſu lentem ex vitro chryſtallino parandam eſſe debere concauam, eam vero, quae ex vitro coronario conficitur, conuexam, prouti eae reuera a Dollondo parantur.

Scholion.

76. Ceterum hic non eſt reticendum, ambas has ſpecies ſummam artificis ſollertiam requirere; ſi enim tantillum in earum conſtructione a menſuris hic praeſcriptis aberretur; fieri poteſt, vt eae minus valcant, quam

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quam ſi lentes adeo ſimplices adhiberentur. Sequentes vero lentes triplicatae multo minorem ſollertiam poſtulant, cum pro ſingulis lentibus ſimplicibus numerus λ vnitati aequetur, ideoque leues errores in conſtructione commiſſi non adeo ſint pertimeſcendi.

Problema 5.

77. Lentem obiectiuam perfectam triplicatam, partim ex vitro coronario $n = 1,53$, partim ex chryſtallino $n = 1,58$ conſtruere.

Solutio.

Pro hoc lentium perfectarum genere ſupra quatuor dedimus ſpecies, quas hic referamus:

I. Lens obiectiua perfecta triplicata, cuius lens prima et tertia ex vitro chryſtallino, media ex coronario eſt parata.

Pro lente

prima, rad. faciei $\begin{cases} \text{anter.} = +0.5039.\ \text{p.} \\ \text{poſter.} = +5.6450.\ \text{p.} \end{cases}$ Flint Glaſſ.

ſecunda, rad. faciei $\begin{cases} \text{anter.} = +0.1364.\ \text{p.} \\ \text{poſter.} = -0.9597.\ \text{p.} \end{cases}$ Crown Glaſſ.

tertia, rad. faciei $\begin{cases} \text{anter.} = +1.0699.\ \text{p.} \\ \text{poſter.} = -0.1404.\ \text{p.} \end{cases}$ Flint Glaſſ.

quae lens capax eſt aperturae, cuius ſemidiameter $x = 0.0341.$ p.

II. Lens obiectiua perfecta triplicata, cuius lens prima et tertia ex vitro chryſtallino, media ex coronario eſt parata.

Pro lente

prima, rad. faciei	anter. $= -0.1762.$ p. poſter. $= -1.9741.$ p.	Flint Glaſſ.
ſecunda, rad faciei	anter. $= +2.5349.$ p. poſter. $= +0.1696.$ p.	Crown Glaſſ.
tertia, rad. faciei	anter. $= +0.6194.$ p. poſter. $= +1.8532.$ p.	Flint Glaſſ.

quae lens capax eſt aperturae, cuius ſemidiameter $x = 0.0424.$ p.

III. Lens obiectiua perfecta triplicata, cuius lens prima et tertia ex vitro coronario, media ex chryſtallino eſt parata.

Pro lente.

prima, rad. faciei	anter. $= +0.5004.$ p. poſter. $= +3.6665.$ p.	Crown Glaſſ.
ſecunda, rad. faciei	anter. $= -0.5107.$ p. poſter. $= -0.4843.$ p.	Flint Glaſſ.
tertia, rad. faciei	anter. $= +0.5219.$ p. poſter. $= +0.4757.$ p.	Crown Glaſſ.

aperturae ſemidiametro $x = 0.1189.$ p.

IV.

IV. Lens obiectiua perfecta triplicata, cuius lens prima et tertia ex vitro coronario, media ex chryſtallino eſt parata.

Pro lente.

prima, rad. faciei { anter. $= +0.2829.$ p. } Crown
{ poſter. $= +2.0729.$ p. } Glaſſ.

ſecunda, rad. faciei { anter. $= -2.1459.$ p. } Flint
{ poſter. $= -0.2955.$ p. } Glaſſ.

tertia, rad. faciei { anter. $= +0.5938.$ p. } Crown
{ poſter. $= +2.5006.$ p. } Glaſſ.

ſemidiametro aperturae $x = 0.0707.$ p.

In his formulis littera p denotat diſtantiam focalem cuiusque lentis perfectae.

Coroll. 1.

78. Inter has quatuor lentes tertia inprimis eſt notatu digna, quod maximam aperturam admittat.

Coroll. 2.

79. Si ergo eiusmodi lens perfecta in quodam teleſcopio loco lentis obiectiuae adhibeatur, in expreſſione pro ſemidiametro confuſionis primus terminus $\mu. \lambda$ prorſus euaneſcit; tum vero etiam in aequatione vltima pro diſperſione deſtruenda terminus primus quoque ad nihilum redigitur.

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Coroll. 3.

80. Huiusmodi igitur lentes perfectae etiam ſpeculis, quibus in teleſcopiis catoptricis vtuntur, longe ſunt anteferendae, cum ſpecula tantum a diſperſione radiorum ſint immunia, neutiquam vero a priori confuſionis genere, quod ab apertura oritur.

CAPVT

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CAPVT III.

DE DISTRIBUTIONE TELESCOPIORUM IN TRIA GENERA PRAECIPUA.

Definitio I.

81.

Imago *vera* eſt, ad quam formandam radii reuera concurrunt indeque porro diffunduntur; dum contra eae imagines *fictae* vocantur, ad quas radii tantum conuergendo diriguntur neque vero ad eas actu formandas concurrunt; vel etiam, ab iis diuergendo vlterius diſcedunt neque tamen ab iis prodierunt.

Corollarium I.

82. Imago igitur vera hac gaudet proprietate, vt ſi in eius loco charta alba eſſet expanſa, ſuper ea effigies a radiis incidentibus exprimeretur, quod in imaginibus fictis vſu non venit.

Coroll. 2.

83. Imagines autem fictae duplicis ſunt generis; vel enim radii inde diuergendo vlterius progrediuntur, cum tamen inde non diſceſſerint, vel ad eas convergendo tendunt, neque tamen eo revera perveniunt, ſed ante ab alia lente aliam directionem accipiunt.

Scholion.

Fig. 15. Tom. I. 84. Ad ea, quae hactenus ſunt propoſita, figuras ita repraeſentauimus, quaſi per ſingulas lentes imagines verae formarentur, ita, vt inter binas quasque lentes ſucceſſiuas imago vera caderet, neque in his figuris vlla imago ficta eſt indicata. Imagines autem illas veras litteris $F\zeta$, $G\eta$, $H\vartheta$ etc. deſignauimus, quae omnes ita ſunt comparatae, vt, ſi ibi charta alba expanderetur, ſuper ea effigies obiecti reuera exprimeretur. Perſpicuum autem eſt, imagines veras neceſſario oriri debere, ſi omnes diſtantiae, quas ſupra poſuimus, determinatrices $aF = \alpha$, $FB = b$; $bG = \beta$; $GC = c$; $cH = \gamma$; $HD = d$ etc. fuerint poſitiuae; imagines autem tum erunt fictae, quando harum diſtantiarum quaedam fiunt negatiuae, id quod in ſequentibus theorematibus fuſius explicabimus.

Theorema I.

85. Si interualli inter binas lentes ſucceſſiuas cuiuscunque v. gr. cD binae partes $cH = \gamma$, et HD-

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$HD = d$, ita vt ſit $cD = \gamma + d$, fuerint poſitiuae: imago vera in puncto H exhibebitur, et contra.

Demonſtratio.

Radii enim per lentem R R refracti ad imaginem $H\theta$ conformandam tendunt et, quia lens ſequens S S vltra locum imaginis H eſt poſita, ab his radiis imago vera in H repraeſentabitur, ita, vt ſi per $H\theta$ charta alba eſſet expanſa, ea iſtos radios reuera exciperet ſuper eaque effigies depingeretur; quod ergo neceſſario ſemper euenire debet, quoties binae partes huius intervalli γ et d fuerint poſitiuae. Ac ſi viciſſim in H repraeſentetur imago vera, manifeſtum eſt, hoc fieri non poſſe, niſi punctum H poſt lentem C cadat, quia alioquin radii eo non porrigerentur; tum vero etiam liquet, hanc imaginem efformari non poſſe, niſi ſequens lens D poſt H cadat. Cum igitur eſſe debeat diſtantia $cH = \gamma$ poſitiua ſimulque diſtantia $cD = \gamma + d > \gamma$, euidens eſt, et diſtantiam d eſſe debere poſitiuam.

Corollarium.

86. Quoniam hactenus ſingula interualla inter binas lentes ſucceſſiuas tanquam ex duabus partibus compoſita ſumus contemplati, inter lentem primam A et ſecundam B imago vera $F\zeta$ cadet, ſi ambae eius partes a et b fuerint poſitiuae; ſimilique modo inter lentem ſecundam B et tertiam C imago vera reperie-

tur, ſi huius interualli B C ambae partes β et c fuerint poſitiuae et ita porro.

Theorema 2.

87. Si binarum partium aliquot, huiusmodi interuallum veluti c D conſtituentium alterutra fuerit negatiua; tum imago H θ lenti C reſpondens erit ficta (fieri enim nequit, vt ambae ſimul ſint negatiuae.)

Demonſtratio.

Cum interuallum c D binis partibus $c\,H = \gamma$, et $H\,D = d$ conſtet, ſumamus primo diſtantiam γ eſſe negatiuam; tum igitur imago H θ ante lentem R R cadet et radii per hanc lentem transmiſſi ita refringentur, quaſi ex iſta imagine eſſent egreſſi, cum tamen inde non emanauerint; quamobrem iſta imago non erit vera, ſed ficta. Sin autem altera pars d fuerit negatiua, imago H θ demum poſt lentem S S caderet, quia autem radii per lentem R R transmiſſi ante quam eo pertingunt per lentem S S de nouo refringuntur, iſtam effigiem non reuera formabunt, ideoque haec imago erit ficta.

Ambae autem partes γ et d ſimul non poſſunt eſſe negatiuae, quia earum ſumma $\gamma + d$ ipſum interuallum c D exprimit, quod ſemper neceſſario eſt poſitiuum.

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Coroll. 1.

88. Si ergo pro primo interuallo A B partium α et b altera fuerit negatiua, inter a et B nulla cadit imago vera; ſi praeterea etiam pro ſecundo interuallo bC partium β et c altera fuerit quoque negatiua, inter a et C nulla cadet imago vera, ac ſi inſuper partium interualli c D, quae ſunt γ et d, altera fuerit negatiua; tum ne quidem in ſpatio a D reperietur imago vera, ſicque fieri poteſt, vt inter plurium lentium ſpatium nulla plane cadat imago vera.

Coroll. 2.

89. Neutiquam ergo numerus imaginum verarum a numero lentium pendet, cum aeque fieri poſſit, vt poſt quamlibet lentem imago vera repraeſentetur atque vt pluribus lentibus nulla plane imago vera reſpondeat.

Coroll. 3.

90. Ex quotcunque igitur lentibus teleſcopium quodpiam fuerit compoſitum fieri poteſt, vt per totum eius ſpatium vel nulla plane imago vera reperiatur vel vnica tantum vel duae vel tres etc. nunquam tamen plures, quam ſunt lentes, vltima demta.

Theorema 3.

91. Poſt quotcunque demum lentes in teleſcopio prima imago vera exhibetur, ea ſemper eſt inuerſa.

Demonſtratio.

Quando ſcilicet imago primae lentis ſtatim eſt vera, perſpicuum eſt, eam quoque eſſe inuerſam; quod autem ea etiam futura ſit inuerſa, ſi demum Fig. 5. Tom. I. poſt plures lentes occurrat, ſequenti modo oſtendi poteſt; conſideretur radius ex centro obiecti E per ſuperius lentis obiectiuae punctum M tranſiens atque iſte radius per ſequentes lentes tranſiens tamdiu ſupra axem verſabitur, donec ad primam imaginem veram pertigerit; quia enim ex axis puncto E eſt egreſſus, vbicunque iterum in axem incideret, ibi exiſteret imago obiecti vera (hic enim ad aberrationem vel diffuſionem radiorum non reſpicimus) ex quo manifeſtum eſt, hunc radium ante non ad axem eſſe peruenturum, quam ad primam imaginem veram pertigerit et quia ex regione ſuperiori hic in axem incidit, ad regionem inferiorem progreſſurus, imago in hoc loco expreſſa erit inuerſa, cum enim ex obiecto ſurſum ſit progreſſus, nunc autem ex imagine deorſum dirigatur, partes obiecti ſurſum vergentes nunc deorſum ſitae conſpicientur.

Coroll. 1.

92. Simili modo intelligere licet, radios illos ex imagine progredientes tamdiu infra axem eſſe verſaturos, donec iterum ad axem pertingant, quod fit in imagine vera ſecunda, vnde iterum in partes axis ſupe-

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ſuperiores tranſeunt vnde patet, ſecundam imaginem ſitum erectum tenere debere, ſicque porro tertia imago vera denuo erit inuerſa, quarta autem erecta et ita porro.

Coroll. 2.

93. Quotcunque ergo fuerint lentes, non tam ad imagines ſingulis lentibus reſpondentes erit reſpiciendum, quam ad imagines veras, cum alternatio ſitus erecti et inuerſi pendeat tantum ab imaginibus veris, dum imagines fictae nihil in hoc ordine turbant.

Scholion.

94. Haec proprietas imaginum verarum tam eſſentialiter naturam teleſcopiorum afficit, vt eorum diſcrimen potiſſimum a numero imaginum verarum petendum eſſe videatur, nulla plane ratione habita imaginum fictarum, quippe quae in hoc negotio parui ſunt momenti. Qui enim voluerit teleſcopia ſecundum lentium numerum in genera diſtribuere, maximis incommodis ſe implicabit, primo enim exigua illa teleſcopia vel potius perſpicilla lente oculari concaua conſtantia et tubos aſtronomicos ad idem genus referre eſſet coactus; dum tamen ſua natura maxime inter ſe diſcrepant, quandoquidem illis obiecta ſitu erecto, his vero ſitu inverſo repraeſentantur, praeterquam quod in loco oculi maxima vtrinque deprehenditur

ditur diuerſitas; deinde ſi cuipiam teleſcopio ſiue ad campum apparentem augendum ſiue ad maiorem diſtinctionis gradum ipſi conciliandum vnica lens inſuper adiungeretur, ſtatim ad longe aliud genus foret referendum, quod certe aeque incongruum videri debet; quibus probe perpenſis non dubito diuerſa teleſcopiorum genera ſecundum numerum imaginum verarum, quae in iis occurrunt, conſtituere, ita, vt primum genus complexurum ſit ea teleſcopia, in quibus nulla plane imago vera occurrit; ſecundum vero ea, in quibus vnica imago vera reperitur, tertium vero ea, quae duas imagines veras continent, ad quae tria genera omnia teleſcopia, quae adhuc excogitata ſunt et elaborata, erunt referenda, ac ſi vlterius progredi velimus, ad quartum genus reuocari conueniet ea teleſcopia, in quibus tres imagines verae deprehenduntur: verum praecedentia iam tam late patent, vt iis omnes plane perfectiones, quae vnquam deſiderari queant, conciliari poſſint, ita, vt nulla plane ratio adſit, cur plures imagines veras ſtatuere velimus. Hanc igitur diuiſionem in ſequentibus problematibus diſtinctius euoluamus.

Problema I.

95. Teleſcopiorum ad primum genus relatorum, in quibus nulla ineſt imago vera, praecipuas proprietates recenſere.

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Solutio.

Cum in his teleſcopiis, quotcunque etiam conſtent lentibus, nulla inſit imago vera, ſingula interualla $aB = \alpha + b$; $bC = \beta + c$; $cD = \gamma + d$ etc., ita ex binis partibus definientur, vt alterutra earum ſit negatiua, idque vsque ad vltimam lentem ocularem. Et quoniam haec eadem interualla neceſſario ſunt poſitiua, facile patet, omnes iſtas fractiones $\frac{\alpha}{b}$; $\frac{\beta}{c}$; $\frac{\gamma}{d}$ etc. debere eſſe negatiuas, in quo character eſſentialis huius generis teleſcopiorum eſt conſtituendus. Viciſſim enim ſi omnes hae fractiones fuerint negatiuae in toto teleſcopio nulla imago vera locum habebit, ideoque ad noſtrum primum genus erit referendum. Alius autem character minus eſſentialis huius generis in hoc conſiſtit, quod haec teleſcopia ſitu erecto obiecta repraeſentent, quia ob nullam imaginem veram ipſa obiecta quaſi immediate adſpicimus.

Coroll. 1.

96. Simpliciſſima ergo ſpecies huius generis duabus conſtabit lentibus et cum ſit $\frac{\alpha}{b}$ quantitas negatiua, fiet ratio multiplicationis $m = \frac{-\alpha}{b}$, vti ſitus erectus poſtulat, hinc neceſſe eſt, vt ſit $\alpha > b$ ideoque α quantitas poſitiua et b negatiua. Cum autem porro eſſe debeat $\beta = \infty$, pro huius lentis ocularis diſtantia focali q habebimus ob $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$ valorem $q = b$ ſicque lens ocularis erit concaua.

Coroll. 2.

97. Cum porro in genere ſit $m = \pm \frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$ etc. cuius factores ſunt noſtrae fractiones, quae omnes eſſe debent negatiuae hinc manifeſtum eſt, cur ſupra ſigna + et − ſint alternantia inuenta, vt ſcilicet pro quouis lentium numero multiplicatio m valorem poſitiuum conſequatur.

Coroll. 3.

98. Oſtendi etiam poteſt, nullam harum litterarum a, b, c, β, γ etc. ſumi poſſe euaneſcentem. Si enim v. c. diſtantia b eſſet minima, quia altera litterarum α et b debet eſſe negatiua, earum ſumma vero $\alpha + b$ poſitiua et finita, neceſſe eſt, vt ſit $\alpha > 0$; $b < 0$; ſit igitur $b = -\omega$, quantitati ſcilicet euaneſcenti et quia eſt $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$ fiet $\beta = \frac{q\omega}{q+\omega} = \omega$ ideoque poſitiuum; foret ergo $c < 0$ hincque $\beta + c$ interuallum c B exprimere non poſſet; vnde patet huiusmodi caſus locum habere non poſſe. Fieri autem poteſt, vt quaepiam harum quantitatum fiat $= \infty$; ſi enim fuerit v. gr. $\beta = \infty$, ob interuallum $\beta + c =$ finito puta $= k$, erit $c = -\infty + k = -\infty$ et $\frac{\beta}{c} = -1$; hoc autem non impedit, quominus ſequens fractio $\frac{\gamma}{d}$ valorem obtineat quemcunque.

Scholion.

99. Notiſſimum eſt hoc teleſcopiorum genus, quippe quod primum ab artifice quodam inuentum per-

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hibetur, dum caſu lentem conuexam cum concaua combinauerat, neque tamen eius eſſentia in hoc eſt ſtatuenda, quod tantum duabus conſtet lentibus. Si enim loco lentis obiectiuae ſimplicis ſubſtituamus duplicatam vel adeo triplicatam; nemo certe putabit, ipſum eius genus mutatum eſſe, quoniam huiusmodi lentes multiplicatae vt ſimplices ſpectari ſolent, ſimili modo lens ocularis poſſet duplicari vel triplicari, ipſo genere non mutato; cum autem nihilominus plures lentes ſimplices adhibeantur, manifeſtum eſt, ipſam generis indolem non a numero lentium pendere, cenſeri poſſe. In ſequentibus autem inprimis operam dabimus, vt nouis lentibus addendis hoc genus ad maiorem perfectionem euehamus.

Problema 2.

100. Teleſcopiorum ad ſecundum genus relatorum, in quibus vnica imago vera occurrit, praecipuas proprietates recenſere.

Solutio.

Ex quotcunque lentibus tale teleſcopium fuerit compoſitum; euidens eſt, non omnes fractiones ex ſingulis lentium interuallis natas $\frac{\alpha}{b}$, $\frac{\beta}{c}$; $\frac{\gamma}{d}$ etc. negatiuas eſſe debere, quia alioquin nulla imago vera eſſet proditura; cum autem vnica adſit vera, neceſſe eſt, vt etiam vnica illarum fractionum fiat poſitiua, quae

ſi fuerit v. c. $\frac{\gamma}{d}$, ambae litterae γ et d poſitiuae eſſe debebunt, dum reliquae fractiones omnes manent, vt ante negatiuae, atque perinde eſt, quaenam illarum fractionum valorem poſitiuum nanciſcatur, dummodo plus vna non ſit poſitiua, atque in hoc conſiſtit character eſſentialis huius generis teleſcopiorum, inter cuius proprietates haec inſuper inprimis eſt notanda, quod obiecta ſitu inuerſo repraeſentet, quandoquidem per huiusmodi teleſcopia non tam ipſa obiecta, quam eorum imaginem veram, quae eſt inuerſa, conſpicere ſumus cenſendi.

Coroll. 1.

101. Si ergo huiusmodi teleſcopium duabus tantum conſtet lentibus, quae ſine dubio ſimpliciſſima huius generis eſt ſpecies, ob vnicum interuallum a B vnica quoque habetur fractio $\frac{\alpha}{b}$, quae propterea poſitiua eſſe debet ideoque etiam vtraque diſtantia α et b; quae cum ob $a = \infty$ et $\beta = \infty$ praebeant diſtantiam focalem vtriusque lentis, manifeſtum eſt, vtramque lentem fore conuexam.

Coroll. 2.

102. Quia igitur huic generi repraeſentatio inuerſa eſt propria, exponens multiplicationis m, quae producto harum fractionum $\frac{\alpha}{b}$. $\frac{\beta}{c}$ etc. aequalis eſt inuenta, valorem negatiuum obtinebit contrarium ſcilicet ei, qui caſu praecedenti prodierat.

Co-

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Coroll. 3.

103. In hoc autem genere euenire poteſt, vt quaepiam quantitatum α, b etc. euaneſcat, quod ſit, ſi in loco ipſius imaginis verae lens conſtituatur. Cadat enim imago vera in ipſam lentem tertiam C, erit $c = 0$, vel potius poſito $c = \omega$, ob $\frac{1}{r} = \frac{1}{c} + \frac{1}{\gamma}$ erit $\gamma = \frac{-r\omega}{r-\omega} = -\omega$ ita, vt ambae quantitates c et γ euaneſcant vnde diſtantiae β et d debent eſſe poſitiuae ſicque patet, fractionum $\frac{\beta}{c}$ et $\frac{\gamma}{d}$ alteram fore poſitiuam, alteram negatiuam, prout voluerimus; quoniam enim imaginem in ipſam lentem R R cadere aſſumimus, perinde eſt, ſiue eam ad interuallum b C ſiue ad interuallum c D velimus referre, vtroque autem caſu etſi fractio $\frac{\beta}{c}$ fiat ∞, fractio vero $\frac{\gamma}{d} = 0$, productum ambarum ſemper eſt $= -\frac{\beta}{d}$.

Scholion.

104. Teleſcopia ad hoc genus pertinentia vocari ſolent aſtronomica, quoniam enim obiecta ſitu inuerſo repraeſentant, potiſſimum ad obſeruationes aſtronomicas adhibentur, vbi parum refert, ſiue obiecta in coelo ſitu erecto ſiue inuerſo conſpiciamus; id quod in obiectis terreſtribus ſecus ſe habet, ad quorum contemplationem quando teleſcopia primi generis non ſufficiunt, ad tertium genus recurrere ſolemus.

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Problema 3.

105. Teleſcopiorum ad tertium genus relatorum, in quibus duae imagines verae occurrunt, praecipuas proprietates recenſere.

Solutio.

Cum hic duae imagines verae occurrant, quotcunque lentes adhibeantur, inter fractiones inde natas $\frac{\alpha}{b}$; $\frac{\beta}{c}$ etc. duae neceſſario debent eſſe poſitiuae, reliquae vero omnes negatiuae, vnde cum duae ad minimum eiusmodi fractiones adeſſe debeant, adeoque etiam duo lentium interualla, euidens eſt, ad huiusmodi teleſcopia tres ad minimum lentes requiri, quo caſu nullae tales fractiones negatiuae habebuntur; vnde fractiones negatiuae eatenus tantum occurrent, quatenus plures tribus lentes in vſum vocantur, atque in hoc eſſentialis character huius generis teleſcopiorum continetur; inter praecipuas autem proprietates haec inprimis eſt notanda, quod per teleſcopia obiecta in ſitu erecto conſpiciantur.

Coroll. 1.

106. Si haec teleſcopia ex tribus lentibus formentur, omnes hae quatuor diſtantiae α, b, β, c eſſe debent poſitiuae et cum diſtantiae a et γ ſint ∞ omnes tres lentes debent eſſe conuexae; ſi enim earum diſtantiae focales ſint p, q et r habebitur 1° $p = \alpha$. 2° $q = \frac{b\beta}{b+\beta}$ et 3° $r = c$. quae omnes ſunt poſitiuae.

Co-

Coroll. 2.

107. Quemadmodum praecedenti caſu licuit in ipſum locum imaginis verae lentem conſtituere, ita etiam hic nulla ratio obſtat, quominus in vtraque imagine vera lentes collocentur; tum autem ea, quae ſupra ſunt de fractionibus modo in infinitum excreſcentibus modo euaneſcentibus tradita, probe ſunt obſeruanda.

Scholion.

108. Hoc genus eum in finem eſt excogitatum, vt tubi aſtronomici ad obiecta terreſtria ſitu erecto contemplanda accommodarentur; quod quidem tribus lentibus fieri poſſe iam annotauimus. Sed quoniam tribus tantum lentibus adhibendis campus apparens fere totus euaneſcit aliaque incommoda ſe inſuper admiſcent, ſtatim quatuor lentes vſurpari ſunt ſolitae quae ita ſunt iunctae, vt duos tubos aſtronomicos connexos referant et tres lentes poſteriores nomine ocularium appellatae ſunt, quibus etiam fere eadem diſtantia focalis tribui poteſt. Ad idem quoque genus referenda ſunt noua illa teleſcopia anglica a Clariſſ. Dollondo nuper inuenta, in quibus praeter lentes obiectiuas duplicatas longe diuerſa lentium ocularium diſpoſitio cernitur. Interim vero haec diſpoſiio infinitis modis variari poteſt, atque adeo debet, vt haec teleſcopia ad ſummum perfectionis gradum euehantur.

Problema 4

109. Teleſcopiorum ad quartum genus relatorum, in quibus tres imagines verae occurrunt, praecipuas proprietates enumerare.

So-

Solutio.

In hoc ergo genere quotcunque lentes adhibeantur, inter fractiones iis reſpondentes $\frac{\alpha}{b}$, $\frac{\beta}{c}$ etc. tres debent eſſe poſitiuae, dum reliquae manent negatiuae, ex quo perſpicuum eſt, ad hoc genus ad minimum opus eſſe quatuor lentibus, et quia vltima imago vera, quae quaſi ab oculo ſpectatur, eſt inuerſa, obiecta quoque per omnia teleſcopia huius generis inuerſa conſpicientur.

Scholion.

110. Quoniam nulla plane ratio ſuadet, vt repraeſentationem praecedentis generis denuo inuertere velimus, atque vti videbimus, omnes perfectiones praecedentibus generibus conferri queunt; nihil aliud lucraremur niſi, vt teleſcopia multo fierent longiora, et numerum lentium ſine vllo vſu multiplicaremus, vt taceam iacturam inſignem radiorum lucidorum, quae ob tot lentes merito eſſet metuenda; atque hanc ob rationem non dubito, genus hoc quartum penitus rejicere, de quo etiam nullum ſupererit dubium, quando tria praecedentia genera ita pertractauerimus, vt omnibus momentis quibus perfectio teleſcopiorum innititur, ſatisfecerimus. Multo magis autem ſequentia, quae conſtitui poſſent genera, nullam plane attentionem merebuntur.

CAPVT

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CAPVT IV.

DE TELESCOPIIS PRIMI GENERIS, QVAE SCILICET IMAGINE VERA DESTITVVNTVR, ET OBIECTA SITV ERECTO REPRAESENTANT.

Problema I.

111.

Si telefcopium primi generis ex duabus tantum lentibus conſtet, obiectiua ſcilicet et oculari, eius conſtructionem euoluere et proprietates exponere.

Solutio.

Cum hic ſit $\frac{\alpha}{b}$ quantitas negatiua et $\alpha + b$ poſitiua, ſi ratio multiplicationis ponatur $= m$, ob $m > 1$ diſtantia α, vt ante vidimus, debet eſſe poſitiua; altera vero b negatiua, vt ſit $b = \frac{-\alpha}{m}$ ſeu diſtantiis focalibus introductis $\alpha = p$, et $q = \frac{-p}{m}$ et interuallum binarum lentium $\alpha + b = (\frac{m-1}{m}).p$ vnde patet ex data multiplicatione m et diſtantia focali p omnia determi-

 nari.

nari. Verum haec diſtantia p tanta eſſe debet, vt lens obiectiua datam admittat aperturam, cuius, ſi claritatis gradus ponatur $= y$, ſemidiameter eſſe debet $x = my$, vnde iam patet, diſtantiam p maiorem eſſe debere, quam $4\,my$ vel $5.\,my$; vnde cum y in partibus digiti dari ſoleat veluti $y = \frac{1}{15}$. dig., vt ſit $x = \frac{m}{15}$ et $p > \frac{m}{15}$ dig. verum hic inprimis ſpectari debet aequatio pro ſemidiametro confuſionis, quae dat

$$\frac{mx^3}{4p^3}\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right) < \frac{1}{4k^3}$$

vnde colligitur $p = kx\sqrt[3]{m\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right)} = km.y\sqrt[3]{m\left(\mu\lambda - \frac{\mu'\lambda'}{m}\right)}$, qui ergo valor, niſi forte minor ſit, quam $5.\,my$, ipſi p tribui debet, vbi vt ſupra notauimus, numerus k poni poteſt vel 30 vel 40 vel 50 prout maior vel minor diſtinctionis gradus deſideratur, atque iam ex datis valoribus λ et λ' cum vitri ſpecie, vnde numeri μ et μ' pendent, ambae lentes conſtrui hincque totum teleſcopium confici poterunt; ad cuius proprietates cognoſcendas quaeramus primo locum oculi eiusue diſtantiam a lente oculari, inuenimusque.

$$O = \frac{m-1}{m}.\,q. \quad \S.\ 30.$$

quae cum ob $q < 0$ ſit negatiua oculum lenti oculari immediate applicari oportet; vnde colligitur ſemidiameter campi ex §. 37. $\Phi = \frac{-\pi}{m-1}$ et $\pi = \frac{+\omega}{q}$, denotante ω ſemidiametrum pupillae; quare ob $q = \frac{-p}{m}$ fiet $\Phi = \frac{+m}{m-1}.\,\frac{\omega}{p}$ vbi inprimis notandum eſt, lentem ocula-

ocularem tantam ſumi debere, vt aperturam admittat, cuius ſemidiameter ſit $= \pi q = \omega$; ex quo neceſſe eſt, vt fiat $-q > 5\,\omega$ vel $4\,\omega$ hincque etiam $p > 4\,m.\,\omega$ vel $> 5.\,m\,\omega$. quae conditio iam in ſe complectitur primam ob $y < \omega$. Quod denique ad alteram confuſionem attinet, cum deſtructio marginis colorati poſtulet, vt ſit § 52.

$$o = \frac{dn}{n-1}.\ B\,\pi.\,p$$

quod cum fieri nequeat, niſi lens obiectiua fuerit perfecta, euidens eſt, marginem coloratum deſtrui non poſſe. Denique pro hac confuſione penitus tollenda eſſe debet

$$o = \frac{dn}{n-1}.\ p + \frac{dn'}{n'-1}.\ q, \quad \text{ſiue}$$

$$\frac{dn}{n-1} = -\frac{1}{m}.\ \frac{dn'}{n'-1}$$

cui caſu adeo, quo lens obiectiua eſt perfecta, ſatisfieri nequit, ob primum terminum euaneſcentem; quia autem m eſt numerus ſatis magnus, alterum membrum per ſe fit ſatis paruum, vt haec confuſio non ſit metuenda.

Coroll. I.

112. Cum diſtantia focalis p maior eſſe debeat, quam $5\,m\,\omega$, pro data multiplicatione m longitudo huius teleſcopii ſemper maior erit, quam $5\,(m-1)\,\omega$ et cum ſit circiter $\omega = \frac{1}{20}$. dig. haec teleſcopii longitudo minor fieri non poterit quam $\frac{m-1}{4}$. dig. ſcilicet

cet ſi velimus, vt ſit $m = 50$, longitudo teleſcopii minor eſſe nequit, quam $12\frac{1}{4}$ dig. etiamſi formula $p = m k y \sqrt[3]{m \mu \lambda - \mu' \lambda'}$ multo minor reddi poſſet.

Coroll. 2.

113. Pro campo apparente inuenimus eius ſemidiametrum $\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p}$ vnde, cum ſit $p > 5 m \omega$, valor ipſius Φ ſemper certe minor erit, quam $\frac{1}{5(m-1)}$ atque in minutis primis erit $\Phi < \frac{687}{m-1}$. minut. quo campo facile contenti eſſe poſſemus, niſi p deberet eſſe multo maius, quam $5 m \omega$.

Coroll. 3.

114. Quoniam margo coloratus tolli non poteſt, niſi lens obiectiua ſit perfecta; hinc ſtatim intelligimus, quanti ſit momenti vſus lentium perfectarum, quas ſupra deſcripſimus; ita, vt earum beneficio his teleſcopiis inſignis gradus perfectionis conciliari poſſit.

Scholion.

115. Solutio huius problematis ita eſt generalis, vt ad omnes vitri ſpecies ex quibus lentes parari poſſunt, pateat; quin etiam loco lentis obiectiuae non ſolum lentes ſimplices, ſed etiam duplicatae vel triplicatae atque adeo perfectae ſubſtitui poſſunt: vnde plurimae ſpecies huius teleſcopii, quod tantum ex

duabus

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duabus lentibus compoſitum ſpectamus, exhiberi poſſunt; quarum praecipuas in ſubiunctis exemplis contemplemur:

Exemplum I.

116. Si ambae lentes fuerint ſimplices atque ex eadem vitri ſpecie confectae, conſtructionem huius teleſcopii definire:

Pro hoc caſu potiſſimum aequatio venit conſideranda:

$$p = m k y \sqrt[3]{\mu (m \lambda - \lambda')}$$

quae diſtantiam focalem primae lentis determinat, ſiquidem valor hinc prodiens maior fuerit, quam $5 . m . \omega$. Videbimus autem ſtatim atque multiplicatio m fuerit notabilis, eius valorem multum eſſe ſuperaturum iſtum limitem $5 . m \omega$ ſeu $\frac{1}{4} m$. dig. ita, vt maximi ſit momenti hanc formulam tam paruam reddere, quam fieri poteſt; quare ſtatim faciamus $\lambda = 1$, vt lens obiectiua ſecundum §. 59. elaborari debeat; quod vero ad lentem ocularem attinet, non convenit $\lambda' = 1$ ponere, ſed potius e re erit, ipſi huic litterae maiorem valorem tribuere, inprimis autem vt haec lens maximae aperturae fiat capax, ea optime vtrinque aeque concava redditur, ex quo numerus $\lambda' = 1.6299$. (§. 61.) pro ea vitri ſpecie, qua $n = 1,55$. et qua artifices plerumque vti ſolent. Pro aliis autem ſpeciebus tantum non differet, vt operae pretium ſit, differentiae

rationem habere; praecipue cum litteras k et y tam adcurate definire non liceat. Sumamus ergo $y = \frac{1}{40}$. dig. vt ſatis magnam claritatem obtineamus, quae in hoc genere neceſſaria videtur; et $k = 40$, vt confuſio ſatis reddatur exigua eritque ob $\lambda = 1$; $\lambda' = 1\frac{5}{9}$

$$p = m.\sqrt[3]{\mu\,(m - 1\tfrac{5}{9})}$$

vnde patet, hic eas vitri ſpecies praeferri debere, quibus maior refractio n reſpondet, quia tum littera μ minores nanciſcitur valores. Cum autem perpetuo μ non multum differat ab vnitate eiusque propterea radix cubica multo minus diſcrepet, quacunque vitri ſpecie vti velimus, tuto ſumere licebit $p = m\sqrt[3]{(m - 1\frac{5}{9})}$ hoc autem caſu circa marginem coloratum nihil efficere licet. Quare ſi hinc diſtantiam focalem lentis obiectiuae debite definiuerimus atque n denotet refractionem vitri, ex quo ambae lentes ſint parandae, conſtructio teleſcopii ſequenti modo ſe habebit:

I°. Lens obiectiua paranda eſt ex formulis §. 59.

II°. Lens ocularis vtrinque aeque concaua conficiatur, ſumendo radium vtriusque faciei $= \frac{-2(n-1)p}{m}$ ob $q = \frac{-p}{m}$.

III°. Hae duae lentes ad diſtantiam A B $= \frac{m-1}{m}.p$ iungantur et tubo inſerantur, vt oculus lenti concauae immediate adplicari poſſit.

IV°.

IV°. Hic tubus campum offeret cuius ſemidiameter erit $\Phi = \frac{m}{m-1} \cdot \frac{\omega}{p} = \frac{3437 \cdot m}{m-1} \cdot \frac{\omega}{p}$ min.

V°. Hoc teleſcopium a vitio marginis colorati liberari nequit.

Coroll. 1.

117. Quodſi multiplicatio tanta ſit, vt fiat $m = 1\frac{5}{8}$, formula p definiens euaneſcit, nihilo vero minus ſumi debet $p = 5 \cdot m\,\omega$ ſeu quaſi $\frac{1}{4} \cdot m$. dig. hocque valore vti licet, etſi m aliquanto ſit maius, dummodo illa formula non excedat $\frac{1}{4} \cdot m$. dig. quod euenit, quamdiu m non ſuperat limitem $1\frac{41}{64}$ qui vix ſuperat valorem $1\frac{5}{8}$; ex quo patet, ſtatim atque multiplicatio m maior ſit, quam $1\frac{5}{8}$, diſtantiam focalem p maiorem capi debere, quam $\frac{1}{4} \cdot m$. dig.

Coroll. 2.

118. Quare ſi verbi gratia debeat eſſe $m = 5$, capi oportet $p = 7\frac{1}{2}$ dig. et $q = -\frac{3}{8}$ vnde ſemidiameter campi apparentis prodit $\Phi = \frac{5}{4} \cdot \frac{2\omega}{15} = \frac{\omega}{6} = \frac{1}{125}$ ob $\omega = \frac{1}{25}$; ſiue $\Phi = \frac{3437}{125}$ min. $= 29$. min. Longitudo autem teleſcopii erit 6 digit.

Coroll. 3.

119. Si multiplicatio deſideretur $m = 10$, reperitur $p = 5\sqrt[3]{6}\,7 = 20\frac{5}{16}$ dig. hincque $q = -2.\frac{1}{32}$ dig. ita, vt longitudo teleſcopii ſit $18\frac{9}{32}$ dig. tum vero ſemidia-

midiameter campi apparentis, qui eſt $\frac{\omega}{p+q}$ fit $\Phi = \frac{32.\omega}{385}$ $= \frac{4}{3033}$ et in minutis $\Phi = 4' \, 42''$, qui campus iam tam eſt exiguus, vt nullo modo tolerari poſſit, quare haec ſpecies teleſcopiorum ne quidem ad multiplicationem $m = 10$ adplicari poteſt.

Exempl. II.

120. Si ambae lentes ex eadem vitri ſpecie parentur, obiectiua vero ſtatuatur duplicata ſec. §. 65. conſtruenda, vt ſit $\lambda = \frac{1-\nu}{4}$ ac ſi vitro communi, pro quo eſt $n = 1.55$, vtamur, erit $\lambda = 0.1918$; ſumtaque iterum vnitate pro $\sqrt[3]{\mu}$ et poſito, vt ante, $\lambda' = 1\frac{5}{8}$ vt lens ocularis fiat aequaliter concaua erit $p = m.\sqrt[3]{(0,1918.m - 1\frac{5}{8})}$ et vt ante, $q = \frac{-p}{m}$. hincque diſtantia lentium $= \frac{m-1}{m} p$ quare ſi inde pro data multiplicatione definiatur valor litterae p, conſtructio ita ſe habebit:

I°. Lens obiectiua paranda eſt ex formulis §. 59. pro $n = 1.55$.

II°. Lens ocularis vtrinque fiat aequaliter concaua, radio exiſtente $= -2.(n-1).\frac{p}{m} = -\frac{11}{10}.\frac{p}{m}$.

III°. Semidiameter campi apparentis erit, vt ante, $\Phi = \frac{m}{m-1}.\frac{\omega}{p} = \frac{1720.m}{m-1}.\frac{1}{p}$ min.

IV°. Aeque parum autem, ac ante, hoc caſu margini colorato remedium afferri poteſt.

Co-

Coroll. 1.

121. Quando autem formula illa praebet $p < \frac{1}{4} m$. dig. nihilo minus ſtatui debet $p = \frac{1}{4} m$. dig. quod inprimis euenit, ſi ſit $m = 8\frac{1}{2}$ circiter; vnde oritur $p = 0$ quare niſi multiplicatio maior deſideretur, ſumi poterit $p = \frac{1}{4} m$. dig. vnde fit $q = -\frac{1}{4}$. dig. et longitudo teleſcopii $\frac{1}{4}(m-1)$ dig. campique apparentis ſemidiameter $\Phi = \frac{689}{m-1}$. minut.

Coroll. 2.

122. Quodſi ergo multiplicatio propoſita ſit $m = 8\frac{1}{2}$, teleſcopium ita erit conſtruendum. I°. ob $p = \frac{17}{8}$ dig. $= 2\frac{1}{8}$ dig. lens obiectiua paretur ſecundum praecepta data. II°. ob $q = -\frac{1}{4}$ dig. radius vtriusque faciei erit $= -\frac{1}{2}(n-1)$ dig. vnde longitudo teleſcopii fit $= 1 \cdot \frac{7}{8}$ dig. campi vero apparentis ſemidiameter $= 1° \; 31'$. quod teleſcopium omni attentione dignum videtur non obſtante margine colorato.

Coroll. 3.

123. Si deſideretur multiplicatio $m = 15$. ſtatim reperitur $p = 16, 15$. dig. hinc $q = -1, 07$. dig. vnde longit. teleſcopii $= 15, 08$. dig. et ſemidiameter campi apparentis erit $= \frac{172}{15,08}$. minut. $= 11' \; 24''$ vnde patet hoc teleſcopium tam ob nimis exiguum campum quam ob nimis magnam longitudinem merito eſſe reiiciendum, dum contra caſus praecedens maxime commendandus videtur.

Exempl. III.

124. Si ambae lentes ex eadem vitri ſpecie conſtent, obiectiua vero ſtatuatur triplicata, ſec §. 66. conſtruenda, vt ſit $\lambda = \frac{8-3\nu}{3+\rho}$; conſtructionem huius teleſcopii definire.

Vtamur vitro communi, pro quo eſt $n = 1,55$ eritque $\lambda = 0,0422$ et maneat $\lambda' = 1,629$, ſumta iterum vnitate pro $\stackrel{3}{V}\mu$ erit

$$p = m.\stackrel{3}{V}(0,0422\,m - 1,629)$$

vnde reliqua, vt in caſibus praecedentibus determinantur.

Inprimis autem hic notari meretur caſus, quo fit $0,0422\,m = 1,629$ ſiue $m = 38\frac{3}{5}$ pro qua ſumi debet lentis obiectiuae diſtantia focalis $p = 9\frac{13}{25}$ dig. manente $q = -\frac{1}{4}$. dig. hincque longitudo tubi $= 9\frac{2}{3}$. dig. ex qua ſemidiameter campi erit $= 18' 17''$ qui quidem campus ſatis eſt paruus, ſed ob tam notabilem multiplicationem facile tolerari poteſt, niſi forte margo coloratus offendat.

Exempl. IV.

125. Si pro lente obiectiua capiatur lens perfecta, ocularis autem maneat ſimplex atque adeo vtrinque aequaliter concaua, conſtructionem teleſcopii deſcribere.

Quo-

Quoniam ſupra huiusmodi lentes perfectas deſcripſimus partim ex vitro coronario, partim ex vitro chryſtallino conficiendas hic ante omnia attendendum eſt, quantae aperturae quaelibet ſit capax; cum enim pro multiplicatione m hic eſſe debeat $x = \frac{m}{45}$ dig. ante omnia videndum eſt, an lens perfecta hic adhibenda tantam aperturam admittat, quae cautela ſedulo eſſet obſeruanda, ſi valor ipſius p quopiam caſu prodiret $= 0$; quo vt ante capi deberet $p = \frac{1}{4}. m$; ita, vt fieret $x = \frac{1}{15}. p$, quod tantum in tertia lente triplicata locum habet. Verum non opus eſt, vt de hoc ſimus ſolliciti, quia ex formula radicali ſuperiori pro hoc caſu nunquam prodire poteſt $p = 0$, quoniam enim lens eſt perfecta, erit per hypotheſin $\lambda = 0$, ita vt fiat $p = m. \sqrt[3]{-1.629}$. vnde patet, ſemper adeo fore $p > m$, ſcilicet $p = 1,17.\, m$. quare ſtatim ſequitur hoc inſigne incommodum, vt mox ac multiplicationi m modicus valor tribuatur, campus apparens tam paruus ſit proditurus, vt teleſcopium fere omni vſu careat; cuius cauſſa cum ſit valor $\lambda = 0$, optandum hic eſſet, vt lens perfecta etiam nunc confuſionem quandam exiguam pareret, vt illa formula pro quapiam multiplicatione praeberet $p = 0$. Secundum praecepta autem ſupra data tales lentes non difficulter inueniri poſſent, quae dum nullam gignerent diſperſionem, aliquam tamen confuſionem producerent; verum eiusmodi inueſtigatio commodius inſtituetur his teleſcopiis vel vnam vel duas lentes nouas adiungendo.

 Scho-

Scholion.

126. Ratio huius inſignis paradoxi, quod lentes perfectae hic minus vtilitatis praeſtent, quam lentes duplicatae et triplicatae praecedentes in hoc manifeſto eſt poſita, quod hic non eiusmodi lente obiectiua egeamus, pro qua ſit $\lambda = 0$, ſed potius tali, vt $\lambda m - \lambda'$ redigi poſſit ad nihilum. Supra autem facile fuiſſet eiusmodi lentes compoſitas inuenire, quae dum confuſioni colorum mederentur, pro priori confuſione datum valorem numeri λ habuiſſent; verum hic non opus eſt, vt illum laborem repetamus; ſed potius alio modo hanc inveſtigationem ad praeſens inſtitutum accommodari conueniet; duas ſcilicet pluresue lentes, quae vnitae lentem perfectam conſtituebant, hic tanquam disiunctas conſideremus quo pacto id commodi aſſequemur, vt non ſolum vtraque confuſio lentem etiam ocularem in calculo comprehendendo penitus tolli, ſed etiam fortaſſe campus apparens vlterius extendi queat; quem in finem ſequens problema praemitti oportet.

Problema 2.

127. Inter lentem obiectiuam et ocularem aliam inſuper lentem inſerere, vt teleſcopium eidem primo generi maneat accenſendum.

Solu-

Solutio.

Ponamus ergo teleſcopium conſtare tribus lentibus PP, QQ, RR, ac primo quidem requiritur, vt hae fractiones $\frac{\alpha}{b}$, $\frac{\beta}{c}$ ſint negatiuae; tum vero vt haec interualla $\alpha + b$; $\beta + c$ ſint poſitiua; exiſtente multiplicatione $m = \frac{\alpha}{b} \cdot \frac{\beta}{c}$ ſiue $m = \frac{\alpha}{c} . B.$ ob $B = \frac{\beta}{b}$, quae proinde quantitas erit poſitiua. Introducamus nunc altera elementa, quae ſupra litteris B, C et indicibus aperturae π, π' cum ſemidiametro campi Φ continebantur, ac pro priori conditione habebimus

$$\frac{\alpha}{b} = \frac{\mathfrak{B}\pi - \Phi}{\Phi} < 0.$$

$$\frac{\beta}{c} = \frac{\mathfrak{C}\pi' - \pi + \Phi}{\mathfrak{B}\pi - \Phi} < 0.$$

vnde cum Φ ex rei natura ſemper ſit poſitiuum, debet eſſe $\mathfrak{B}\pi - \Phi$ negatiuum, at vero $\mathfrak{C}\pi' - \pi + \Phi$ poſitiuum; et quia $\gamma = \infty$; ideoque $C = \infty$ et $\mathfrak{C} = 1$ vnde poſterior conditio dat $\pi' - \pi + \Phi > 0.$ Pro campo autem apparente inuenimus $\Phi = \frac{-\pi + \pi'}{m - 1}$, vnde cum Φ et $m - 1$ ſint quantitates poſitiuae, debet eſſe $-\pi + \pi'$ quantitas poſitiua, qua praecedens etiam conditio ſponte continetur. Vt autem praeterea interualla lentium fiant poſitiua, has duas conditiones adipiſcimur ex §. 16.

$$1^\circ. \ \frac{\mathfrak{B}\pi p}{\mathfrak{B}\pi - \Phi} > 0.$$

vnde cum denominator ſit negatiuus, etiam numerator debet eſſe negatiuus ſeu $\mathfrak{B}\pi p < 0$ prouti ergo

 quan-

quantitas p fuerit vel poſitiua vel negatiua, debet eſſe $\mathfrak{B}\,\pi$ vel negatiuum vel poſitiuum.

$$2^\circ.\ \frac{B\Phi\,(\pi' - (1 - \mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\pi' - \pi + \Phi)} > 0$$

vbi cum Φ ſit poſitiuum, totus vero denominator negatiuus, etiam pro numeratore $Bp\,(\pi' - (1 - \mathfrak{B})\,\pi)$ debet eſſe < 0.

Ex his igitur conditionibus ſi loco Φ valorem inuentum ſubſtituamus, ſequentes concluſiones conſequemur

1°. $\pi' - \pi > 0$.

2°. ob $\mathfrak{B}\,\pi - \Phi < 0$, debet eſſe
$(m-1)\,\mathfrak{B}\pi - \pi' + \pi < 0$ ſeu $\pi' - \pi >$
$(m-1)\,\mathfrak{B}\pi$ ſiue $\pi' > ((m-1)\,\mathfrak{B} + 1)\,\pi$

3°. $\mathfrak{B}\,\pi\,p < 0$

4°. $Bp\,(\pi' - (1 - \mathfrak{B})\,\pi) < 0$.

quia hic igitur formulae 3 et 4 ambae ſunt negatiuae, haec per illam diuiſa

$$\frac{B(\pi' - (1 - \mathfrak{B})\,\pi)}{\mathfrak{B}\,\pi} > 0$$

vnde ſi denominator fuerit poſitiuus etiam numerator debet eſſe poſitiuus et contra. Conſideremus nunc ambos caſus extremos, alterum, quo media lens lenti obiectiuae vnitur, alterum, quo ea lenti oculari vnitur. Priori caſu, quo ſcilicet $a + b = 0$, fit $\pi = 0$, quemadmodum ſupra iam notauimus pro lentibus quot-

cunque

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cunque cum obiectiua lente coaleſcentibus. Poſteriore caſu, quo $\beta + c = 0$, debet $\pi' - (1 - \mathfrak{B})\pi = 0$ ſeu $\pi' = (1 - \mathfrak{B})\pi$, qui valor in conditione ſuperiore ſecunda poſitus dabit $m\mathfrak{B}\pi < 0$ ſeu $\mathfrak{B}\pi < 0$. Cum autem campus apparens potiſſimum a lente oculari pendeat, cui reſpondet littera π', haec littera π' neceſſario eſt poſitiua quare vt campus ob lentem mediam non minuatur, ſed potius augeatur, numerum π negatiuum eſſe oportet, ex quo ſuperiores conditiones propius hoc modo definientur

1ma. ſcilicet $\pi' - \pi$ iam ſponte fit > 0 ideoque omitti poteſt.

2da. eſt $\pi' > ((m-1)\mathfrak{B} + 1)\pi$

ex 3^{tia}. autem ſequitur $\mathfrak{B}p > 0$

et 4to. $\frac{B(\pi' - (-\mathfrak{B})\pi)}{\mathfrak{B}\pi} > 0$.

Conſideretur adhuc locus oculi, cuius diſtantia a lente oculari fit $O = \frac{\pi'}{m\Phi} \cdot r$ quae ob $\frac{\pi'}{m\Phi}$ poſitiuam fieret poſitiua, ſi modo r eſſet poſitiuum at cum ſit $r = c$ ob $C = \infty$ et $\mathfrak{C} = 1$ erit $r = \frac{B\Phi}{\pi' - \pi + \Phi}$ cuius denominator cum ſit poſitiuus examinandum eſt, vtrum Bp ſit poſitiuum an negatiuum; at ſi Bp eſſet poſitiuum; diſtantia O quoque foret poſitiua, ſin autem Bp eſſet negatiuum, foret quoque diſtantia O negatiua, oculusque lenti tertiae immediate applicari deberet, de quo caſu praecepta ſupra data ſunt obſeruanda.

Co-

Coroll. 1.

128. Quia ſtatim ac multiplicatio m fit modicae quantitatis, Φ multo minus eſt, quam π, cum $\mathfrak{B}\pi - \Phi$ ſit negatiuum, quantitas $\mathfrak{B}\pi$ fiet quoque negatiua et ob $\pi < 0$ erit $\mathfrak{B}$ poſitiuum. Hinc pro tertia conditione $\mathfrak{B}\pi p < 0$ debebit eſſe p poſitiuum (excepto ſcilicet caſu, quo π quam minimum habet valorem ideoque p etiam negatiuum eſſe poſſet) et per tertiam et quartam conditionem coniunctim erit ob denominatorem negatiuum etiam numerator $B(\pi' - \pi + \mathfrak{B}\pi)$ negatiuus ſi ergo fuerit $\pi' - \pi + \mathfrak{B}\pi > 0$ erit $B < 0$; contra vero $B > 0$.

Coroll. 2.

129. Hae igitur conditiones impleri poſſunt pluribus modis, dum plura elementa manent indeterminata, ſtatim enim patet, quantitatem α ſeu p tam affirmatiuum, quam negatiuum valorem accipere poſſe; at quia $\mathfrak{B}p > 0$ ob $\pi < 0$, ſi p ſtatuamus poſitiuum, etiam $\mathfrak{B}$ debet eſſe poſitiuum; ſin autem p ſumatur negatiue, etiam $\mathfrak{B}$ debet eſſe negatiuum; interim tamen cum ſit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$, etiamſi ſit $\mathfrak{B}$ poſitiuum, littera B etiam nunc eſſe poteſt tam poſitiua, quam negatiua; altero vero caſu, quo $\mathfrak{B}$ eſt negatiuum, ſemper etiam B ſit negatiuum.

Scho-

Scholion.

130. Eodem modo, quo hoc problema reſoluimus, conditiones etiam inueniri poſſunt, quando duae pluresue lentes inter obiectiuam et ocularem inſeruntur ſeu quando huiusmodi teleſcopium ex quatuor pluribusue lentibus eſt compoſitum; ponamus enim quatuor id lentibus conſtare atque ſequentes ſex conditiones erunt adimplendae.

$1°.\ \frac{\alpha}{b} < 0;\quad 2°.\ \frac{\beta}{c} < 0.\quad 3°.\ \frac{\gamma}{d} < 0$

$4°.\ \alpha + b > 0;\ 5°.\ \beta + c > 0.\ 6°.\ \gamma + d > 0$

exiſtente $\delta = \infty$ ideoque $D = \infty$ et $\mathfrak{D} = 1$. vnde ſi loco harum litterarum valores ſupra dati introducantur, hae ſex conditiones praebebunt ſequentes formulas, in quibus Φ ſemper vt poſitiuum ponitur

$1°.\ \mathfrak{B}\pi - \Phi < 0.$

$2°.\ \frac{\mathfrak{C}\pi' - \pi + \Phi}{\mathfrak{B}\pi - \Phi} < 0.$

$3°.\ \frac{\pi'' - \pi' + \pi - \Phi}{\mathfrak{C}\pi' - \pi + \Phi} < 0.$

quae tres conditiones commodius ita referuntur.

$1°.\ \mathfrak{B}\pi - \Phi < 0$

$2°.\ \mathfrak{C}\pi' - \pi + \Phi > 0$

$3°.\ \pi'' - \pi' + \pi - \Phi < 0.$

Pro tribus reliquis conditionibus, quia in ſingulis denominatores ſunt negatiui, etiam numeratores opor-

tet eſſe negatiuos vnde ſequentes conditiones erunt adimplendae.

$$4^{\circ}.\ \mathfrak{B}\,\pi\,p < 0.$$

$$5^{\circ}.\ B\,p\,(\mathfrak{C}\,\pi' - (1 - \mathfrak{B})\,\pi) < 0$$

$$6^{\circ}.\ B\,C\,p\,(\pi'' - (1 - \mathfrak{C})\,\pi') < 0.$$

quae prout p fuerit vel poſitiuum vel negatiuum duplici modo conſiderari poterunt; in hoc negotio autem inprimis conſideranda eſt expreſſio pro campo apparente, quae eſt $\Phi = \frac{-\pi + \pi' - \pi''}{m-1}$ quae quia tam magna deſiderari ſolet, quam fieri poteſt, curandum eſt, vt fractiones π et π'' obtineant valores negatiuos eosque maximos, qui tamen $\frac{1}{4}$ vel $\frac{1}{3}$ ſuperare nequeunt, ac ſi forte hoc fieri nequeat, et alteruter debeat eſſe poſitiuus, tum vt is fiat quam minimus, erit efficiendum.

Scholion. 2.

131. His iam praemiſſis videamus, quo modo ſuperiori incommodo, quo lentes perfectae pro hoc teleſcopiorum genere ineptae ſunt deprehenſae, remedium afferri poſſit. Conſiderabimus igitur teleſcopium vt tribus lentibus compoſitum, ac duas priores prorſus vniamus vt interuallum $\alpha + b$ euaneſcat ſicque lens obiectiua fiat duplicata, verum nunc ſingula elementa ita definiamus, vt non pro ſola obiectiua vtraque confuſio deſtruatur, ſed pro toto teleſcopio.

Quo-

Quoniam vero ad hoc duplici vitri ſpecie opus eſt, adhibere cogimur binas illas ſpecies anglicas, ſcilicet vitrum coronarium et chryſtallinum. Vnde duo potiſſimum problemata naſcuntur, prout vel prima lens ex coronario, ſecunda vero ex chryſtallino, vel contra prior ex chryſtallino, ſecunda vero ex coronario fuerit paranda; de tertia autem lente oculari perinde fere erit, ſiue eam ex vitro coronario ſiue ex chryſtallino conficere velimus, dummodo ea vtrinque aeque concaua reddatur, quandoquidem ea hoc modo maximam aperturam admittit, a qua campus apparens dependet.

Problema 3.

132. Si teleſcopii lens obiectiua ſit duplicata ac prior quidem ex vitro coronario, poſterior vero ex chryſtallino parata, lens autem ocularis etiam ex vitro coronario; conſtructionem huius teleſcopii pro quauis multiplicatione m deſcribere.

Solutio.

Cum igitur hic ſit $a + b = 0$; ſiue $\alpha = -b$; et $\frac{\alpha}{b} = -1$ erit multiplicatio $m = -\frac{\beta}{c}$ ſeu $c = -\frac{\beta}{m}$ vbi littera β exprimit diſtantiam focalem ipſius lentis obiectiuae duplicatae ideoque, vt ex probl. 1 patet, debet eſſe poſitiua; vnde lens ocularis erit concaua. Cum igitur ſit $b = \frac{\beta}{B}$; $q = \mathfrak{B}b = \frac{\mathfrak{B}\beta}{B} = \frac{\beta}{B+1}$ erit $\alpha = \frac{-\beta}{B}$

et litterae μ et ν, vna cum μ'', ex refractione $n = 1,53$; litterae vero μ' et ν' ex refractione $n = 1,58$ ſunt ſumendae; vnde pro confuſione ex apertura lentium deſtruenda habebimus hanc aequationem:

$$\mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu\lambda''}{mB^3} - \frac{\mu'\nu'}{\mathfrak{B}B} = 0$$

cum autem ſit $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 7 : 10$ atque $n'' = n$ ob valorem diſtantiae O negatiuum pro margine colorato tollendo nanciſcimur hanc aequationem:

$$\pi'(3B + 10) = 10.\pi.$$

deinde vero pro hac confuſione penitus tollenda ſatisfieri oportet huic aequationi:

$$0 = -7 + \frac{10(B+1)}{B} - \frac{7}{mB}$$

ſeu $0 = -7B + 10.(B+1) - \frac{7}{m}$

vnde reperitur $B = \frac{7-10.m}{3.m}$, $\mathfrak{B} = \frac{10m-7}{7m-7}$, ex qua littera B perfecte determinatur, ita, vt ex prima aequatione tantum litterae λ et λ' definiendae reſtent, quia ob lentem ocularem vtrinque aequalem, λ'' iam definitur. Inde igitur commodiſſime definitur numerus λ':

$$\lambda' = \frac{\mu\mathfrak{B}^3\lambda}{\mu'} + \frac{\mu\mathfrak{B}^3\lambda''}{m\mu'B^3} - \frac{\nu'\mathfrak{B}^2}{B}$$

in qua quidem aequatione λ pro lubitu accipi poſſet, ſed ne λ' vnitatem nimis ſuperet, conueniet ſumi $\lambda = 1$ ſicque omnia iam erunt determinata, ita, vt nihil amplius ſuperſit, quod ex aequatione media poſſet determinari,

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minari, quia ratio litterarum π et π' ex praemiſſis iam datur. Cum enim ſit $b = \frac{\beta}{B} = \frac{\pi\Phi}{\mathfrak{B}\pi - \Phi} = \frac{-\beta\Phi}{B(\mathfrak{B}\pi - \Phi)}$ hincque $\pi = 0$, et cum pro campo apparente ſit $\Phi = \frac{-\pi + \pi'}{m-1}$ erit $\pi' = (m-1)\Phi$ vnde pro ſecunda aequatione prodit

$$0 = (m-1)\Phi(3B + 10)$$

quod cum fieri nequeat, praeter caſum $3B + 10 = 0$ ſeu [illegible] margo coloratus tolli nequit, niſi multiplicatio ſit maxima ideoque pro maioribus multiplicationibus erit inſenſibilis, ad quem caſum cum haec teleſcopia accommodari conueniat, margo coloratus non erit metuendus, ſufficietque, ſi primae et tertiae aequationi ſatisfecerimus. Inuentis igitur quantitatibus B, λ et λ' pro data multiplicatione m gradus claritatis y aſſumatur, quo contenti eſſe voluerimus; indeque habebitur ſemidiameter aperturae primae lentis x. Si deinde diſtantiam focalem totius lentis obiectiuae, quae eſt aequalis β, vt indefinitam ſpectemus; habebimus inde 1° diſtantiam focalem prioris lentis; $\alpha = \frac{-\beta}{B}$ et pro poſteriore diſtantias determinatrices $b = \frac{\beta}{B}$ et β; ex quibus cum numeris λ et λ' vtramque lentem poterimus conſtruere; in qua conſtructione notetur minimus radius ſiue conuexitatis ſiue concauitatis eiusque parti quintae vel etiam quartae aequetur $x = my$; vnde ipſa quantitas β in digitis determinabitur. Hinc porro colligimus

 diſtan-

diſtantiam focalem lentis ocularis $= c = \frac{-\beta}{m}$; ex qua ſi huic lenti vtrinque figura aequalis tribuatur, vt ſcilicet maximae aperturae fiat capax radius iſtius curuaturae erit $= -\frac{2(n-1)\beta}{m}$ vti ſupra iam oſtendimus §. 61, vbi etiam inuenimus pro hac lente fore $\sqrt{(\lambda'' - 1)} = \frac{\sigma-\rho}{2\tau}$; vnde valor ipſius λ'' definitur.

Coroll. 1.

Cum hic diſt[illegible] poſt vltimam lentem O'' fiat negatiua; ideoque oculus huic lenti immediate adplicari debeat, in formula campum apparentem declarante $\Phi = \frac{-\pi+\pi'}{m-1}$ fractio π' ſumi debet $= \frac{\omega}{e}$ vt ſcilicet campum inueniamus, quem vno obtutu conſpicimus; expediet autem, aperturam iſtius lentis tantam fieri, quantam curuatura facierum admittit, ſicque nihil obſtat, quominus ipſi π' valor $= \frac{1}{4}$ vel $= \frac{1}{3}$ tribuatur.

Coroll. 2.

134. Quod hic de valore vltimae litterarum π, π', π'' etc. notauimus, latiſſime patet, vt ſcilicet ei ſemper valor $\frac{1}{3}$ vel $\frac{1}{4}$ tribui poſſit, dummodo in computo campi apparentis eius valor ad $\frac{\omega}{e}$ imminuatur, ſi quidem hic fuerit minor; quippe quo modo campus vno obtutu conſpectus definitur. Quando autem apertura lentis ocularis maior fuerit pupilla, tum pupilla eam quaſi peragrando ſucceſſiue totum campum con-

conſpiciet, quem verus valor ipſius π' definit ſicque in poſterum hanc limitationem a pupilla petitam penitus omittere poterimus; dummodo notetur, caſu, quo π' maius, quam $\frac{\omega}{c}$, hunc campum non vno obtutu apparere.

Coroll. 3.

135. Hoc igitur pacto teleſcopium adipiſcimur primi generis, quod obiecta ſine vlla confuſione ſiue ab apertura lentium ſiue a diuerſa radiorum natura oriunda repraeſentabit, ita, vt in illo nihil amplius poſſit deſiderari, niſi quod campus apparens nimis ſit exiguus; quo tamen defectu omnia teleſcopia tam Newtoniana, quam Gregoriana aeque laborant.

Scholion. I.

136. Si haec ad praxin accommodare velimus, inchoandum erit a valore litterae B, quem tertia aequatio ſuppeditat, ſcilicet $B = \frac{7 - 10 m}{3 m}$, qui ſtatim atque m ſit numerus modice magnus, abit in $B = -\frac{10}{3}$ quia autem hic valor $-\frac{7}{3}$ deriuatus eſt ex Dollondi experimentis, vnde rationem $\frac{dn}{n-1} . \frac{dn'}{n'-1} = 7 : 10$ deduximus, nemo certe arbitrabitur, hanc rationem tam exacte veritati reſpondere, vt non ſatis notabiliter ab ea diſcrepare poſſit; quam ob cauſſam ridiculum plane foret, ſi circa valorem huius litterae B nimis ſcrupuloſi eſſe vellemus; neque etiam res ipſa tantam preciſionem exigere

gere videtur, cum iam plurimum praeſtitiſſe is ſit cenſendus, qui hanc confuſionis ſpeciem, quae hactenus nullo plane modo imminui poſſe eſt credita, plurimum imminuere potuerit, etiamſi ad nihilum non reduxerit, audacter igitur ſtatuere poterimus, $B = -\frac{10}{9}$ pro quacunque multiplicatione, indeque tantum ſupereſt, vt formula pro λ' inuenta euoluatur; in quo nihil omnino negligere licebit; quoniam vt ſupra iam inuenimus ſolus terminus $\frac{\mu\mathfrak{B}'\lambda''}{m\mu'\mathfrak{B}^3}$ tanti erat momenti, vt a lente obiectiua perfecta optatus effectus exſpectari non potuerit.

Scholion 2.

137. Quoniam in ſequentibus plurimum intererit, vt lentibus ocularibus eiusmodi figura tribuatur, quae maximae aperturae ſit capax, hocque manifeſto eueniat, ſi ambae huius lentis facies reddantur aequales: pro huiusmodi lente valor litterae λ ita definietur, vt fiat $\sqrt{\lambda - 1} = \frac{\sigma - \varrho}{2\tau}$ quem igitur pro praecipuis vitri ſpeciebus hic exhibeamus.

n.	$\sqrt{\lambda - 1}$.	λ.
1. 53	0, 77464.	1. 60006.
1. 55	0, 79367.	1. 62991.
1. 58	0, 82125.	1. 67445.

Cum igitur nunc habeamus valorem $\lambda'' = 1,60006$, per ea, quae in problemate ſunt conſtituta, habebimus

$\mu = 0.$

$\mu = 0.9875$; $\mu' = 0.8724$; $\nu' = 0.2529$. ſumto $B = -\frac{10}{3}$ et $\mathfrak{B} = +\frac{10}{7}$; aequatio prima reſoluenda induet hanc formam:

$$\lambda' = 3,3001.\lambda - \frac{0.1428}{m} + 0.1548$$

ex qua ne valor ipſius λ' praeter neceſſitatem nimis magnus prodeat, ſtatuamus $\lambda = 1$, fietque

$$\lambda' = 3,4549 - \frac{0.1428}{m}$$

cuius aequationis vſum in aliquot exemplis oſtendamus.

Exempl. I.

138. Huiusmodi teleſcopium conſtruere, quod obiecta vicies quinquies aucta repraeſentet, ſeu ſit $m = 25$. Cum ſit $\lambda = 1$, erit $\lambda' = 3,4492$ et $\lambda' - 1 = 2,4492$ et $\surd(\lambda' - 1) = 1,5649$; atque hinc ſequens ſingularum lentium conſtructio colligetur:

I. Pro lente prima ex vitro coronario facta

ob eius diſtantiam focalem $p = a = +\frac{3\beta}{10}$ et $\surd(\lambda - 1) = 0$ fiet

$$\text{radius faciei} \begin{cases} \text{anter.} = 0.1807.\beta \\ \text{poſter.} = 1.3239.\beta \end{cases}$$

II. Pro ſecunda lente ex vitro chryſtallino

cum ſint diſtantiae determinatrices $b = \frac{\beta}{B} = -\frac{3}{10}\beta$, et litterae $\varrho = 0.1413$, $\sigma = 1,5827$, $\tau = 0,8775$

et $\sqrt{(\lambda' - 1)} = 1,5649$ ſi pro radiis anterioris et poſterioris faciei ponamus litteras F et G, habebimus

$$F = \frac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda'-1)}}$$

$$G = \frac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda'-1)}}$$

atque hinc

$$\frac{1}{F} = \frac{3\sigma - 10.\rho \mp 7.\tau.\sqrt{(\lambda'-1)}}{3\beta}$$

$$\frac{1}{G} = \frac{3\rho - 10.\sigma \pm 7.\tau.\sqrt{(\lambda'-1)}}{3\beta}$$

quibus euolutis prodit

$$\frac{1}{F} = \frac{3.3351 \mp 9.6124}{3\beta}$$

$$\frac{1}{G} = \frac{-15.4031 \pm 9.6124}{3\beta}$$

vt igitur radii non nimis fiant parui, vti oportet ſignis ſuperioribus, vnde obtinebimus

$$\frac{1}{F} = \frac{-6.2773}{3\beta};\quad F = -0.4779.\beta$$

$$\frac{1}{G} = \frac{-5.7907}{3\beta};\quad G = -0.5180.\beta$$

III. Pro tertia lente oculari

ex vitro coronario paranda conſtructio eſt facillima, dum vtriusque faciei radius eſſe debet $= 2(n-1)r = -1,06.r = -0,00424.\beta$.

Binae priores lentes ſibi inuicem immediate iunguntur, vt vnam quaſi lentem conſtituant, cuius aperturae ſemidiameter maior eſſe nequit, quam quarta circi-

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cititer pars radii minimi quae eſt $= 0.0452\,\beta$, & habebimus $x = 0.0452\,\beta$. Debet autem eſſe $x = my$, denotante y gradum claritatis atque iam notauimus ſtatui poſſe $y = \frac{1}{25}$. dig. ita, vt hoc caſu habeamus $x = \frac{1}{2}$ dig. quo circa valor ipſius β ita determinabitur, vt ſit $\beta = 11, 1$ dig. ſaltim β hoc limite non debet capi minus vnde ſuperiores menſurae abſolute innoteſcunt. Campi autem apparentis ſemidiameter ob $\pi = 0$ erit $\Phi = \frac{\pi'}{m-1} = \frac{\pi'}{24}$; ſumtoque $\pi' = \frac{1}{4}$ erit in minutis primis $35\frac{3}{4}$ min. quem campum oculus vno obtutu cerneret, ſi ſemidiameter pupillae eſſet $\pi' r. = 0.1120$. Quanto autem eſt minor, tanto minorem quoque campum vno obtutu videbit. Longitudo autem huius teleſcopii erit $= 10\frac{3}{4}$ digit.

Scholion.

139. Hoc ergo teleſcopium ad praxin ſatis accommodatum videtur, cum eius longitudo minor ſit vndecim digitis et tamen vicies quinquies obiecta augeat, campo apparente non adeo exiguo exiſtente; hincque etiam patet quantum lens perfecta hic immutari debuerit, vt etiam confuſionem a lente oculari oriundam tolleret. Verum hic notandum eſt, conſtructionem huius inſtrumenti ſummam artificis ſollertiam requirere, minimumque errorem commiſſum totum opus irritum reddere quare non niſi poſt plura tentamina ſucceſſus ſperari poterit. Multo maiore autem ſollertia erit opus, ſi maiorem quoque multipli-

cationem deſideremus, vti ex ſequenti exemplo erit manifeſtum.

Exemplum II.

140. Huiusmodi teleſcopium conficere, quod obiecta quinquagies multiplicet ſeu ſit $m = 50$.

Erit pro hoc caſu $\lambda' = 3.4521$ et $\sqrt{(\lambda' - 1)} = 1.5659$ qui valor praecedentem ſuperat $\frac{1}{1504}$ hoc eſt, ſui parte $\frac{1}{1504}$, ita, vt ſuperior formula $\sqrt{(\lambda' - 1)}$ per $1 + \frac{1}{1504}$ multiplicata praebeat praeſentem valorem et cum reliqua elementa maneant, vt ante, erit

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = 0.1807.\ \beta \\ \text{poſter.} = 1.3239.\ \beta \end{cases}$

II. Pro ſecunda lente

habebimus

$$\frac{1}{F} = \frac{3.5351 \mp 0.6185}{3\beta}$$

$$\frac{1}{G} = \frac{-15.4032 \pm 0.6185}{3\beta}$$

ſumtisque ſignis ſuperioribus habebimus

$$\frac{1}{F} = \frac{-6.2834}{3\beta};\ F = -0.4774\ \beta$$

$$\frac{1}{G} = \frac{-5.7846}{3\beta};\ G = -0.5186.\ \beta$$

quae duae lentes iunctae aperturam admittent, cuius ſemidiameter $= 0.0452.\ \beta$; quo ſcilicet maior non debet

debet eſſe valor $x = my = 1$. dig. quare capi debebit β maius, quam 22, 1. dig.

III. Pro lente oculari,

cuius diſtantia focalis eſt $= \frac{-\beta}{m} = \frac{-\beta}{50}$ radius vtriusque faciei erit $= -\frac{2(n-1)\beta}{50} = -0,0212$. & ſumto autem $\pi' = \frac{1}{4}$ erit aperturae eius ſemidiameter $x = +\frac{\beta}{858}$ $= 0,110$. dig. vnde ſemidiameter campi apparentis fit $\Phi = \frac{\pi'}{m-1} = \frac{1}{196}$ ſiue angulus $\Phi = 17\frac{1}{2}$ min. prim. Longitudo denique huius teleſcopii erit $= \beta + r = 21,658$ ſiue $21\frac{2}{3}$ dig.

Scholion.

141. In hoc exemplo conſtructio lentis ſecundae vix diſcrepat a praecedente; vnde patet, quam adcurate menſurae inuentae obſeruari debeant vt effectus voto reſpondeat facillimeque euenire poſſe, vt quae lens obiectiua datae cuidam multiplicationi deſtinatur, ea longe alii multiplicationi inſeruiat; quare quantamcunque etiam ſollertiam artifex adhibuerit, multiplicatio cui conuenit, explorari debet, dum ſcilicet ei ſucceſſiue aliae atque aliae lentes oculares adiunguntur; tum enim pro certa quadam multiplicatione fieri poterit, vt teleſcopium egregium effectum producat, hanc ob cauſſam ſuperſedeamus altero caſu ſupra memorato, quo pro lente obiectiua lens prior ex vitro chryſtallino, poſterior ex coronario parari debebat, quoniam

 haec

haec quae euoluimus, ſufficere videntur et multo magis expediet pro lente obiectiua lentem triplicatam exhibere eamque talem, cuius prima et tertia lens ex vitro chryſtallino, media ex coronario ſit confecta, quia iam ſupra hinc aptiſſima lens perfecta eſt nata.

Problema 3.

142. Si lens obiectiua teleſcopii ſit triplicata, cuius prima et tertia lens ex vitro chryſtallino, media vero ex coronario ſit conficienda, lens autem ocularis etiam ex vitro coronario; huius teleſcopii conſtructionem deſcribere, vt omni confuſione careat.

Solutio.

Hoc igitur teleſcopium ex quatuor omnino lentibus conſtabit, pro quibus erit $n = 1,58$; $n' = 1,53$; $n'' = n$ et $n''' = n'$. et quia tres priores lentes in vnam quaſi coaleſcere debent, erit $\alpha + b = 0$; et $\mathfrak{B} + c = 0$; ſiue $\frac{\alpha}{b} = -1$; et $\frac{\beta}{c} = -1$; quare cum ſit multiplicatio $m = -\frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$ erit $m = \frac{-\gamma}{d}$ ſeu $d = \frac{-\gamma}{m}$ reliquae vero litterae ſimili modo per γ exprimi poterunt, ſcilicet $c = \frac{\gamma}{C}$; $\mathfrak{B} = \frac{-\gamma}{C}$; $b = \frac{-\gamma}{BC}$ et $\alpha = \frac{\gamma}{BC}$, ex quibus diſtantiae focales oriuntur

$$p = \frac{\gamma}{BC};\ q = \frac{-\mathfrak{B}\gamma}{BC};\ r = \frac{\mathfrak{C}\gamma}{C};\ s = \frac{-\gamma}{m}.$$

Quibus praemiſſis pro confuſione ex apertura lentium orta deſtruenda habebimus hanc aequationem:

μλ-

$$\mu\lambda - \frac{\mu'}{\mathfrak{B}}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{\mu''}{\mathfrak{C}.B^5}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu'}{C}\right) - \frac{\mu'''\lambda'''}{B^3C^3.m} = 0$$

quae ob $\mu'' = \mu$; $\nu'' = \nu$ et $\mu''' = \mu'$ euoluta dabit:

$$0 = \begin{cases} \mu\lambda - \frac{\mu'\lambda''}{\mathfrak{B}^3} + \frac{\mu\lambda''}{B^3\mathfrak{C}^3} - \frac{\mu'\lambda'''}{B^3C^3.m} \\ \quad - \frac{\mu'\nu''}{B\mathfrak{B}^3} + \frac{\mu\nu}{B^3C\mathfrak{C}}. \end{cases}$$

Ne nimis rationi $7 : 10$, qua ante vſi ſumus, inhaereamus, ponamus in genere $\frac{dn}{n-1} = \zeta$, et $\frac{dn'}{n'-1} = \eta$, vt ſit circiter $\zeta : \eta = 10 : 7$; deinde quia noſtro caſu ſit $\pi = 0$ et $\pi' = 0$ pro margine colorato abolendo habebimus

$$0 = \zeta - \frac{\eta(B+1)}{B} + \frac{\zeta(C+1)}{BC}$$

ſiue:

$$\zeta(1 + C + BC) = \eta(B + 1)C$$

ex qua quid concludere liceat deinceps videbimus: Tertiam aequationem nobis praebet deſtructio tota huius confuſionis, ſcilicet iſtam:

$$0 = BC + C + \frac{\zeta m - \eta}{(\zeta - \eta)m}$$

Ponatur breuitatis gratia $\frac{\zeta m - \eta}{(\zeta - \eta)m} = \vartheta$, eritque

$$0 = BC + C + \vartheta$$

vnde prodit $C = \frac{-\vartheta}{B+1}$

vel $B = -1 \frac{-\vartheta}{C}$

Cum autem ſecunda aequatio abeat in hanc formam:

B C-

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$$BC + C + \frac{\zeta}{\zeta-\eta} = 0$$

ambabus ſimul ſatisfieri nequit, niſi ſit $\vartheta = \frac{\zeta}{\zeta-\eta}$; hoc eſt niſi ſit $\frac{\zeta-\eta}{m} = \zeta$, ſiue $\zeta m - \zeta m = \eta = 0\, m$; ſiue $m = \infty$, prorſus vt in caſu praecedente. Regrediemur igitur ad noſtram aequationem primam, in qua ſiue loco B ſiue loco C valorem debitum ſubſtituamus. Cum autem rationem $\zeta : \eta$ non tam exacte noſſe licet, ſufficiet valores proximos ſumſiſſe, hunc in finem, in tertia aequatione terminum per m diuiſum negligamus et habebimus:

$$0 = BC + C + \frac{\zeta}{\zeta-\eta};\ \text{ſiue}$$

$$0 = BC + C + \tfrac{10}{3};\ \text{hincque}$$

$$C = \frac{-10}{3(B+1)}\ \text{et}$$

$$C + 1 = \frac{3B-7}{3(B+1)}$$

quibus ſubſtitutis et diuiſione facta per $(B+1)^3$ prodit

$$\begin{aligned} 0 = & -1000\,\mu\lambda.\,\mathfrak{B}^3 + 1000\,\mu'\lambda' \\ & + \mu\lambda''(10\mathfrak{B}-7)^3 - \frac{27\,\mu\,\lambda'''}{m} \\ & + 1000.\,\mu'\nu'\mathfrak{B}(1-\mathfrak{B}) \\ & - 30\,\mu\nu(10\mathfrak{B}-7)(1-\mathfrak{B}) \end{aligned}$$

quae ſumto $\lambda'' = \lambda$ fit aequatio quadratica, ex qua $\mathfrak{B}$ definitur.

Ex

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Ex hac autem aequatione cognoſcimus, huiusmodi ſubſtitutionem etiam in genere ſuccedere; cum enim ſit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ ob $B + 1 = \frac{1}{1-\mathfrak{B}}$ fiet $C = -\vartheta(1-\mathfrak{B})$ et $C + 1 = 1 - \vartheta + \vartheta\mathfrak{B}$ hincque $\mathfrak{C} = \frac{-\vartheta(1-\mathfrak{B})}{1-\vartheta+\vartheta\mathfrak{B}}$ et ipſa aequatio prima reducetur ad hanc formam, ſi ſcilicet per $\mathfrak{B}^3$ multiplicetur:

$$0 = \mu\lambda\mathfrak{B}^3 - \mu'\lambda' - \mu\lambda''(\mathfrak{B} - 1 + \tfrac{1}{\vartheta})^3$$
$$+ \frac{\mu''\lambda'''}{n.\vartheta^3} - \mu'\nu'\mathfrak{B}(1 - \mathfrak{B})$$
$$+ \frac{\mu\nu(1-\mathfrak{B})(\mathfrak{B} - 1 + \frac{1}{\vartheta})}{\vartheta}$$

exiſtente $\vartheta = \frac{\zeta m - \eta}{(\zeta - 1)m}$; ac ſi ponatur $\vartheta = \frac{10}{3}$ praecedens aequatio ſponte prodit.

Statuamus igitur $\lambda'' = \lambda$ et euolutio huius aequationis ſequentem praebebit aequationem quadraticam ſecundum poteſtates litterae $\mathfrak{B}$ diſpoſitam

$$\mathfrak{B}^2[3\mu\lambda[1 - \tfrac{1}{\vartheta}] + \mu'\nu' - \tfrac{\mu\nu}{\vartheta}]$$
$$+ \mathfrak{B}[-3\mu\lambda[1 - \tfrac{1}{\vartheta}]^2 - \mu'\nu' + \tfrac{\mu\nu}{\vartheta}[2 - \tfrac{1}{\vartheta}]]$$
$$+ \mu\lambda[1 - \tfrac{1}{\vartheta}]^3 - \mu'\lambda' + \frac{\mu'\lambda'''}{n\vartheta^3} - \mu\nu[1 - \tfrac{1}{\vartheta}] = 0.$$

ex qua $\mathfrak{B}$ definiri debet.

Nunc igitur ſtatuamus $\vartheta = \frac{10}{3}$; tum vero $\lambda = \lambda' = \lambda'' = 1$. et pro lente oculari ſit $\lambda''' = 1.60006$. tum vero $\mu = 0.8724$ et $\nu = 0.2529$ $\mu' = 0.9875$, $\nu' = 0$.

$\nu' = 0.2196$; vnde fit $l\mu\nu = 9.3436645$;
$l\mu'\nu' = 9.3361694$.

Pro termino $\mathfrak{B}^2$.

$$\mathfrak{B}^2 \left(\tfrac{21}{10}\mu + \mu'\nu' - \tfrac{1}{10}\mu\nu\right)$$

$$\mathfrak{B}^2 (+1.83204 - 0.06618)$$

$$\begin{array}{r} +0.21685 \\ \hline 2.04889 \\ 0.06618 \\ \hline 1.98271 \end{array}$$

$$+1.98271.\ \mathfrak{B}^2 - 1.38675\ \mathfrak{B} - 0.84271 + 0.\tfrac{01266}{m} = 0.$$

qua diuiſa per 1.98271 fiet

$$\mathfrak{B}^2 = 0.69942.\ \mathfrak{B} + 0.42503 - \tfrac{0.02152}{m}$$

cuius reſolutio ſuppeditat

$$\mathfrak{B} = 0.34971 \pm \sqrt{\left(0.54736 - 0.\tfrac{02152}{m}\right)} \text{ vel}$$

$$\mathfrak{B} = 0.34971 \pm \left(0.73983 - 0.\tfrac{01454}{m}\right)$$

vnde bini ipſius $\mathfrak{B}$ valores erunt

$$\text{I. } \mathfrak{B} = 1.08954 - 0.\tfrac{01454}{m}$$

$$\text{II. } \mathfrak{B} = -0.39012 + 0.\tfrac{01454}{m}$$

Coroll. I.

143. Tribus igitur prioribus lentibus immediate coniunctis exiſtit lens obiectiua triplicata, cuius diſtan-

tia

tia focalis erit aequalis γ, ex qua radios ſingularum facierum definire oportet, inter quos notetur minimus, qui ſit $= i\gamma$, cuius pars quarta $= \frac{1}{4} i\gamma$ dabit ſemidiametrum aperturae, quam iſta lens obiectiua admittit.

Coroll. 2.

144. Porro vero ex multiplicatione m data et gradu claritatis y definitur ſemidiameter aperturae lentis obiectiuae $x = my$ idque in digitis, ſumendo v. gr. $y = \frac{1}{50}$ dig. vnde habebitur iſta aequatio $my = \frac{1}{4} i . \gamma$, ex qua per menſuram abſolutam colligitur $\gamma = \frac{4 . my}{i}$.

Coroll. 3.

145. Cum autem lens ocularis debeat eſſe vtrinque aeque concaua, vt ſit $\lambda''' = 1,60006$, erit eius diſtantia focalis $= d = \frac{-\gamma}{m}$; vnde radius vtriusque faciei ſtatui debet $= -\frac{2(n'-1)}{m} . \gamma = \frac{-1,2}{m}\gamma$, cuius aperturae ſemidiameter ſumi poteſt quater minor, vt ſit $x = \frac{\gamma}{4m}$.

Exempl. I.

146. Poſita multiplicatione $m = 25$ conſtruere huiusmodi teleſcopium ex valore priore pro littera $\mathfrak{B}$ inuento.

Cum igitur ſit $m = 25$, erit $\mathfrak{B} = +1,08896$ ex quo ſequitur $B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -12,24100$ et log. B $= 1.0878169$. Porro $C = -\vartheta(1-\mathfrak{B}) = 0,2965$ hincque ob $BC + C + \vartheta = 0$ colligimus $BC = -C - \vartheta$ $= +\vartheta - \vartheta\mathfrak{B} - \vartheta = -\vartheta\mathfrak{B} = -3,6298$ et $\mathfrak{C} = \frac{C}{C+1} = 0.22869$.

Sint nunc radii faciem primae lentis F et G; ſecundae F′ et G′ ac tertiae F″ et G″ ob diſtantias determinatrices.

$$a = \infty;\ b = \frac{-\gamma}{BC};\ c = \frac{\gamma}{C}$$

$$\alpha = \frac{\gamma}{BC};\ \beta = \frac{-\gamma}{C};\ \gamma = \gamma$$

et numeros $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$ erit

$$F = \frac{\alpha}{\sigma} = \frac{\gamma}{BC.\sigma};\ G = \frac{\alpha}{\varrho} = \frac{\gamma}{BC\varrho}$$

$$F' = \frac{b\beta}{\varrho'\beta + \sigma' b} = \frac{-\gamma}{BC\varrho' + C\sigma'}$$

$$G' = \frac{b\beta}{\varrho' b + \sigma'\beta} = \frac{-\gamma}{BC\sigma' + C\varrho'}$$

$$F'' = \frac{c\gamma}{\varrho\gamma + \sigma c} = \frac{\gamma}{C\varrho + \sigma}$$

$$G'' = \frac{c\gamma}{\sigma\gamma + \varrho c} = \frac{\gamma}{C\sigma + \varrho}$$

Cum igitur ſit $\varrho = 0,1413$; $\sigma = 1,5827$ et $\varrho' = 0,2266$; $\sigma' = 1,6602$ calculo inſtituto obtinebimus:

$$F = -0,1740\,\gamma;\ G = -1,9497\,\gamma$$

$$F' = +3,0276\,\gamma;\ G' = +0,1678\,\gamma$$

$$F'' = 0,6155\,\gamma;\ G'' = +1,6378\,\gamma$$

At pro lente oculari radius vtriusque faciei erit $= -0,0424\,\gamma$. Inter illos autem radios minimus eſt $0,1678\,\gamma$, cuius parti quartae $0,0419\,\gamma$ ſi aequetur $x = my = 25\,y = \frac{1}{2}$ dig. prodibit $\gamma = 12$ dig. Longitudo teleſcopii $\gamma\left(1 - \frac{1}{m}\right) = 11,52$ dig. et ſemidiameter campi apparentis ob $\pi = 0$ et $\pi' = 0$ fiet $\Phi = -\frac{\pi''}{m-1}$ et ſumto $\pi'' = -\frac{1}{4}$ erit $\Phi = \frac{1}{96}$ in part. rad.

rad. vel $\Phi = 35\frac{3}{4}$ min. prim. quem oculus vno obtutu conſpiceret, ſi ſemidiameter pupillae aequalis eſſet ſemidiametro aperturae lentis ocularis hoc eſt $= \frac{1}{4}\frac{\gamma}{m} = \frac{\gamma}{105} = \frac{1}{24}$ dig. alioquin ſi pupilla minor eſſet, in eadem ratione campus deberet imminui.

Exempl. II.

147. Poſita multiplicatione $m = 50$ conſtruere huiusmodi teleſcopium ex valore priore ipſius $\mathfrak{B}$.

Cum ſit $m = 50$ erit $\mathfrak{B} = +1,08925$ ex quo ſequitur $B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -12,2045$ et log. $B = 1.0865194$. Porro $C = 0.2975$ et $BC = -3,6308$. Cum igitur praecedentes formulae etiam nunc locum habeant, radii ſingularum facierum ita reperiuntur expreſſi:

$$F = -0,1740.\gamma;\ G = -1.9493.\gamma.$$
$$F' = +3.0410.\gamma;\ G' = +0.1677.\gamma.$$
$$F'' = 0,6155.\gamma;\ G'' = +1.6337.\gamma.$$

Horum radiorum minimus eſt 0,1677, cuius parti quartae 0.0419 γ aequalis ſtatui debet ſemidiameter aperturae $x = my = 1$ dig. ex quo definitur $\gamma = \frac{1}{0,0419} = 23,86$ dig. ita, vt ſtatui poſſit $\gamma = 24$ dig. Tum autem erit diſtantia focalis lentis ocularis $= \frac{\gamma}{m} = \frac{12}{25}$ dig. radiusque vtriusque faciei $1,06.\frac{12}{25} = 0.508$ dig.

Longitudo ergo huius teleſcopii erit $= \gamma(1-\frac{1}{m}) = 23,04$ dig. et ſemidiameter campi apparentis $\Phi = \frac{-\pi\gamma'}{49} = \frac{11}{196}$ et in minutis primis $\Phi = 17\frac{1}{2}$ minut.

Scholion.

148. Ad maiorem multiplicationem hunc calculum non proſequor, quia differentia prodiret tam exigua, vt ab artificibus vix videatur exſequenda; quare eadem exempla etiam ab altero valore pro $\mathfrak{B}$ inuento euoluamus.

Exempl. III.

149. Poſita multiplicatione $m = 25$, conſtruere huiusmodi teleſcopium ex valore poſteriore ipſius $\mathfrak{B}$.

Cum ſit $m = 25$, erit $\mathfrak{B} = -0,38954$ et $1 - \mathfrak{B} = 1,38954$, vnde fit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}} = -0.280369$ et log. $B = 9.4476810$; deinde fiet $C = -\vartheta(1-\mathfrak{B})$ $= -4.6318$ et $BC = +1,29850$.

Quia igitur formulae pro radiis facierum manent, vt ſupra, inueniemus eos, vt ſequitur:

$F = 0\ 486585\,\gamma$; $G = 5,45024\,\gamma$.
$F' = +0.13521.\,\gamma$; $G' = -0.90400.\,\gamma$
$F'' = +1.07723.\,\gamma$; $G'' = -0.13909.\,\gamma$.

Inter hos radios minimus eſt $0,13521\,\gamma$ cuius parti quartae $0,03380.\,\gamma$ aequari debet ſemidiameter aperturae $x = my. = \frac{1}{5}$ dig. vnde $\gamma = \frac{1}{5 \cdot 0,06780} = 15$ dig.; ita vt teleſcopii longitudo $= \gamma(1 - \frac{1}{m}) = 14\frac{2}{5}$ dig. diſtantia autem focalis lentis ocularis erit $= -\frac{3}{5}$ dig. ita, vt radius faciei vtriusque $= 0.6360.$ dig. et ſemidiameter campi apparentis erit vt ſupra, $\Phi = 35\frac{3}{4}$ min. qui ab oculo vno obtutu vel ſaltim ſucceſſiue conſpici poterit.

Exem-

Exempl. IV.

150. Poſita multiplicatione $m = 50$ conſtruere huiusmodi teleſcopium ex valore poſteriore ipſius $\mathfrak{B}$.

Cum ſit $m = 50$, erit $\mathfrak{B} = -0.38983$ et $1 - \mathfrak{B} = 1.38983$; vnde colligitur $C = -4,6328$ et $BC = +1,2995$.

Cum igitur formulae pro radiis facierum maneant eaedem, ex iis facto calculo nanciſcemur:

$F = 0.48621.\gamma$; $G = 5,44604.\gamma$
$F' = 0.13519.\gamma$; $G' = -0,90285.\gamma$
$F'' = 1.07747.\gamma$; $G'' = -0.13906.\gamma$.

Inter quos radios minimus eſt $0.13519.\gamma$ cuius parti quartae $0.03379.\gamma$ aequari debet ſemidiameter aperturae $x = my = 1.$ dig. vnde $\gamma = 29$ dig. et diſtantia focalis lentis ocularis $= -0,58$. dig. et radius vtriusque faciei $= 0,6148$. Longitudo ergo teleſcopii erit $= 28,42$ dig. et ſemidiameter campi $\Phi = 17\frac{1}{2}$ minut.

Scholion.

151. Etſi haec teleſcopia quatuor lentibus reuera conſtant, ea tamen quaſi tantum ex duabus lentibus compoſita ſpectare licet, propterea quod tres priores lentes in vnam coaluerunt, vt lens obiectiua fieret triplicata et meliore ſucceſſu loco lentium triplicatarum perfectarum ſupra traditarum vſurpanda; quandoquidem iam vidimus, lentibus illis perfectis ſolam ipſarum confuſionem vtriusque generis annihilari,

lari, ita, vt confuſio lentis ocularis etiam nunc tota ſubſiſteret, quamobrem lentes triplicatas hic in vſum vocatas data opera ita inſtruximus, vt non eſſent perfectae ſed vt iis etiam confuſio lentis ocularis ad nihilum redigeretur, quae ſi modo artifex exactiſſime perficere poſſet, nihil amplius deſiderari poſſe videretur. Verum duabus adhuc difficultatibus haec teleſcopia premuntur; altera eſt, quod tribus huiusmodi lentibus coniungendis craſſities ita fiat modica, vt non amplius tanquam euaneſcens ſpectari poſſit, quemadmodum calculus noſter poſtulat; vnde etiamſi artifex noſtras menſuras exactiſſime exſequi valeret, neutiquam tamen perfectus conſenſus inter theoriam et praxin ſperari poſſet; altera difficultas in anguſtia campi apparentis eſt poſita, maximeque eſt optandum, vt campo maior amplitudo concilietur; quo igitur huic duplici incommodo conſulamus, in ſequenti capite hanc inueſtigationem vlterius proſequamur, dum huius generis teleſcopiis reuera plures duabus lentes tribuemus, quae omnes a ſe inuicem certis interuallis ſint disiunctae, vbi inprimis in hoc erit inquirendum, num hoc modo etiam vtriusque generis confuſio aeque feliciter tolli poſſit; deinde vero num hoc modo campus apparens magis amplificari poſſit, ac ſi praeterea longitudo horum teleſcopiorum minor prodiret; tum certe iis ſummus perfectionis gradus conciliatus eſſet cenſendus.

CAPVT

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CAPVT V.

DE VLTERIORE TELESCOPIORVM PRIMI GENERIS PERFECTIONE VNA PLVRIBVSVE LENTIBVS ADJICIENDIS.

Problema 1.

152.

Si huiusmodi teleſcopium primi generis ex tribus lentibus a ſe inuicem ſeparatis ſit conficiendum, inueſtigare momenta, quibus ei maximus perfectionis gradus conciliari queat.

Solutio.

Manentibus perpetuo omnibus elementis vti in principio ſunt conſtituta, conſideremus primo aequationem $m = \frac{\alpha}{b} \cdot \frac{\beta}{c}$ in qua ambae fractiones $\frac{\alpha}{b}$ et $\frac{\beta}{c}$ debent eſſe negatiuae ac praeterea internalla $a + b$ ac $\beta + c$ poſitiua et quoniam nunc non debet eſſe $\frac{\alpha}{b} = -1$, ne binae priores lentes coaleſcant, ſtatuamus $\frac{\alpha}{b} = -k$, vt fiat $m = -k. \frac{\beta}{c}$ hincque $\mathfrak{b} = \frac{-mc}{k}$ ſiue $c = \frac{-\beta k}{m}$ et $\alpha = -bk$ vnde ob $\mathfrak{b} = \mathrm{B}\, b$ omnes hae diſtantiae per α ſequenti modo determinantur, $b = -\frac{\alpha}{k}$; $\mathfrak{b} = -\frac{\mathrm{B}\alpha}{k}$ et

et $c = + \frac{B\alpha}{m}$ exiſtente $\gamma = \infty$. Hinc igitur eſſe oportebit $\alpha\,(1 - \frac{1}{k}) > 0$; $\alpha\,B\,(\frac{1}{m} - \frac{1}{k}) > 0$. ſeu, quia m et k ſunt poſitiua $\alpha\,(k - 1) > 0$ et $\alpha\,B\,(k - m) > 0$ adeoque etiam $\frac{B(k-m)}{k-1}$ debet eſſe > 0, quocirca duo caſus erunt perpendendi, *prior Caſus*, quo α eſt quantitas poſitiua, tum debet eſſe $k > 1$; tum vero vel $k > m$ ſi B ſit poſitiuum vel $k < m$, ſi $B < 0$. *Altero caſu*, quo α eſt negatiuum, debet eſſe $k < 1$; tum vero vel $k > m$, ſi B ſit negatiuum vel $k < m$, ſi B ſit poſitiuum, vbi ob $m > 1$ illa conditio $k > m$ ſponte cadit. His igitur praemiſſis primo ad nihilum redigamus formulam pro ſemidiametro confuſionis ſupra datam:

$$0 = \mu\lambda + \frac{\mu' \cdot q}{\mathfrak{B}^2 \cdot p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''\lambda''}{B \cdot m}.$$

ſiue ob $q = -\frac{\alpha\mathfrak{B}}{k}$ et $p = \alpha$

$$0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right) + \frac{\mu''\lambda''}{B \cdot m}$$

quae redit ad hanc formam

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3 \cdot k} + \frac{\mu''\lambda''}{B^3 \cdot m} - \frac{\mu'\nu'}{\mathfrak{B}Bk}$$

Deinde vt margo coloratus tollatur ob $O = 0$ haec habetur aequatio

$$0 = \frac{dn}{n-1}\,B\,\pi' - \frac{dn'}{n'-1} \cdot \frac{1}{k}\left((B + 1)\,\pi' - \pi\right)$$

atque vt haec confuſio penitus euertatur habetur ex § 54.

$0 =$

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$$o = \frac{dn}{n-1} \cdot \frac{1}{p} + \frac{dn'}{n'-1} \cdot \frac{1}{k^2} \cdot \frac{1}{q} + \frac{dn''}{n''-1} \cdot \frac{1}{m^2} \cdot \frac{1}{r}$$

Ad has aequationes reſoluendas primo ratio inter π et π' debet definiri, id quod facillime praeſtabitur per formulas fundamentales in ipſo initio propoſitas: $\frac{\pi}{\Phi} = \frac{a+b}{q} = \frac{1-k}{\mathfrak{B}}$ et $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{Bx}{c} = m$ ex quibus colligitur $\pi = \frac{(1-k)\Phi}{\mathfrak{B}}$ et $\pi' = (m-1)\Phi + \pi = \frac{1-k+(m-1)\mathfrak{B}}{\mathfrak{B}}\Phi$ ita, vt ſit $\pi : \pi' = 1-k : 1-k+(m-1)\mathfrak{B} = 1 : 1 + \frac{m-1}{1-k} \cdot \mathfrak{B}$. deinde breuitatis gratia ſtatuamus

$$\frac{dn}{n-1} = N; \quad \frac{dn'}{n'-1} = N'; \quad \frac{dn''}{n''-1} = N''.$$

atque hinc IIda et IIItia aequatio transformabuntur in ſequentes:

$$\text{II.} \quad o = N.\, B(1-k+(m-1)\mathfrak{B})$$
$$- N'.\, \tfrac{1}{k}(B(1-k) + (B+1)(m-1)\mathfrak{B})$$

ſiue

$$o = (m-1)N\mathfrak{B} + (1-k)N - \frac{m-k}{k}N'.$$

$$\text{III.} \quad o = N - \frac{N'}{k\mathfrak{B}} + \frac{N''}{mB}$$

Ex vtraque harum aequationum definiri poteſt valor ipſius $\mathfrak{B}$.

Ex IIda

$$\mathfrak{B} = \frac{m-k}{(m-1)k} \cdot \frac{N'}{N} - \frac{1-k}{m-1}$$

Ex IIItia vero ſequitur

$$\mathfrak{B} = \frac{mN' - N''}{k(mN - N'')}$$

 videa-

Videamus, an poſterior valor ipſius $\mathfrak{B} = \frac{mN' - kN''}{k(mN - N')}$ cum conditione ante inuenta $\frac{B(k-1)}{k-1} > 0$ ſubſiſtere poſſit. Hunc in finem ob $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ quaeramus $1 - \mathfrak{B}$ fietque $1 - \mathfrak{B} = \frac{m(kN - N')}{k(mN - N')}$ eritque $B = \frac{mN' - kN''}{m(kN - N')}$ vnde conditio noſtra poſtulat, vt ſit $\frac{(mN' - kN'')(k - m)}{m(kN - N')(k - 1)} > 0$ quae ſi eſſet $N = N' = N''$ abiret in hanc $\frac{-(k-m)^2}{m(k-1)^2} > 0$ quod eſt impoſſibile, eatenus igitur tantum haec conditio locum habere poterit, quatenus litterae N, N′, N″ ſunt inaequales, id quod eueniet, ſi numerator prodeat poſitiuus, quod fit, ſi vterque eius factor vel fiat poſitiuus vel vterque negatiuus; priori caſu $mN' - kN'' > 0$ adeoque $k < m . \frac{N'}{N''}$ et $k > m$, quod fieri poteſt, ſi modo ſit $\frac{N'}{N''} > 1$ ſiue $N' > N''$.

Pro altero vero caſu, quo vterque factor eſt negatiuus, erit $k < m$ et $k > \frac{N'}{N''} . m$, quod fieri poteſt, ſi modo ſit $\frac{N'}{N''} < 1$ ſeu $N' < N''$; vnde patet pro vtroque caſu litteras N′ et N″ inaequales eſſe debere ſeu lentem IIdam et IIItiam ex diuerſis vitri ſpeciebus confici debere. In genere autem patet, k non multum ab m differre poſſe. Sequuntur haec ſi numerator ſtatuatur poſitiuus; ſi vero numerator ſit negatiuus, etiam denominatorem oportet eſſe negatiuum, pro quo etiam duos caſus habemus. Pro priori caſu ſi ſit $k > 1$ debet eſſe $kN < N'$ adeoque $k < \frac{N'}{N}$ pro poſteriori ſi $k < 1$ debet ſimul eſſe $k > \frac{N'}{N}$, pro quorum vtroque prima et ſecunda lens debent eſſe ex di-

uerſo

verſo vitro formatae. Verum ex his quatuor caſibus eum eligi conuenit, qui ambos valores pro $\mathfrak{B}$ inuentos proxime aequales reddat; denique autem poſtquam $\mathfrak{B}$ et k conuenienter definiuerimus, ex prima aequatione ſiue λ ſiue λ' quaeri debet, quia λ'' iam inde datur, quod lens ocularis debeat eſſe vtrinque aequaliter concaua.

Coroll. 1.

153. Quatuor illi caſus pro determinatione litterae k facile ad duas ſequentes conditiones reducuntur, nam

vel 1° k ſumi debet intra limites 1 et $\frac{N'}{N}$

vel 2° k ſumi debet intra limites m et $\frac{N'}{N''}$. m

ita vt numerus iſte k proxime vel vnitati vel multiplicationi m aequalis accipi debeat, quoniam fractiones $\frac{N'}{N}$ et $\frac{N'}{N''}$ parumper tantum ab vnitate differunt.

Coroll. 2.

154. Operae igitur pretium erit inueſtigare, caſus, quibus k ipſi alterutri limiti aequalis ſtatuitur

1°. Si $k = 1$ foret interuallum inter primam et ſecundam lentem $= 0$ et $\mathfrak{B} = \frac{mN'-N''}{mN-N''}$ et $B = \frac{mN'-N''}{m(N-N'')}$ et inter 2 et 3tiam lentem $= B\alpha\left(\frac{1-m}{m}\right)$ vnde colligitur vtrum α poſitiuum an negativum ſumi debeat.

 2°. Si

2°. Si $k = \frac{N'}{N}$ fit interuallum inter primam et ſecundam lentem $= \frac{N'-N}{N}\alpha$ quod cum poſitivum eſſe debeat, patet, vtrum α poſitiue an negatiue ſumi oporteat tum vero erit $\mathfrak{B} = 1$ et $B = \infty$ vnde $\beta + c$ ſeu diſtantia inter 2 et 3 lentem fieret $= \infty$.

3°. Si $k = m$ interuallum ſecundae et tertiae lentis euaneſcet fietque $\mathfrak{B} = \frac{N'-N''}{mN-N''}$; et $B = \frac{N'-N''}{mN-N'}$ interuallum vero inter 1 et 2 lentem $= \alpha(k-1) = \alpha(m-1)$, vbi manifeſto α debet eſſe quantitas poſitiua.

4°. Si $k = \frac{N'}{N''}.m$, fiet $\mathfrak{B} = 0$ et $B = 0$; vnde fieret diſtantia inter 2 et 3 lentem $= 0$.

Cum igitur neque lentium diſtantias nullas neque infinitas admitti conueniat numerum k nulli limitum prorſus aequalis ſumi poterit.

Coroll. 3.

155. Quod porro ad campum apparentem attinet, qui pendet a formula $\pi' - \pi$ quia inuenimus $\pi : \pi' = 1 : 1 + \frac{m-1}{1-k}\mathfrak{B}$ erit pro memoratis quatuor caſibus

1°. Si $k = 1$ erit $\pi : \pi' = 1 : \infty$ hinc $\pi = 0$; ita, vt pro campo apparente haberetur $\Phi = \frac{\pi'}{m-1}$

2°. Si

2°. Si $k = \frac{N'}{N}$; fiet $\pi : \pi' = 1 : 1 + \frac{mN - N}{N - N'} = 1 : \frac{mN - N'}{N - N'}$ hincque $\pi = \frac{N - N'}{mN - N'} . \pi'$ et $\pi' - \pi = \frac{N(m-1)}{mN - N'} \pi'$ ſicque pro campo apparente fiet $\Phi = \frac{N}{mN - N'} . \pi'$. ſicque Φ maius euadet, ſi $\frac{N}{mN - N'} > \frac{1}{m-1}$ hoc eſt, ſi $N < N'$, quod ergo eueniet, ſi prima lens ex vitro coronario, ſecunda ex chryſtallino paretur.

3°. Si $k = m$, erit $\pi : \pi' = 1 : \frac{mN - N'}{mN - N'}$, ſeu $\pi = \frac{mN - N'}{mN - N'} . \pi'$. Vnde colligitur campum apparentem maiorem fieri, quam in capite praecedente, ſi fuerit $\pi < 0$; quod cum hic fieri nequeat, in hoc caſu campus maior non eſt exſpectandus.

4°. Si $k = \frac{N'}{N''} m$ erit $\pi : \pi' = 1 : 1$, vnde $\pi = \pi'$ et $\pi' - \pi = 0$, quo ergo caſu campus apparens plane euaneſceret.

Coroll. 4.

156. Hinc ergo concludimus, vt maiorem campum obtineamus, quam ante, neceſſario requiri, vt ſit $N' > N$ atque k capi debere intra limites 1 et $\frac{N'}{N}$, qui poſterior limes cum ſit vnitate maior, etiam k erit maius vnitate; ex quo ſequitur diſtantiam α capi debere poſitiuam, quia $\alpha(k-1) > 0$.

Corol-

Coroll. 5.

157. Cum igitur ob eam cauſſam potiſſimum plures lentes adhibeamus, vt maiorem campum obtineamus, ex pluribus illis caſibus, prout litterae N, N′, N″ inter ſe variare poſſunt, hic vnicus nobis relinquitur, quo $N' > N$ atque k inter limites 1 et $\frac{N'}{N}$ ſumitur.

Scholion. 1.

158. In his corollariis vſi ſumus eo valore ipſius 𝔅, quem ex tertia aequatione deduximus. Supra autem iam obſeruauimus, hanc aequationem ita eſſe comparatam, vt de ea nunquam omnino certi eſſe queamus; cum enim valores litterarum N, N′, N″ etc. ex nulla theoria adhuc definiri poſſint, ſed tantum per experimenta, qualia a Dollondo ſunt inſtituta, concludantur; quantacunque cura et ſollertia in iis adhibeatur, nunquam tamen tantum praeciſionis gradum ſperare licet, vt non error ſatis notabilis ſit pertimeſcendus; quam ob cauſſam etiam valor ipſius 𝔅 inde deductus pro vero haberi non poterit, ſed contentos nos eſſe oportet, ſi modo hunc valorem propemodum cognouerimus; id quod ipſa etiam rei natura confirmatur, quia enim aequatio noſtra tertia ſpatium diffuſionis, per quod imagines diuerſicolores ſunt diffuſae, prorſus ad nihilum redigit; facile intelligitur, ad praxin ſufficere, dummodo hoc ſpatium reddatur ſatis exi-

exiguum, praecipue poſtquam id praeſtiterimus, vt margo coloratus diſpareat inprimis igitur valor litterae $\mathfrak{B}$ ex ſecunda aequatione determinari debet, qui ſi ita fuerit comparatus, vt tantum praeterpropter tertiae aequationi ſatisfaciat, confuſio inde oriunda eo magis negligi poterit, quod etiam in teleſcopiis ex vna vitri ſpecie paratis non adeo nocere deprehenditur. Verum ex ſecunda aequatione valorem ipſius $\mathfrak{B}$ pro eo etiam caſu definire licet, quo omnes lentes ex eadem vitri ſpecie eſſent confectae, ita, vt foret $N = N' = N''$ tum enim concluderetur

$$\mathfrak{B} = \frac{m-k}{(m-1)k} - \frac{1-k}{m-1} = \frac{m-2k+k^2}{(m-1)k}$$

quo valore ſi velimus vti, vt conditio ſupra praeſcripta $B.\frac{k-m}{k-1} > 0$ adimpleatur, cum inde ſit

$$1 - \mathfrak{B} = \frac{mk-m+k-k^2}{(m-1)k} = \frac{(k-1)(m-k)}{(m-1)k}$$

erit $B = \frac{m-2k+k^2}{(k-1)(m-k)}$ hincque conditio $\frac{-m+2k-k^2}{(k-1)^2} > 0$; in qua cum denominator certe ſit poſitiuus, etiam numerator talis eſſe debet, adeoque $1 - m - (k-1)^2 > 0$, quod fieri nequit. Ex quo perſpicuum eſt, hoc caſu marginem coloratum plane tolli non poſſe. Videamus igitur, ſi diuerſo vitro vtamur, num hoc vitium effugere queamus. Hunc in finem ponamus breuitatis gratia $\frac{N'}{N} = \xi$ vt ξ ſit numerus vnitatem vel tantillum ſuperans vel ab ea deficiens, et cum ſit

$$\mathfrak{B} = \frac{(m-k)\xi + k(k-1)}{(m-1)k} \text{ erit } B = \frac{(m-k)\xi + k(k-1)}{(m-k)(k-\xi)}$$

vnde conditio noſtra poſtulat, vt ſit $\frac{(k-m)\xi - k(k-1)}{(k-1)(k-\xi)} > 0$. Hic duo caſus ſunt conſiderandi.

I°. Si denominator ſit poſitiuus, quod ſit vel ſi $k > \xi$ et $k > 1$ vel ſi $k < \xi$ et $k < 1$. Tum enim eſſe debet $(k-m)\xi - k(k-1) > 0$

ſiue $\frac{(1+\xi)^2}{4} - m\xi > (k - \frac{1}{2}(1+\xi))^2$

quod cum m notabiliter ſuperet vnitatem, ξ vero ab vnitate parum differat, manifeſto fieri nequit.

II°. Si denominator ſit negatiuus quod ſit, ſi k continetur intra limites ξ et 1. Tum vero numerator debet etiam eſſe negatiuus ſeu $(k-m)\xi - k(k-1) < 0$

ſiue $\frac{(1+\xi)^2}{4} - m\xi < (k - \frac{1}{2}(1+\xi))^2$

quod ſponte euenit, cum pars prior manifeſto ſit negatiua. Hic igitur caſus, vt iam notauimus, ſolus eſt, qui attentionem meretur, cum hoc modo etiam tertiae aequationi ſaltim proxime ſatisfiat.

Scholion 2.

159. Quodſi ergo nobis propoſitum ſit, marginem coloratum tollere, quae proprietas potiſſimum deſiderari ſolet, primo tenendum eſt, hoc nullo modo lentibus ex vna vitri ſpecie factis praeſtari poſſe, ſed ſaltem primam et ſecundam lentem ex diuerſo vitro conſtare debere, ita, vt poſito $\frac{N'}{N} = \xi$ ſiue $N = 1$, $N' = \xi$, littera ξ ab vnitate differat, dum pro N'' ſiue

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ſiue vnitas ſiue ξ pro lubitu accipi poterit, deinde vidimus, numerum k intra limites 1 et ξ ſumi debere, quo facto erit $B = \frac{(m-k)\xi + k(k-1)}{(m-k)(k-\xi)}$ et $\mathfrak{B} = \frac{(m-k)\xi + k(k-1)}{(m-1)k}$; vnde diſtantiae determinatrices erunt

$$b = -\frac{\alpha}{k}.$$

$$\beta = \frac{-(m-k)\xi - k(k-1)}{k(m-k)(k-\xi)}\alpha$$

$$c = \frac{(m-k)\xi + k(k-1)}{m(m-k)(k-\xi)}.\alpha$$

hincque lentium interualla

$$a + b = \alpha\left(\frac{k-1}{k}\right)$$

$$\beta + c = \frac{-(m-k)\xi - k(k-1)}{(m-k)(k-\xi)}.\frac{m-k}{mk}.\alpha$$

$$= \frac{\pm(m-k)\xi + k(k-1)}{mk(\xi-k)}.\alpha$$

hincque tota teleſcopii longitudo erit

$$= \frac{m-1}{m}\left(\frac{\xi-k+1}{\xi-k}\right)\alpha$$

Porro maxime intereſt in campum apparentem inquirere, quod fit determinando valorem $\pi = \frac{\pi'}{1 + \frac{m-1}{1-k}\mathfrak{B}}$, qui abit in ſequentem $\pi = \frac{k(1-k)}{(m-k)\xi}.\pi'$. vnde adipiſcimur $\Phi = \frac{\pi'-\pi}{m-1} = \frac{\pi'}{m-1}\left(1 + \frac{k(k-1)}{(m-k)\xi}\right)$. Cum igitur maxime interſit, campum, quantum fieri poteſt, augeri, hinc obtinemus iſtam concluſionem, numerum k vnitate maiorem eſſe debere, vnde cum k contineatur intra limites 1 et ξ, haec porro regula obſeruetur, *litteram*

ξ *vnitate maiorem eſſe debere*; vnde ſequitur, lentem ſecundam ex vitro chryſtallino, primam vero ex communi eſſe parandam; quo pacto alter caſus, quo fieret $\xi < 1$ penitus e praxi excluditur. Quare cum ſit $k > 1$ diſtantia α, quae adhuc incerta eſt relicta, debet eſſe poſitiua.

Nunc demum conſideremus aequationem tertiam, qua confuſio colorum penitus tollitur, et videamus, quanta ea nunc ſit proditura. Illa autem tertia aequatio nunc ſit

$$0 = 1 - \frac{\xi}{k\mathfrak{B}} + \frac{N''}{m\mathfrak{B}}$$

quae nunc induet hanc formam:

$$0 = \frac{-m(k-1)(\xi-k) - (m-k)(\xi-k) \cdot N''}{m((m-k)\xi + k(k-1))}$$

quae quantitas vtique non erit aequalis nihilo, ſed cum k, ξ et N'' parum ab vnitate differant, ſemper erit valde parua, id quod clarius inde perſpicitur, quod numerator habeat factorem minimum $\xi - k$, denominator autem ſemper ſit ſatis magnus eoque maior, quo maior fuerit multiplicatio. Ex quo manifeſtum eſt hanc confuſionem nunquam fore perceptibilem. Praeterea autem cum haec confuſio plane euaneſceret, ſi caperetur $\xi = k$, conſultum quidem videtur numerum k limiti ξ propiorem capere, quam vnitati, quandoquidem ipſi limiti ξ aequari nequit, quia interuallum inter IIdam et IIItiam lentem fieret infinitum, vti et longitudo teleſcopii; quare ne

ea

ea nimis magna prodeat, contrarium potius ſuadendum eſt, vt littera k a limite ξ, quantum fieri poteſt remoueatur et vnitati propius capiatur. Conſequenter vnicus caſus, qui euolui meretur, in hoc conſiſtet, vt numero k valor vnitati proximus aſſignetur et exceſſus tam ſit exiguus, quam craſſities lentium admittere ſolet. Si enim k ipſi vnitati aequaretur, haberemus caſum praecedentis capitis, quo interuallum lentium plane nullum eſt poſitum, quod incommodum hic evitare conſtituimus.

Problema 2.

160. Si prima lens ex vitro coronario, ſecunda vero ex chryſtallino paretur, et inter eas interuallum tam exiguum ſtatuatur, quam craſſities lentium admittit, regulas determinare, quas in conſtructione huius teleſcopii obſeruare oportet.

Solutio.

Hic ergo ex Dollondi experimentis ſtatui debebit $\xi = \frac{10}{7}$ et quia k limiti 1 propius accipi conuenit, quam alteri limiti $\frac{10}{7}$, ſumamus $k = \frac{8}{7}$ et quae in praecedentibus ſcholiis ſunt tradita, ſequentes nobis ſuppeditant determinationes.

I. Pro diſtantiis determinatricibus.

$$b = -\tfrac{7}{8}.\,\alpha;\quad \beta = \frac{35(7m-8)+28}{8(7m-8)}\,\alpha$$

$$\text{vel } \beta = \frac{7(35m-36)}{8(7m-8)}.\,\alpha$$

$$c = \frac{-35m+36}{m(7m-8)}.\,\alpha$$

 II. Pro

II. Pro interuallis lentium.

$$a + b = \tfrac{1}{8}\,\alpha$$

$$\mathfrak{b} + c = \tfrac{35m - 36}{8m}\cdot\alpha$$

et longitudo teleſcopii $= \frac{9(m-1)}{2m}\cdot\alpha$

Hic obſeruandum eſt, cum α ſit diſtantia focalis primae lentis eiusque ſemidiameter aperturae eſſe debeat $x = m y = \frac{m}{50}$. dig. iſtam diſtantiam α minorem eſſe non poſſe, quam $5\,x$ ſeu $\frac{m}{10}$. dig. ita vt ſit $\alpha > \frac{m}{10}$ dig. quare ſi capiatur verbi gratia $m = 50$, longitudo teleſcopii prodiret maior, quam $\frac{9\cdot49}{20}$; maior quam 22 dig. et ſi fieri debeat $m = 100$, ea maior eſſe deberet, quam $\frac{9\cdot99}{20}$. dig. maior quam 44 dig. quae diſtantia cum facile tolerari queat, manifeſtum eſt, haec teleſcopia etiam ad maiores multiplicationes adhiberi poſſe; pro minoribus autem multiplicationibus eximium certe vſum praeſtant, cum ſi ſtatuatur $m = 5$, longitudo prodeat $> \frac{36}{20}$ dig. maior, quam $\frac{9}{5}$ dig., ſumtoque $m = \frac{5}{2}$, ea prodeat $> \frac{27}{40}$ dig.

Pro campo autem apparente habebimus eius ſemidiametrum

$$\Phi = \frac{\pi'}{m-1}\left(1 + \frac{4}{5(7m-1)}\right).$$

ideoque aliquantum maior, quam caſu praecedente, praeſertim ſi multiplicatio fuerit exigua. Notentur etiam diſtantiae focales harum lentium p, q, r et cum ſit

$\mathfrak{B} =$

$$\mathfrak{B} = \frac{35m - 36}{20(m-1)} \text{ et } B = \frac{35m - 36}{-7m + 8}$$

erit $p = \alpha$; $q = -\frac{(35m - 36)\alpha}{32(m-1)}$; $r = \frac{-(35m - 36)\alpha}{m(7m - 8)}$

et ſemidiameter aperturae ſecundae lentis ex §. 23

$$= \frac{m(35m - 36).\pi'}{400(m-1)(7m-8)} + \frac{14m(m-1)}{25}$$

Denique pro conſtructione harum lentium numeri λ, λ' et λ'' ita accipi debent, vt ſatisfiat primae noſtrae aequationi, quae erat

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3 k} + \frac{\mu''\lambda''}{B^3 . m} - \frac{\mu'\nu'}{\mathfrak{B}Bk}$$

vbi notandum eſt, vt lens ocularis vtrinque fiat aeque concaua, ſtatui debere $\lambda'' = 1.60006$. ſi haec lens ſit ex vitro coronario ideoque $\mu'' = \mu$ ſin autem ſit ex vitro chryſtallino ideoque $\mu'' = \mu'$, fore $\lambda'' = 1.67445$. haec autem aequatio non niſi caſibus particularibus pro data multiplicatione euolui poterit; vbi meminiſſe iuuabit, fore,

$\mu = 0.9875$; $\mu' = 0.8724$.

$\nu' = 0.2529$; $\mu'\nu' = 0,2206$.

Exempl. I.

161. Si multiplicatio ſit $m = \frac{5}{2}$, teleſcopium huius generis ex tribus lentibus conſtans deſcribere.

Cum ſit $m = \frac{5}{2}$, erunt diſtantiae determinatrices
$b = -\frac{7}{8}.\alpha$; $\mathfrak{C} = \frac{721}{175}.\alpha$
$c = -\frac{206}{95}.\alpha$. $B = -\frac{103}{15}$

cr

et interualla lentium

$$a + b = \frac{1}{1}\cdot a;\ \mathfrak{C} + c = \frac{103}{43}\cdot a$$

et longitudo teleſcopii $= \frac{27}{13}\cdot a$ atque pro campo apparente fiet $\Phi = \frac{2\pi'}{3}\left(1 + \frac{8}{95}\right)$ ſumto $\pi' = \frac{1}{4}$ et multiplicando per 3437 minut erit angulus $\Phi = 10^\circ\, 21'\, 2''$

Cum nunc ſit $B = \frac{-103}{19}$ erit $\mathfrak{B} = \frac{103}{84}$

et habebimus

$$\text{Log.}\,(-B) = 0.7340836\,(-)$$

$$\text{et Log.}\ \mathfrak{B} = 0.0885580$$

aequatio autem pro confuſione prima tollenda, ſi lentem ocularem ex vitro coronario faciamus, vt ſit

$\mu'' = \mu$ et $\lambda'' = 1.60006$, erit

$$0 = 0.9875.\,\lambda - 0.4140\ \lambda' - 0.00396$$
$$+\,0.02903$$

$$0 = 0.9875.\,\lambda - 0.4140\ \lambda' + 0.02507$$

vnde quaeratur λ', et habebitur

$$\lambda' = 2,3852.\ \lambda + 0.06055$$

Si ergo hic capiatur $\lambda = 1$, fiet

$$\lambda' = 2,4457$$

vnde fit $\lambda' - 1 = 1,4457$
et log. $\sqrt{(\lambda' - 1)} = 0.0800391$

vnde conſtructio ſingularum lentium ſequenti modo ſe

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ſe habebit, ſiquidem radii facierum primae lentis ſint F et G; ſecundae F′ et G′ et tertiae F″ et G″.

I. Pro prima lente ex vitro coronario.

$$F = \frac{\alpha}{\sigma} = 0.6023.\ \alpha$$

$$G = \frac{\alpha}{\varrho} = 4.4131.\ \alpha$$

II. Pro ſecunda lente ex vitro chryſtallino.

$$\frac{1}{F'} = \frac{\varrho\beta + \sigma b \mp \tau.(b+\beta)\sqrt{\lambda'-1}}{b\beta}$$

$$\frac{1}{G'} = \frac{\sigma\beta + \varrho b \pm \tau(b+\beta)\sqrt{\lambda'-1}}{b\beta}$$

cum nunc ſit $\log.\ \frac{b}{\alpha} = \log.\ (-\frac{7}{8}) = 9.9420080(-)$

et $\log.\ \frac{\beta}{\alpha} = 0.6760917.$

et $\log.\ \frac{(b+\beta)}{\alpha} = \log.\ \frac{147}{38} = 0.5875336.$

$\log.\ \sigma = 0.1993986.$

$\log.\ \varrho = 9.1501422.$

$\log.\ \tau = 9.9432471.$

Vnde inuenitur

$$\frac{1}{F'} = \frac{-0.7147 \mp 4.0815}{b\beta}.\ \alpha$$

$$\frac{1}{G'} = \frac{+7.3838 \pm 4.0815}{b\beta}.\ \alpha$$

Vt maiores numeri euitentur, ſumantur ſigna inferiora, fietque

$$F' = \frac{b\beta}{3.3663} = -1,2327.\ \alpha$$

$$G' = \frac{b\beta}{3.3023} = -1,2568.\ \alpha$$

Pro lente oculari ex vitro coronario paranda, cum ea vtrinque ſit aeque concaua, eiusque diſtantia focalis ſit $c = -\frac{106}{95}.\alpha$, radius concauitatis pro vtraque facie erit $= 2(n-1)c = -\frac{106 \cdot 106}{95}\alpha = -2,2985.\alpha$.

Prima lens admittit aperturam, cuius ſemidiameter $x = 0.1506.\alpha$. Nunc vero claritas poſtulat, vt ſit $x = \frac{m}{50}$. dig. $= \frac{1}{25}$. dig. vnde α maius, quam $\frac{1}{5}$ dig. Sumatur ergo $\alpha = \frac{1}{2}$ dig. et conſtructio teleſcopii ita ſe habebit:

I. Pro lente prima

rad. faciei { anter. $= +0.3012$. dig. } Crown
{ poſter. $= +2,2065$. dig. } Glaſſ.

II. Pro lente ſecunda

rad. faciei { anter. $= -0,6163$. dig. } Flint
{ poſter. $= -0,6284$. dig. } Glaſſ.

III. Pro lente tertia

radius vtriusque faciei $= -1.1492$. dig.
quae paratur ex Crown Glaſſ.

Tum interuallum ſtatuatur:

Inter

I. et II. $= \frac{1}{16}$. dig. $= 0,0625$. dig.

II. et III. $= \frac{103}{80}$ dig. $= 1,2875$. dig.

ita, vt tota teleſcopii longitudo ſit futura
$= 1,3500$. dig. $= 1\frac{1}{3}$ dig.

ſpatii vero viſi ſemidiameter erit $= 10°. 21' 2''$.

Exem-

Exemplum II.

162. Si multiplicatio $m = 5$. teleſcopium huius generis ex tribus lentibus conſtans deſcribere.

Cum ſit $m = 5$, erit $7m - 8 = 27$ et $35m - 36 = 139$, vnde diſtantiae determinatrices fient

$$b = -\tfrac{7}{8}.\ \alpha = -0,8750.\ \alpha$$

$$\beta = \tfrac{973}{216}\ \alpha = +4,5046.\ \alpha$$

$$c = \tfrac{-139}{5.27}\ \alpha = -1,0296.\ \alpha$$

Ex quibus fiunt interualla

$$\alpha + b = \tfrac{1}{8}\ \alpha;\ \beta + c = 3,4750.\ \alpha$$

vnde teleſcopii longitudo $= 3,6.\ \alpha$.

Pro campo autem apparente fiet $\Phi = \frac{\pi'}{4}\left(1 + \frac{4}{5.27}\right)$ ſumtoque $\pi' = \frac{1}{4}$ et multiplicando per 3437 min. erit $\Phi = 3^\circ\ 41'$.

Cum iam ſit $\mathfrak{B} = \frac{139}{112}$ et $B = \frac{-139}{27}$, ſumtisque logarithmis

$$\text{Log. } \mathfrak{B} = 0.0937968$$

$$\text{Log. } B = 0.7116510\ (-)$$

et aequatio pro confuſione prima tollenda, ſi lentem ocularem ex vitro coronario paremus, vt ſit $\mu'' = \mu$ et $\lambda'' = 1.60006$, erit

$$0 = 0.9875.\ \lambda - 0.39933.\ \lambda' - 0.002316$$
$$+ 0.030211$$
$$\overline{+ 0.027895}$$

 ex

ex qua iterum quaeratur

$$\lambda' = 2,4729\, \lambda + 0.06985.$$

Hic non, vt ante ſumamus $\lambda = 1$ ſed, vt prima lens maximae aperturae fiat capax, ideoque diſtantia α minor accipi poſſit, capiatur $\lambda = 1,60006$, vt haec lens fiat vtrinque aequaliter conuexa, habebiturque

$$\lambda' = 4.0266 \text{ et } \lambda' - 1 = 3.0266.$$

et $\text{Log.}\, \sqrt{(\lambda' - 1)} = 0.2404775.$

atque hinc obtinebimus:

I. Pro prima lente ex vitro coronario:

radius vtriusque faciei $= 2(n - 1).\, \alpha = 1,06.\, \alpha$; quae aperturam admittit, cuius ſemidiameter $x = 0,26.\, \alpha$.

II. Pro ſecunda lente ex vitro chryſtallino:

ob $\text{Log.}\, \frac{(b+\beta)}{\alpha} = 0.5598588.$

calculus ita ſe habebit:

$$\frac{1}{F'} = \frac{-0.7170 \pm 0.5109}{b\beta}.\, \alpha$$

$$\frac{1}{G'} = \frac{+5.0357 \mp 5.5400}{b\beta}.\, \alpha$$

valeant hic ſigna ſuperiora eritque:

$$F' = \frac{b\beta}{4,795\, \alpha} = -0.8223.\, \alpha$$

$$G' = \frac{b\beta}{1,4648\, \alpha} = -2.6908.\, \alpha$$

III. Pro

III. Pro tertia lente ex vitro coronario.

erit radius vtriusque faciei $= 2(n-1)c = 1,06\,c$ $= -1.0920.\ \alpha$.

Cum nunc ob claritatem eſſe debeat $x = \frac{m}{50}$ dig. $= \frac{1}{15}$. dig. fiet α maius, quam $\frac{1}{5}$ dig.

Sumi igitur poterit $\alpha = \frac{1}{2}$ dig. et conſtructio teleſcopii ita ſe habebit.

I. Pro lente prima Crown Glaſſ.

rad. faciei vtriusque $= 0,5300$. dig.

II. Pro lente ſecunda Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0.4111.\ \text{dig.} \\ \text{poſter.} = -1.3454.\ \text{dig.} \end{cases}$

III. Pro lente tertia Crown Glaſſ.

rad. vtriusque faciei $= -0,5460$. dig.

Tum vero ſtatuatur interuallum

I. et II. $= \frac{1}{16}$ dig.

II. et III. $= 1,7375$. dig.

ita, vt tota longitudo ſit $= 1,8$. dig. ideoque nondum duos adaequet digitos.

Spatii tandem viſi ſemidiameter erit $= 3^\circ\ 41'$.

Corollarium.

163. Teleſcopia igitur in his duobus exemplis conſtructa aptiſſima videntur ad vſum vulgarem quoniam ea facile quis ſecum gerere poteſt iisque in ſpectaculis praeſertim vti. Sequentia autem exempla ad maiores multiplicationes accommodemus.

Exempl. III.

164. Sit multiplicatio $m = 25$, teleſcopium huius generis tribus lentibus conſtans deſcribere.

Cum ſit $m = 25$, erit $7m - 8 = 167$ et $35m - 36 = 839$, eruntque diſtantiae determinatrices

$$b = -\tfrac{7}{8}.\alpha = -0,875.\alpha;$$

$$\beta = \tfrac{7.839}{8.167}.\alpha = 4,3960.\alpha;$$

$$\text{Log.}\ \tfrac{-b}{\alpha} = 9.9420081\ (-)$$

$$\text{Log.}\ \tfrac{\beta}{\alpha} = 0.6430535.$$

$$c = -\tfrac{835.\alpha}{35.167} = -0.2009.\alpha$$

$$b + \beta = 3.5210.\alpha$$

$$\text{Log.}\ (b + \beta) = 0.5466660.$$

Hinc prodeunt lentium interualla

$$a + b = \tfrac{1}{8}\alpha;\ \beta + c = 4.1951.\alpha$$

et tota longitudo $= 4,3201.\alpha$

Pro

Pro campo autem apparente fiet $\Phi = \frac{\pi'}{84}(1 + \frac{4}{3,187})$ hincque angulus $\Phi = 36$ min. prim. circiter.

Cum iam porro ſit $\mathfrak{B} = \frac{139}{11,14}$

et Log. $\mathfrak{B} = 0.0963927$

Log. $- B = 0.7010454\,(-)$

perueniatur ad ſequentem aequationem

$$0 = 0,9875\lambda - 0.3922\lambda' - 0,0004984 + 0.03077$$

ſeu $0 = 0.9875\lambda - 0.3922\lambda' + 0,03028.$

Quoniam vidimus, valorem $\lambda = 1,60006$ longitudinem teleſcopii haud mediocriter diminuiſſe, ſtatim ponamus $\lambda = 1,60006$ eritque

$$0 = 1,6103 - 0,3922\lambda'.$$

vnde prodit

$$\lambda' = 4,1057; \text{ et } \lambda' - 1 = 3.1057$$

$$\text{et log. } \sqrt{(\lambda' - 1)} = 0,2460797.$$

vnde conſtructio ſingularum lentium ita ſe habebit:

I. Pro prima lente ex vitro coronario radius vtriusque faciei $= 1,06\,\alpha$ quae ergo aperturam admittit, cuius ſemidiameter $x = 0,265.\,\alpha.$

II. Pro ſecunda lente calculus ita ſe habebit:

$$\frac{1}{F'} = \frac{-[illegible].7633 \pm 5.4449}{b\beta}\alpha.$$

$$\frac{1}{G'} = \frac{6.[illegible] \mp 5.440}{b\beta}\alpha.$$

Valeant

Valeant ſigna ſuperiora, eritque

$F' = \frac{b\beta}{4,7816\,\alpha} = -0,8216\,\alpha$

$G' = \frac{b\beta}{1,7449.\alpha} = -2,7694.\,\alpha$

III. Pro tertia lente ex vitro coronario.

radius vtriusque faciei $= 1,06.\,c = -0,21295.\,\alpha$

Claritas autem poſtulat $= x = \frac{m}{5}$ dig. $= \frac{1}{5}$ dig. vnde concluditur $\alpha > 1,88$ ſumatur ergo $\alpha = 2$ et conſtructio haec erit

I. Pro lente prima

rad. vtriusque faciei $= 2,12$ dig. Crown Glaſſ.

II. Pro lente ſecunda

rad. faciei $\begin{cases} \text{anter.} = -1,6432 \text{ dig.} \\ \text{poſter.} = -5,5388 \text{ dig.} \end{cases}$ Flint Glaſſ.

III. Pro lente tertia

radius vtriusque faciei $= -0,42590$ dig. Crown Gl.

Tum ſtatuatur interuallum lentium

I. et II. $= \frac{1}{4}$ dig.

II. et III. $= 8,3902.$ dig.

et tota longitudo $= 8.64.$ dig. campique apparentis ſemidiameter $x = 36'$. circiter.

Exem-

Exempl. IV.

165. Si m debeat eſſe $= 50$, erit $7m - 8 = 342$; $35m - 36 = 1714$ adeoque diſtantiae

$$b = -\tfrac{7}{8}\alpha = -0,875.\alpha$$

$$\beta = 4,3852.\alpha.;\quad c = -0,10023.\alpha$$

$$\log.\tfrac{\beta}{\alpha} = 0.6419928$$

$$\log.\tfrac{-b}{\alpha} = 9.9420081\ (-)$$

$$\log.\tfrac{(b+\beta)}{\alpha} = 0.5453319.$$

$$\log.\tfrac{b\beta}{\alpha^2} = 0.5840009\ (-)$$

Tum vero interualla lentium erunt

$$\alpha + b = \tfrac{1}{8}\alpha = 0,125\,\alpha$$

$\beta + c = 4,2850.\alpha$ hincque

tota longitudo $= 4,4100\,\alpha$.

Porro reperitur

$$\mathrm{Log.} - B = 0,6999847.$$

$$\mathrm{Log.}\ \mathfrak{B} = 0,0966567.$$

Pro campo apparente reperitur $\Phi = \frac{\pi'}{49}\left(1 + \frac{4}{5.342}\right)$ ſeu angulus $\Phi = 17\tfrac{1}{2}$ minut.

Pro confuſione tollenda ſtatuatur ſtatim in aequatione inuenta $\lambda = 1,60006$ eritque

$$0 = 1,5801 - 0.3915.\lambda' - 0,00025 \\ + 0,03083$$

ſiue $0,3915\,\lambda' = 1,6107$

vnde $\lambda' = 4,1141$; hinc $\lambda' - 1 = 3.1141$ et

log. $\sqrt{(\lambda' - 1)} = 0.2466663$

vnde conſtructio ſingularum lentium ita ſe habebit.

I. Pro lente prima

radius vtriusque faciei $= 1,06\,\alpha$ quae ergo aperturam admittit, cuius ſemidiameter $= 0,265.\,\alpha$.

II. Pro lente ſecunda

$$\frac{1}{F'} = \frac{-0,7653 \pm 5,4355}{b\beta}\,\alpha$$

$$\frac{1}{G'} = \frac{+6,3161 \mp 5,4355}{b\beta}\,\alpha$$

Valeant ergo ſigna ſuperiora eritque

$$F' = \frac{b\beta}{4,6702\,\alpha} = -0,8216.\,\alpha$$

$$G' = \frac{b\beta}{1,3414.\,\alpha} = -2,7776.\,\alpha$$

III. Pro lente tertia

erit radius vtriusque faciei $=$

$$2(n-1)c = 1,06.\,c = -0,10624.\,\alpha$$

Claritas autem poſtulat, $x = \frac{m}{35} = 1$ dig. vnde ſequitur $\alpha > 3,8$, ſumto ergo $\alpha = 4$, habebitur ſequens teleſcopii conſtructio.

I. Pro lente prima: Crown Gl.

radius vtriusque faciei $= 4,24$. dig.

II. Pro

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II. Pro lente ſecunda

radius facici $\left\{\begin{array}{l} \text{anter.} = -3,2864 \\ \text{poſter.} = -11,1104 \end{array}\right\}$ Flint Glaſſ.

III. Pro lente tertia

radius vtriusque facici $= -0,42496$ Crown Glaſſ.

Tum vero ſtatui debet interuallum lentium

I. et II. $= 0,5$ dig.

II. et III. $= 17,1400.$ dig.

adeoque teleſcopii longitudo $= 17,6400.$ dig.

Campi denique viſi ſemidiameter inuentus eſt $17\frac{1}{2}$ min.

Scholion.

166. Cum in his ſolutionibus littera λ indeterminata ſit relicta, in tribus poſterioribus exemplis eius loco non vnitatem poſuimus, vt ante fecimus, ſed potius ei tribuimus illum valorem, quo ambae eius facies inter ſe aequales redderentur hocque modo inſigne commodum ſumus nacti, vt lens prima fere duplo maiorem aperturam admitteret hincque diſtantia α fere ad dimidium reduci poſſet. Vt autem in genere quaepiam lens cuius diſtantiae determinatrices ſunt a et α ambas ſuas facies obtineat aequales, ſupra vidimus, capi debere $\sqrt{(\lambda-1)} = \frac{(\sigma-\rho)(a-\alpha)}{2\tau(a+\alpha)} = \frac{2(2n-1)}{n\cdot\sqrt{(4n-1)}}\cdot\left(\frac{1-k}{1+k}\right)$

ob $\alpha = A a$, vnde fit $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} \cdot \frac{(1-A)^2}{(1+A)^2}$ quare ſi vel a vel α fuerit infinitum, vti fit tam in lente obiectiua, quam in lente oculari, habebitur $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)}$. Sin autem velimus, vt alia quaepiam lens obtineat ambas ſuas facies inter ſe aequales; tum ob $\frac{(1-A)^2}{(1+A)^2} = 1 - \frac{4A}{(1+A)^2}$ capere debemus $\lambda = 1 + \frac{4(nn-1)^2}{n^2(4n-1)} - \frac{16(nn-1)^2 A}{n^2(4n-1)(1+A)^2}$. Cum autem in noſtra expreſſione pro ſemidiametro confuſionis tum occurrat talis forma $\lambda (A+1)^2 + \nu A$, valor iſtius formulae fiet $= (A+1)^2 + \frac{4(nn-1)^2(A+1)^2}{n^2(4n-1)} - \frac{16(nn-1)^2 A}{nn(4n-1)} + \frac{4(n-1)^2 A}{4n-1}$.

Commodius autem erit, hoc caſu valorem ipſius λ pro facilitate calculi ita exprimere $\lambda = 1 + \frac{(\sigma-\rho)^2(1-A)^2}{4\tau^2(1+A)^2}$.

Exempl. V.

167. Si multiplicatio m debeat eſſe valde magna vel ſaltim maior, quam 25, huius generis teleſcopia ex tribus lentibus conſtantia deſcribere. Hic ſtatim obſeruo, ſumta prima lente vtrinque aequaliter conuexa, fore radium vtriusque curuaturae, vt ante, $= 1,06 . \alpha$, quae admittet aperturam, cuius ſemidiam. $= \frac{1}{4} . \alpha = x$, cum autem ob claritatem ſumi debeat $x = \frac{m}{50}$. dig. hinc intelligimus, ſemper ſtatui poſſe $\alpha = \frac{2m}{25} = 0,08 . m$. dig. et pro campo apparente $\Phi = \frac{\pi'}{m-1}$; ſumtoque $\pi' = \frac{1}{4}$, erit $\Phi = \frac{850}{m-1}$. min.

Nunc

Nunc autem ante, quam reliquas partes conſtructionis definiamus, contemplemur caſum, quo $m = \infty$ eritque

$$b = -\tfrac{7}{8}\,\alpha;\ \mathfrak{b} = \tfrac{35}{8}.\,\alpha;\ c = \tfrac{-1}{m}.\,\alpha;\ = -\tfrac{2}{5}.\ \text{dig.}$$

Diſtantiae porro lentium $\alpha + \mathfrak{b} = \frac{1}{8}\,\alpha$.

et $\mathfrak{b} + c = \left(\frac{35.\alpha}{8} - \frac{2}{5}\right)$ dig.

et $\mathfrak{B} = \frac{5}{4}$; $B = -5$.

Pro ſequente calculo ſtatim ſumamus $\lambda = 1{,}60006$. et aequatio prodibit

$$0 = 1{,}5801 - 0.3908.\ \lambda' + 0.03088$$

vnde inuenitur

$$\lambda' = 4.1220. \text{ et } \lambda' - 1 = 3.1220.$$

$$\text{et log. } \sqrt{(\lambda' - 1)} = 0.2472164$$

$$\text{Hinc ob Log.} - \tfrac{b}{\alpha} = 9.9420081\ (-)$$

$$\text{Log.}\ \tfrac{\beta}{\alpha} = 0.6409781.$$

$$\text{Log.}\ \tfrac{b+\beta}{\alpha} = 0.5440680.$$

$$\text{Log.}\ \tfrac{b\beta}{\alpha^2} = 0.5829862 -$$

Ex quibus pro ſecunda lente habebimus

$$\frac{1}{F'} = \frac{-0.7662 \pm 5.4266}{b\beta}.\,\alpha$$

$$\frac{1}{G'} = \frac{+6.8006 \mp 5.4266}{b\beta}.\,\alpha$$

 ſeu

ſeu ſumtis ſignis ſuperioribus

$$F' = \frac{b\beta}{4,36537 . \alpha} = -0.8214 . \alpha$$

$$G' = \frac{b\beta}{1,3747 . \alpha} = -2.7861 . \alpha$$

qui valores pro multiplicatione infinita locum habent; at nunc pro multiplicatione quacunque m ſtatuatur

$$F' = -\left(0.8214 + \frac{f}{m}\right)\alpha$$

$$G' = -\left(2,7861 + \frac{g}{m}\right)\alpha$$

vbi valores litterarum f et g ex caſu praecedente $m = 50$ vel etiam, ſed minus tuto, ex caſu $m = 25$ erui debent, hocque modo reperitur $f = 0.01$ et $g = -0.4250$ ita, vt ſit in genere

$$F' = -\left(0,8214 + \frac{0,01}{m}\right)\alpha$$

$$G' = -\left(2,7861 - \frac{0,4250}{m}\right)\alpha$$

Deinde cum ſupra iam inuenta ſit diſtantia focalis lentis tertiae $= -\frac{3}{5}$ dig. pro $m = \infty$, ſtatuamus pro quauis multiplicatione m eſſe $c = -\frac{3}{5} - \frac{h}{m}$ eritque

$$c = -\left(\frac{3}{5} + \frac{1,2480}{m}\right) \text{dig.}$$

cuius ergo radius vtriusque faciei erit

$$-\left(0.4240 + \frac{1,0728}{m}\right) \text{dig.}$$

Cum igitur ſit $\alpha = 0.08\, m$. dig.

Con-

Conſtructio teleſcopii ſequenti modo ſe habebit

I. Pro lente prima Crown Glaſſ.

rad. faciei vtriusque $= 0.0848.m.$ dig.

II. Pro lente ſecunda Flint Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = -(0.0657.m + 0.0008)\text{dig.} \\ \text{poſter.} = -(0.2228.m - 0.0340)\text{dig.} \end{cases}$

III. Pro lente tertia Crown Glaſſ.

radius vtriusque faciei $= -(0.4240 + \frac{0.1228}{m})$

Tum vero interualla erunt

$\alpha + b = 0.01.m;$

$\beta + c = (0,35\,m - 0.36)$ dig.

hincque tota longitudo

$= (0,36\,m - 0.36)$ dig.

campique viſi ſemidiameter $= \frac{159}{m-1}$. minut. prim.

Corollarium.

168. Si ergo teleſcopium deſideretur, quod centies multiplicet, id ita ſe habebit

I. Pro lente prima. Crown Glaſſ.

radius vtriusque faciej $= 8,48.$ dig.

II. Pro

II. Pro lente ſecunda. Flint Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = -6.57. \text{ dig.} \\ \text{poſter} = -22,24 \text{ dig.} \end{cases}$

III. Pro lente tertia.

radius vtriusque faciei $= -0.43$. dig.

Interuallum erit lentis

I. et II. $= 1$. dig.

II. et III. $= 34,64$. dig.

hincque longitudo teleſcopii

$= 35,64$. dig.

campique viſi ſemidiameter

$= 8\frac{1}{2}$ min.

Problema 3.

169. Si huiusmodi teleſcopium primi generis ex quatuor lentibus a ſe inuicem ſeparatis ſit conſtruendum, inueſtigare momenta, quibus ei maximus perfectionis gradus conciliatur.

Solutio.

Hic igitur iſtarum trium fractionum $\frac{\alpha}{b}$; $\frac{\beta}{c}$; et $\frac{\gamma}{d}$ ſingulae debent eſſe negatiuae; ponamus ergo $\frac{\alpha}{b} = -k$ et $\frac{\beta}{c} = -k'$. et cum ſit $m = -\frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$ habebimus

$b =$

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$b = -\frac{\alpha}{k}$; $\beta = -\frac{B\alpha}{k}$; $c = -\frac{\beta}{k'} = +\frac{B\alpha}{k.k'}$ et $\gamma = +\frac{BC\alpha}{k.k'}$ et $m = \frac{-k.k'.\gamma}{d}$ hinc $d = -\frac{BC\alpha}{m}$; vnde interualla lentium $\alpha + b = \alpha(1 - \frac{1}{k})$ $\beta + c = B\alpha(\frac{1}{kk'} - \frac{1}{k})$ et $\gamma + d = BC\alpha(\frac{1}{kk'} - \frac{1}{m})$ quae cum debeant eſſe poſitiua aeque ac numeri k, k' et m, bina poſteriora per primum diuiſa dabunt has duas conditiones

$$1^\circ.\ \frac{B(1-k')}{k'(k-1)} > 0.$$

$$2^\circ.\ \frac{BC(m-kk')}{mk'(k-1)} > 0.$$

Iam conſideremus aequationem, qua margo coloratus tollitur, pro caſu, quo diſtantia O eſt negatiua: ponendo, vt ante $\frac{dn}{n-1} = N$; $\frac{dn'}{n'-1} = N'$, $\frac{dn''}{n''-1} = N''$. $\frac{dn'''}{n'''-1} = N'''$ eritque

$$0 = N.BC\pi''.\alpha + N'.b((B+1)C\pi'' - \pi) + N''.c\left(\frac{(C+1)\pi'' - \pi'}{B}\right)$$

ſeu
$$0 = NBC\pi'' - \frac{N'}{k}((B+1)C\pi'' - \pi) + \frac{N''}{k.k'}((C+1)\pi'' - \pi')$$

quem in finem inueſtigare oportet relationes inter litteras π, π', π'', eſt vero ex capite 1.

Imo. $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = -k$

vnde $\pi = \frac{1-k}{\mathfrak{B}}.\ \Phi.$

IIdo. $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{B\alpha}{c} = kk'$

vnde $\pi' = (\frac{1}{\mathfrak{B}} - \frac{k}{\mathfrak{B}} + kk')\frac{\Phi}{\mathfrak{C}}.$

IIItio. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{BCz}{a} = -m;$

vnde ob $\mathfrak{D} = 1$ fiet

$$\pi'' = \left(-m + \frac{1}{BC} - \frac{k}{\mathfrak{B}C} + \frac{kk'}{\mathfrak{C}}\right)\Phi$$

vnde aequatio noſtra erit

$$0 = N\left(-BCm + 1 - \frac{Bk}{\mathfrak{B}} + \frac{BC.k.k'}{\mathfrak{C}}\right)$$

$$-\frac{N'}{k}\left(\frac{-BCm}{\mathfrak{B}} - \frac{Bk}{\mathfrak{B}} + \frac{BC.kk'}{\mathfrak{B}\mathfrak{C}}\right)$$

$$+\frac{CN''}{\mathfrak{C}kk'}(-m + kk');$$ vnde fit

$$C = N - N(B+1)k + NBkk' + N'(B+1) - N'(B+1)k'$$

$$-\frac{N''.m}{kk'} + N''$$

diuiſum per

$$NBm - NBkk' - \frac{N'(B+1)m}{k} + N'(B+1)k'$$

$$+\frac{N''m}{kk'} - N''$$

vel ſuccinctius

$$C = Nkk'(1 - k - Bk(1-k')) + N'kk'(B+1)(1-k')$$

$$-N''(m - kk')$$

diuiſum per

$$(m - kk')(Nkk'B - N'k'(B+1) + N'')$$

adeoque

$$1 + C = Nkk'(1 - k + B(m-k))$$

$$-N'(B+1)k'(m-k)$$

diui-

diuiſum per

$$(m - k\,k')\,(\mathrm{N}\,k\,k'\mathrm{B} - \mathrm{N}'\,k'\,(\mathrm{B} + 1) + \mathrm{N}'')$$

atque hinc

$$\mathfrak{C} = \mathrm{N}\,k\,k'\,(1 - k - \mathrm{B}\,k\,(1 - k'))$$
$$+ \mathrm{N}'\,k\,k'\,(\mathrm{B} + 1)\,(1 - k') - \mathrm{N}''\,(m - k\,k')$$

diuiſum per

$$\mathrm{N}\,k\,k'\,(1 - k + \mathrm{B}\,(m - k)) - \mathrm{N}'\,(\mathrm{B}+1)\,k'\,(m-k)$$

ſed facile patet, hoc modo nobis vix vlterius progredi licere ob harum formularum complicationem, niſi pro k et k' et pro N, N′, N″ valores ſubſtituantur interim tamen haec methodus etiam ſucceſſura videtur, ſi eam ad plures adhuc lentes applicare vellemus, ceterum haud abs re erit hoc negotium etiam alio modo tentaſſe.

Ex praecedentibus ſcilicet aequationibus non litteras π, π', π'' quaeri, ſed potius his quaſi datis ſpectatis litteras $\mathfrak{B}$ et $\mathfrak{C}$ definiri conueniet; vnde ſtatim obtinemus

$$\mathfrak{B} = \frac{1 - k}{\pi}.\,\Phi;\quad \mathfrak{C} = \frac{(k\,k' - 1)\,\Phi + \pi}{\pi'}$$

ex tertia denique aequatione ob $\mathfrak{D} = 1$ colligitur

$$\Phi = \frac{-\pi + \pi' - \pi''}{m - 1};$$

ita, vt et Φ quaſi datum ſpectari queat. Hinc cum ſit

 B =

$$B = \frac{\mathfrak{B}}{1-\mathfrak{B}} \text{ et } C = \frac{\mathfrak{C}}{1-\mathfrak{C}}$$

habebimus

$$B = \frac{(1-k)\Phi}{\pi-(1-k)\Phi}\,;\; C = \frac{(k.k'-1)\Phi+\pi}{\pi'-\pi-(kk'-\ )\Phi}.$$

Nunc cum prior conditio poſtulet, vt ſit $\frac{B(1-k')}{k'(k-\ )} > 0$; altera vero per hanc diuiſa $\frac{C(m-kk')}{1-k'} > 0$ valoribus illis ſubſtitutis hae duae conditiones abibunt in ſequentes:

$$1^\circ.\ \frac{(k'-1)\Phi}{(\pi-(1-k)\Phi)k'} > 0$$

ſeu $\frac{(k'-1)\Phi}{\pi-(1-k)\Phi} > 0$

$$2^\circ.\ \frac{((kk'-1)\Phi+\pi)(m-kk')}{(1-k')(\pi'-\pi-(kk'-1)\Phi)} > 0$$

quae per illam multiplicata dat

$$\frac{-\Phi(m-kk')(\pi+(kk'-1)\Phi)}{(\pi-(1-k)\Phi)(\pi'-\pi-(kk'-1)\Phi)} > 0$$

ſi hic loco Φ eius valor ſubſtituatur, qui cum ſemper ſit poſitiuus ob $m-1$ etiam poſitiuum, dat primo

$$-\pi+\pi'-\pi'' > 0;$$

tum vero binae iſtae conditiones dabunt

$$1^\circ.\ \frac{k'-1}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''} > 0$$

$$2^\circ.\ \frac{(m-kk')((m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi'')}{(k'-1)((m-kk')\pi-(m-kk')\pi'-(kk'-1)\pi'')} > 0$$

tum vero B et C ita definientur:

$$B = \frac{(k-1)(\pi-\pi'+\pi'')}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}$$

$$C = \frac{(m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi''}{-(m-kk')\pi+(m-kk')\pi'+(kk'-1)\pi''}$$

Verum

Verum ſi hos valores ſubſtituere vellemus ſiue in aequatione pro margine colorato vitando ſiue inprimis pro ſemidiametro confuſionis ad nihilum redigendo in multo maiores ambages incideremus, quam priore methodo euenit; quocirca aliam adhuc methodum quaerere debemus; nulla autem alia nobis relinquitur, niſi vt ex ſuperioribus aequationibus litteras k et k' inueſtigemus; quo pacto noſtra inueſtigatio ſatis plana reddetur.

Hanc viam ſequentes ſtatim habemus $k = \frac{\Phi - \mathfrak{B}\pi}{\Phi}$ et $k k' = \frac{\Phi - \pi + \mathfrak{C}\pi'}{\Phi}$; exiſtente $\Phi = \frac{-\pi + \pi' - \pi''}{m - 1}$ vnde cum k et k' ſint numeri poſitiui, pariter atque angulus Φ habemus ſtatim iſtas conditiones:

$$\Phi - \mathfrak{B}\pi > 0.$$

$$\Phi - \pi + \mathfrak{C}\pi' > 0.$$

$$-\pi + \pi' - \pi'' > 0.$$

Porro cum hinc interualla lentium fiant

$$1^\circ.\ \alpha + b = \frac{-\mathfrak{B}\pi}{\Phi - \mathfrak{B}\pi} \cdot \alpha > 0$$

$$2^\circ.\ \beta + c = \frac{(\mathfrak{B}\pi - B\mathfrak{C}\pi')\alpha\Phi}{(\Phi - \pi + \mathfrak{C}\pi')(\Phi - \mathfrak{B}\pi)} > 0$$

$$3^\circ.\ \gamma + d = \frac{(m-1)\Phi + \pi - \mathfrak{C}\pi'}{m(\Phi - \pi + \mathfrak{C}\pi')} \cdot B C \alpha > 0.$$

inde colligimus has nouas conditiones:

$$-\mathfrak{B}\pi . \alpha > 0.$$

$$(\mathfrak{B}\pi - B\mathfrak{C}\pi')\alpha > 0.$$

$$((m-1)\Phi + \pi - \mathfrak{C}\pi') B C \alpha > 0.$$

T 3 ideo-

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idèoque etiam harum quoti poſitiui eſſe debent

$$\frac{\mathfrak{B}\pi - B\mathfrak{C}\pi'}{-\mathfrak{B}\pi} > 0$$

$$\frac{((m-1)\Phi + \pi - \mathfrak{C}\pi')BC}{\mathfrak{B}\pi - B\mathfrak{C}\pi'} > 0$$

quae poſterior ob $(m-1)\Phi = -\pi + \pi' - \pi''$ abit in hanc

$$\frac{(\mathfrak{C}\pi' - C\pi'')B}{\mathfrak{B}\pi - B\mathfrak{C}\pi'} > 0.$$

ſicque quinque habentur conditiones ab α liberae, quibus ſatisfieri oportet. Nunc autem aequatio pro deſtruendo margine colorato ita ſe habebit:

$$0 = NBC\pi'' - \frac{N'\Phi}{\Phi - \mathfrak{B}\pi}((B+1)C\pi'' - \pi)$$
$$+ \frac{N''\Phi}{\Phi - \pi + \mathfrak{C}\pi'}((C+1)\pi'' - \pi')$$

His quomodocunque obſeruatis perpendatur aequatio vltima pro confuſione penitus deſtruenda

$$0 = N - \frac{N'}{k\mathfrak{B}} + \frac{N''}{kk'B\mathfrak{C}} - \frac{N'''}{m.BC.}$$

ob $p = \alpha, q = \mathfrak{B}b = -\frac{\mathfrak{B}\alpha}{k}$; $r = c = \frac{B\mathfrak{C}\alpha}{kk'}$

et $s = d = -\frac{BC\alpha}{m}$

num ei vel abſolute vel ſaltim proxime ſatisfieri queat.

Denique vt etiam confuſio prior tollatur ſatisfiat huic aequationi:

$$\mu\lambda - \frac{\mu'\lambda'}{k\mathfrak{B}^3} + \frac{\mu''\lambda''}{kk'B^3\mathfrak{C}^3} - \frac{\mu'''\lambda'''}{B^3C^3.m} - \frac{\mu'\nu'}{k\mathfrak{B}B} + \frac{\mu''\nu''}{kk'B^2C\mathfrak{C}} = 0.$$

Scho-

Scholion.

170. Cum hic in genere vix vlterius progredi liceat, ad caſus particulares erit deſcendendum, et quia in capite praecedente lentes perfectae triplicatae id optatum vſum non praeſtiterant, quod confuſio a lente oculari oriunda ab iis non deſtruebatur, hic lentem obiectiuam iterum triplicatam ſtatuamus, vt bina priora interualla euaneſcant, eius vero tres lentes ita definiamus, vt iis etiam confuſio a lente oculari oriunda deſtruatur; quo facto deinceps forte via patebit inter tres lentes priores exigua interualla ſtatuendi. Semper enim in huiusmodi disquiſitionibus arduis expedit a caſibus facilioribus exordiri, quoniam inde ratio perſpicitur difficultates ſuperandi, quae primo intuitu inuincibiles erant viſae.

Problema 4.

171. Si tres lentes priores inter ſe immediate iungantur, vt lentem obiectiuam triplicatam conſtituant, quarta vero lens ſit ocularis, regulas pro conſtructione huiusmodi teleſcopii exponere.

Solutio.

Cum hic ſint interualla tam $\alpha + b = o$, quam $\mathfrak{G} + c = o$ fiet ſtatim $k = 1$ et $k' = 1$, vnde ſequuntur diſtantiae

$b =$

$$b = -\alpha;\ \mathfrak{b} = -B\alpha;\ c = B\alpha,$$
$$\gamma = BC\alpha \text{ et } d = -\frac{BC\alpha}{m}$$

hincque interuallum

$$\gamma + d = BC\alpha\left(1 - \frac{1}{m}\right) = \frac{m-1}{m} \cdot BC\alpha$$

quod debet eſſe poſitiuum. Pro litteris autem π, π', π'' habebimus

$$1^{\circ}.\ \pi = 0;\quad 2^{\circ}.\ \pi' = 0.$$
$$3^{\circ}.\ \pi'' = -(m-1)\Phi.$$

Atque hinc aequatio pro tollendo margine colorato erit

$$0 = NBC - N'(B+1)C + N''(C+1)$$

vnde elicimus

$$C = -\frac{N''}{NB - N'(B+1) + N''}$$

vnde interuallum $\gamma + d$ ſit

$$= -\frac{(m-1)}{m} \cdot \frac{N''.B\alpha}{NB - N'(B+1) + N''}.$$

quod cum eſſe debeat poſitiuum, duo caſus ſunt perpendendi.

Alter, quo $\alpha > 0$. tum eſſe debet

$$\frac{N''.B}{NB - N'(B+1) + N''} < 0.$$

ideoque

$$N - \frac{N'}{B}(B+1) + \frac{N''}{B} < 0.$$

ſiue

ſiue $N - N' + \frac{1}{B}(N'' - N') < o.$

Altero caſu, ſi $\alpha < o$; contrarium euenire debet, ſcilicet

$$N - N' + \frac{1}{B}(N'' - N') > o.$$

Conſideretur nunc aequatio, qua iſta confuſio penitus tollitur, ſcilicet

$$o = N - \frac{N'}{\mathfrak{B}} + \frac{N''}{B\mathfrak{C}} - \frac{N'''}{mBC}$$

ex qua per B C multiplicata, vt ſit

$$o = NBC - N'(B+1)C + N''(C+1) - \frac{N'''}{m}$$

quoniam a praecedente aequatione non differt, niſi vltimo termino $\frac{N'''}{m}$ qui prae reliquis eſt valde paruus, concludimus, ſi illi fuerit ſatisfactum, ſimul quoque huic proxime ſatisfieri idque eo magis, quo maior fuerit multiplicatio m, quae concluſio nititur fundamento, quod numeri N, N′, N″, N‴ parum ab vnitate differunt.

Pro priore autem confuſione tollenda inſuper ſatisfieri debet huic aequationi

$$o = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^2} + \frac{\mu''\lambda''}{B\,\mathfrak{C}^2} - \frac{\mu'''\lambda'''}{mB\,C^2} - \frac{\mu'\nu'}{\mathfrak{B}B} + \frac{\mu''\nu''}{B\,C\mathfrak{C}}$$

in qua loco C eius valor ſupra inuentus

$$C = \frac{-N''}{B(N - N') - N' + N''}$$

ſubſtitui debet, id quod in genere ad formulam valde moleſtam deduceret, quare ſolutio non niſi caſibus particularibus abſolui poterit.

Coroll. 1.

172. Etſi conditiones pro littera B ſunt datae, haec tamen littera prorſus indeterminata relinquitur, dummodo notetur

1°. ſi fuerit

$$N - N' + \tfrac{1}{B}(N'' - N') < 0$$

tum capi debere α poſitiuum.

2°. Sin autem fuerit

$$N - N' + \tfrac{1}{B}(N'' - N') > 0$$

tum capi debere α negatiuum.

Corollarium 1.

Cum inuenerimus $C = \frac{-N''}{B(N-N') - N' + N''}$

erit $1 + C = \frac{B(N-N') - N'}{B(N-N') - N' + N''}$

hincque $\mathfrak{C} = \frac{-N''}{B(N-N') - N'}$.

Coroll. 2.

173. Si velimus loco B introducere $\mathfrak{B}$ ponendo $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$; tunc conſequemur

$$C = \frac{-N''(1-\mathfrak{B})}{(N-N'')\mathfrak{B} - N' + N''} \quad \text{et} \quad \mathfrak{C} = \frac{-N''(1-\mathfrak{B})}{N\mathfrak{B} - N'}$$

quibus obſeruatis ſubſtitutio poſtrema facilius expedietur: fiet enim poſtrema aequatio

$0 =$

$$0 = \begin{cases} \mu\lambda\mathfrak{B}^3 - \mu'\lambda' - \mu'\nu'\mathfrak{B}(1-\mathfrak{B}) \\ -\frac{\mu''\lambda''(N\mathfrak{B}-N')^3}{(N'')^3} \\ +\frac{\mu''\nu''(1-\mathfrak{B})(N\mathfrak{B}-N')((N-N'')\mathfrak{B}-N'+N'')}{(N'')^2} \\ +\frac{\mu'''\lambda'''((N-N'')\mathfrak{B}-N'+N'')^3}{m(N'')^3} \end{cases}$$

Coroll. 3.

174. Reſpectu campi apparentis cum ſit $\pi'' = -(m-1)\Phi$, ſi ſtatuamus, vt hactenus $\pi'' = -\frac{1}{4}$; prodibit ſemidiameter $\Phi = \frac{1}{4(m-1)}$ et in min. primis $\Phi = \frac{859}{m-1}$ min. prim. ſiquidem lens ocularis fiat vtrinque aequaliter concaua, quod vti oſtendimus fiet ſi $\lambda''' = 1.60006$. hac ſcilicet lente ex vitro coronario parata. Sin autem eam ex vitro chryſtallino parare velimus, poni debet $\lambda''' = 1.67445$.

Exemplum I.

175. Si prima et tertia lens fuerit ex vitro coronario, media ex chryſtallino, ex hisque lens obiectiua conſtituatur, lens vero ocularis ex vitro coronario paretur, pro quauis data multiplicatione teleſcopium conſtruere.

Hoc exemplum ideo affero, quod hic caſus in capite praecedente eſt praetermiſſus, quem autem hic alio modo tractabo, vt longitudo minor prodeat.

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Cum igitur hic ſit $n = 1.53$; $n' = 1.58$; $n'' = 1.53$; $n''' = 1.53$ erit, vti vidimus, $N = 7$, $N' = 10$; $N'' = 7$; $N''' = 7$ ita, vt ſit $\mu'' = \mu$; $\nu'' = \nu$; $\mu''' = \mu$. Ex his colligitur $C = + \frac{7}{3}(1 - \mathfrak{B})$ $\mathfrak{C} = \frac{-7(1-\mathfrak{B})}{7\mathfrak{B} - 10}$. Tantum ergo reſtat haec aequatio reſoluenda

$$0 = \begin{cases} \mu \lambda \mathfrak{B}^3 - \mu' \lambda' - \mu' \nu' \mathfrak{B}(1 - \mathfrak{B}) \\ - \frac{\mu \lambda'' (7\mathfrak{B} - 10)^3}{7^3} \\ - \frac{3\mu\nu(1-\mathfrak{B})(7\mathfrak{B}-10)}{7^2} - \frac{27 \cdot \mu \lambda'''}{7^3 \cdot n} \end{cases}$$

quodſi ergo ſtatuamus $\lambda'' = \lambda$ et $\lambda''' = 1.60006$. haec aequatio induet hanc formam

$$\begin{aligned} &\mu \lambda \left(\tfrac{30}{7} \mathfrak{B}^2 - \tfrac{300}{49} \mathfrak{B} + \tfrac{1000}{343}\right) \\ &- \mu' \lambda' + \mu' \nu' (\mathfrak{B}^2 - \mathfrak{B}) \\ &+ \mu \nu \left(\tfrac{3}{7} \mathfrak{B}^2 - \tfrac{51}{49} \mathfrak{B} + \tfrac{30}{49}\right) \\ &- \frac{27 \cdot \mu \lambda'''}{343 \cdot m} = 0. \end{aligned}$$

quae tantum eſt aequatio quadratica, ex qua valor ipſius $\mathfrak{B}$ erui debet; terminis igitur ſecundum poteſtates ipſius $\mathfrak{B}$ diſpoſitis habebitur:

$$\begin{aligned} &\mathfrak{B}^2 \left(\tfrac{30}{7} \mu \lambda + \mu' \nu' + \tfrac{3}{7} \mu \nu\right) \\ &+ \mathfrak{B} \left(- \tfrac{300}{49} \mu \lambda - \mu' \nu' - \tfrac{51}{49} \mu \nu\right) \\ &+ \tfrac{1000}{343} \mu \lambda - \mu' \lambda' + \tfrac{30}{49} \mu \nu \\ &- \frac{27 \cdot \mu \lambda'''}{343 \cdot m} = 0. \end{aligned}$$

Reſo-

Reſolutionem autem huius aequationis ita inſtituamus, vt lens obiectiua maiorem aperturam admittat, quem in finem, non vt ante, $\lambda = 1$, ſed $\lambda = 1.60006$ ſtatuamus, vt prima lens vtrinque ſibi ſimilis euadat; quare cum ſit

$$\log. \mu = 9.9945371$$

$$\log. \mu\nu = 9.3360593. \quad \text{l.}\ \mu' = 9.9407157$$

$$\log. \mu'\nu' = 9.3436055. \quad \mu'\nu' = 0.2206$$

$$\text{et Log.}\ \lambda = \text{Log.}\ \lambda''' = 0.2041363$$

at pro ſecunda lente ponatur non, vt ante, $\lambda = 1$, ſed hanc litteram indeterminatam relinquamus; vnde noſtra aequatio in numeris ita erit comparata:

$$0 = 7.0852\,\mathfrak{B}^2 - 10.1201\,\mathfrak{B} + 4.7393 - \mu'\lambda' - 0.\frac{12477}{m}.$$

quae reducitur ad hanc

$$\mathfrak{B}^2 = \frac{10.1201}{7.0852}\mathfrak{B} - \frac{4.7393}{7.0852} + \frac{\mu'\lambda'}{7.0852} + \frac{0.12477}{m.7.0852}.$$

$$\mathfrak{B}^2 = 1.4283\,\mathfrak{B} - 0.6689 + 0.1231\,\lambda' + 0.\frac{01755}{m}.$$

Vnde inuenitur

$$\mathfrak{B} = 0.7142 \pm \sqrt{\left(-0.1589 + 0.1231\,\lambda' + \frac{0.01755}{m}\right)}$$

 Vnde

Vnde patet, λ' capi debere vnitate maius. Statuatur ergo $\lambda' = 1\frac{1}{2}$ eritque

$$\mathfrak{B} = 0.7142 \pm \sqrt{(0.0257 + \frac{0.01755}{m})}$$

hinc autem vlterius progredi non licet, niſi litterae m valores determinatos tribuendo; quem in finem ſequentes caſus adiungimus.

Caſus I.

$m = 10.$

176. Erit hoc caſu $\mathfrak{B} = 0.7142 \pm 0.1657$ ſumtoque ſigno inferiore

$$\mathfrak{B} = 0.5485.$$

vel ſumto ſuperiore ſigno

$$\mathfrak{B} = 0.8799.$$

Sin autem velimus, vt pro $\mathfrak{B}$ vnicus valor $= 0.7142$ prodeat, capi deberet

$$\lambda' = \frac{0.1572}{0.1231} = 1\frac{341}{1231}$$

hocque caſu hic vtamur.

Cum igitur ſit $\lambda' = 1\frac{341}{1231}$ erit $\lambda' - 1 = \frac{341}{1231}$.

$$\text{Log.}\ \sqrt{\lambda' - 1} = 9.7208976$$

$$\text{Log.}\ (\lambda' - 1) = 9.4417952.$$

Cum nunc pro omni multiplicatione ſit $\mathfrak{B} = 0.7142$, ſi quidem capiamus

$\lambda' =$

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$\lambda' = 1.2908 - \frac{0.1425}{m}$

$\lambda' - 1 = 0.2908 - \frac{0.1425}{m}$

hincque erit $1 - \mathfrak{B} = 0.2857$

$B = 2.4998.\ C = 0.6666 = \frac{2}{3}.$

Vnde obtinemus diſtantias

$b = -\alpha;\ \beta = -2.4998.\alpha = -2\tfrac{1}{2}\alpha$

$c = +2.4998.\alpha.$

$\gamma = 1.6665.\alpha,\ d = -0.16665\,\alpha$

Cum nunc ſit $\lambda = 1.60006.$

et $\lambda' = 1.2766$

$\lambda'' = 1.60006 = \lambda'''.$

erit

I. Pro Ima lente vtrinque aequaliter conuexa radius vtriusque faciei $= 1.06.\alpha.$

II. Pro IIda lente ex vitro chryſtallino

$$\frac{1}{F'} = \frac{\varrho\beta + \sigma b \mp \tau(b+\beta)\sqrt{\lambda'-1}}{b\beta}$$

$$\frac{1}{G'} = \frac{\sigma\beta + \varrho b \pm \tau(b+\beta)\sqrt{\lambda'-1}}{b\beta}$$

$$\frac{1}{F'} = \frac{-1.0960 \mp 1.6152}{b\beta}\,\alpha$$

$$\frac{1}{G'} = \frac{-1.0080 \pm 1.6152}{b\beta}.\ \alpha$$

ſum-

ſumtisque ſignis ſuperioribus erit

$$F' = \frac{-b\beta}{2.5512.\alpha} = -0.7039\,\alpha$$

$$G' = \frac{-b\beta}{2.4828\,\alpha} = -1.0070\,\alpha.$$

III. Pro tertia lente ex vitro coronario.

Cum hic ſit $\tau.\sqrt{\lambda'' - 1} = \frac{\sigma - \varrho}{2}$

tum vero $c = 2\tfrac{1}{3}\,\alpha.$ et $\gamma = 1.\tfrac{2}{3}\,\alpha.$ erit $c + \gamma = 4\tfrac{1}{3}\,\alpha.$

$$\frac{1}{F'} = \frac{4.5280 \pm 2.0870}{c\gamma}\,\alpha$$

$$\frac{1}{G'} = \frac{7.3725 \pm 2.0870}{c\gamma}\,\alpha$$

et ex ſignis inferioribus

$$F' = \frac{c\gamma}{1.5410\,\alpha} = 2.7039.\,\alpha$$

$$G' = \frac{c\gamma}{6.3205\,\alpha} = 0.6592.\,\alpha.$$

Hae ergo tres lentes ſibi iunctae aperturam admittent, cuius ſemidiameter aeſtimari poteſt

$$x = 0.1648.\,\alpha. = \tfrac{1}{7}\,\alpha \text{ circiter.}$$

Cum autem ob claritatem eſſe debeat $x = \frac{m}{50}$ dig. $= \frac{1}{5}$ dig. capi debebit circiter $\alpha = \frac{7}{5}$ dig. vnde teleſcopii longitudo $= 1.4999\,\alpha = 1\tfrac{1}{2}\,\alpha = 2,1.$ dig.

IV. Pro quarta lente aequaliter vtrinque concaua. erit rad. vtriusque faciei $=$

$$1.06.\,d. = -1.06.(0.1666)\,\alpha$$
$$= -0.1766.\,\alpha$$
$$= -0.2472.\text{ dig.}$$

Co-

Corollarium 1.

177. Si ergo hoc modo valor ipſius λ' definiatur, praecedentes determinationes pro omnibus multiplicationibus valebunt, excepta ſola lente ſecunda; tum autem pro quarta lente ſemidiameter vtriusque eius faciei capi debet $= -(1.06.).\frac{5}{3}.\frac{\alpha}{m}$ ſiue $= -1.7666.\frac{\alpha}{m}$.

Coroll. 2.

178. Conſtructio autem ſecundae lentis a multiplicatione pendebit, quia valor litterae λ' multiplicationem inuoluit, cum ſit $\lambda' = 1.2908 - \frac{0.1425}{m}$.

Scholion.

179. Haud difficile autem erit, pro quauis multiplicatione ſecundam lentem definire, poſtquam ea iam pro caſu $m = 10$ eſt inuenta; ſtatuatur enim $m = \infty$ erit $\lambda' = 1.2908$. hinc $\lambda' - 1 = 0.2908$. et Log. $\sqrt{(\lambda' - 1)} = 9.7317972$ vnde membrum ambiguum erit $= 1.6562\,\alpha$. vnde pro lente ſecunda erit

$$\frac{1}{F'} = \frac{-1.9360 \mp 1.6562}{b\beta}.\ \alpha$$

$$\frac{1}{G'} = \frac{-1.0080 \mp 1.6562}{b\beta}.\ \alpha$$

ſumtis ergo ſignis ſuperioribus

$$F' = \frac{-b\beta}{3.5922.\alpha} = -0.6959.\alpha$$

$$G' = \frac{-b\beta}{2.4418.\alpha} = -1.0239.\alpha$$

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Nunc igitur ponamus pro multiplicatione quacunque m eſſe

$$F' = -\left(0.6959 + \frac{f}{m}\right)\alpha$$

$$G' = -\left(1.0239 + \frac{g}{m}\right)\alpha$$

et quia poſito $m = 10$.

$$0.6959 + \frac{f}{10} = 0.7039$$

et $$1.0239 + \frac{g}{10} = 1.0070$$

reperitur $f = 0.0800$

$$g = -0.1690$$

quibus inuentis adipiscimur ſequentem teleſcopii conſtructionem.

I. Pro prima lente Crown Glaſſ.

radius vtriusque faciei $= + 1.06\,\alpha$.

II. Pro ſecunda lente Flint Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = -\left(0.6959 + \frac{0.0800}{m}\right)\alpha \\ \text{poſter.} = -\left(1.0239 - \frac{0.1690}{m}\right)\alpha \end{cases}$

III. Pro tertia lente Crown Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = + 2.7039\,\alpha \\ \text{poſter.} = + 0.6592.\,\alpha \end{cases}$

IV. Pro quarta lente Crown Glaſſ.

rad. vtriusque faciei $= - 1.7666.\,\frac{\alpha}{m}$

quibus

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quibus lentibus paratis ternae priores ſibi invicem iungantur, poſt quas tertia collocetur intervallo $= \frac{m-1}{m} \cdot \frac{5}{3} \cdot \alpha$.

Cum porro ſit $x = \frac{m}{50}$ dig. et invenerimus $x = 0.1648\,\alpha$, hinc colligitur fore $\alpha = \frac{m}{8,2400}$, ita, vt ſtatui poſſit $\alpha = \frac{4}{33} \cdot m$ ſeu $\alpha = \frac{12}{100} \cdot m$. Quare habetur

Conſtructio teleſcopii primi generis:

I. Pro prima lente Crown Glaſſ.

rad. fac. vtriusque $= 0.1272 \cdot m$.

II. Pro ſecunda lente Flint Glaſſ.

rad. fac. $\begin{cases} \text{anter.} = -0.0835 \cdot m - 0.0096 \\ \text{poſter.} = -0.1229 \cdot m + 0.0202. \end{cases}$

III. Pro tertia lente Crown Glaſſ.

rad. fac. $\begin{cases} \text{anter.} = 0.3244 \cdot m \\ \text{poſter.} = 0.0791 \cdot m \end{cases}$

quibus tribus lentibus immediate iunctis poſtea intervallo $= \frac{1}{5}(m-1)$ dig. ſtatuatur lens ocularis.

IV. Pro quarta lente. Crown Glaſſ.

radius vtriusque faciei $= -0.2119$.

CASUS 2.

180. Cum pro omnibus multiplicationibus conſtructio teleſcopii ſit tradita, ſolutionem exempli ſupra allati alio modo expediamus. Scilicet cum hic

pro $\mathfrak{B}$ valorem vnitate minorem ſimus conſecuti, qui ſupra vnitate maior prodierat, notatu dignus videtur caſus $\mathfrak{B} = 1$, quem hic euoluamus. Tum autem erit $B = \infty$ et quia diſtantiae determinatrices ſunt α; $b = -\alpha$; $\beta = -B\alpha$; $c = B\alpha$; $\gamma = BC\alpha$, $d = \frac{-BC\alpha}{m}$ neceſſe eſt, vt ſit BC quantitas finita ideoque $C = 0$ et $\mathfrak{C} = C = 0$. Quare ſtatuamus $BC = \vartheta$, vt fiat $\gamma = \vartheta\alpha$ et $d = \frac{-\vartheta\alpha}{m}$ hincque teleſcopii longitudo $= \frac{m-1}{m}.\vartheta\alpha$. His poſitis aequatio pro margine colorato tollendo dabit ob $N = 7$, $N' = 10$, $N'' = N''' = 7$. et $\pi = 0$, $\pi' = 0$, $\pi'' = -(m-1)\Phi$.

$$0 = N\vartheta - N'\vartheta + N''$$

$$0 = 7\vartheta - 10\vartheta + 7; \text{ hincque } \vartheta = \tfrac{7}{3}.$$

Nunc autem aequatio pro confuſione primae ſpeciei tollenda fiet

$$0 = \mu\lambda - \mu'\lambda' + \frac{27\mu\lambda''}{343} - \frac{27\mu\lambda'''}{343\,m}$$

quae per μ diviſa ob $\frac{\mu'}{\mu} = 0.8834$ abit in hanc

$$0 = \lambda - 0.8834\,\lambda' + 0.0787\,\lambda'' - \frac{0.0787.\lambda'''}{m}$$

facta autem lente oculari vtrinque aequali erit

$$\lambda''' = 1.60006 \text{ et}$$

$$0 = \lambda - 0.8834\,\lambda' + 0.0787.\lambda'' - \frac{0.1259}{m}$$

ex qua litteras λ ita definiri conuenit, vt vnitatem minimum ſuperent ſtatuamus ergo $\lambda = 1$ et $\lambda'' = 1$ erit

erit $0.8834\,\lambda' = 1.0787 - \frac{0.1250}{m}$

vnde colligitur

$$\lambda' = 1.2210 - \frac{0.1425}{m}$$

ſola ergo lens ſecunda a multiplicatione m pendet, quam deinceps ſeorſim euoluamus

Calculum ergo pro prima, tertia et quarta inſtituamus

Pro prima autem eſt

$$F = \frac{\alpha}{\sigma} = 0.6023.\,\alpha$$

$$G = \frac{\alpha}{\varrho} = 4.4111.\,\alpha.$$

Pro tertia lente ob $\lambda'' = 1.$ et $c = \infty$,

$$F = \frac{\gamma}{\sigma} = 1.4055.\,\alpha$$

$$G = \frac{\gamma}{\varrho} = 10.2925.\,\alpha.$$

Pro quarta lente.

radius vtriusque faciei $= 1.06.\,d$

$$= -\frac{2.4733.\,\alpha}{m}$$

et tota teleſcopii longitudo $= \frac{m-1}{m}.\,\frac{7}{3}.\,\alpha.$

Pro ſecunda autem lente cum in genere ſit

$$F = \frac{b\beta}{\varrho\beta + \sigma b \pm \tau(b+\beta)\sqrt{\lambda - 1}}.$$

$$G = \frac{b\beta}{\sigma\beta + \varrho b \mp \tau(b+\beta)\sqrt{\lambda - 1}}$$

ob $\beta = \infty$ et $b = \mp \alpha$ erit

$$F = \frac{-\alpha}{\varrho \pm \tau \sqrt{(\lambda - 1)}}.$$

$$G = \frac{-\alpha}{\sigma \mp \tau \sqrt{(\lambda - 1)}}$$

pro quo duo caſus ſunt euoluendi alter, quo $m = 10$ et alter, quo $m = \infty$.

Priore erit ob $m = 10$

$\lambda' = 1.2068$; $\lambda' - 1 = 0.2068$.

et Log. $\sqrt{(\lambda' - 1)} = 9.6577753$
Log. $\tau = 9.9432471$

9.6010224

cui logarithmo reſpondet 0.3990 ideoque

$$F = \frac{-\alpha}{0.1414 \pm 0.3990}$$

$$F = \frac{-\alpha}{0.5404} = -1.8505.\alpha$$

$$G = \frac{-\alpha}{1.5427 \mp 0.3990}$$

$$G = \frac{-\alpha}{1.1437} = -0.8467.\alpha.$$

Altero caſu ob $m = \infty$ erit

$\lambda' = 1.2210$ et $\lambda' - 1 = 0.2210$

et Log. $\sqrt{(\lambda' - 1)} = 9.6721961$
Log. $\tau = 9.9432471$

9.6154432

hinc $\tau.\sqrt{\lambda' - 1} = 0.4125$.

Vnde

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Vnde ſit

$$F = \frac{-\alpha}{0.1414 \pm 0.4125} = \frac{-\alpha}{0.5539}$$

$$G = \frac{-\alpha}{1.5827 + 0.4125} = \frac{-\alpha}{1.1702}$$

hincque

$$F = -1.8054.\alpha$$

$$G = -0.8545.\alpha$$

quare ſtatuamus pro multiplicatione quacunque m

$$F = -\left(1.8054 + \frac{f}{m}\right)\alpha$$

$$G = -\left(0.8545 + \frac{g}{m}\right)\alpha$$

et ex caſu $m = 10$ elicimus

$$f = 0.4510;\ g = -0.0780$$

ita, vt ſit

$$F = -\left(1.8054 + \frac{0.4510}{m}\right)\alpha$$

$$G = -\left(0.8545 - \frac{0.0780}{m}\right)\alpha$$

pro α autem definiendo conſideretur radius minimus in lente hac obiectiua triplicata occurrens 0.6023α, cuius pars quarta $0.1506.\alpha = \frac{m}{50}$; ſicque prodibit $\alpha = \frac{m}{7.5330}$ dig. Sumatur ergo $\alpha = \frac{2m}{15}$ dig. et habetur ſequens

Con-

Conſtructio Teleſcopii primi generis.

I. Pro prima lente Crown Gl.

radius faciei $\begin{cases} \text{anter.} = +\, 0.0803.\, m.\ \text{dig.} \\ \text{poſter.} = +\, 0.5882.\, m.\ \text{dig.} \end{cases}$

II. Pro ſecunda lente Flint Glaſſ.

rad. fac. $\begin{cases} \text{anter.} = (-0.2407.m - 0.0601)\ \text{dig.} \\ \text{poſter.} = (-0.1139.m + 0.0104)\ \text{dig.} \end{cases}$

III. Pro tertia lente Crown Glaſſ.

radius faciei $\begin{cases} \text{anter.} = +\, 0.1874.\, m.\ \text{dig.} \\ \text{poſter.} = +\, 1.3723.\, m.\ \text{dig.} \end{cases}$

IV. Pro quarta lente Crown Glaſſ.

radius vtriusque faciei $= 0.3298.$ dig.

Tribus prioribus lentibus inuicem iunctis quartae ab iis interuallum erit

$= \frac{14}{45}(m-1)$ dig.

et campi apparentis ſemidiameter erit, vt hactenus, $\Phi = \frac{850}{m-1}$. min. prim.

Scholion.

181. Quia in hac ſolutione poſuimus $\lambda = 1$ et $\lambda'' = 1$, conſuluimus potiſſimum artifici, quia hoc caſu errores in exſecutione commiſſi non admodum negotium turbant, ſed longitudo horum teleſcopiorum prodiit aliquanto maior, propterea quod radius ſatis exi-

exiguus in determinatione lentis obiectiuae occurrebat, huic autem incommodo medelam afferemus, ſi pro prima et tertia lente ſtatuamus $\lambda = 1.60006$, quo facto obtinebitur $\lambda' = 1.9536 - \frac{0.1125}{m}$ qui numerus tantum in ſecundam lentem influit, cuius conſtructionem deinceps inueſtigemus.

Iam vero erit

Pro prima lente Crown Glaſſ.

radius vtriusque faciei $= 1.06.\,\alpha$.

Pro tertia lente Crown Glaſſ.

radius vtriusque faciei $= 1.06.\,\gamma$
$= 2.473\,\alpha$.

Pro quarta lente

radius vtriusque faciei $= -\frac{2.14757.\alpha}{m}$.

Reſtat igitur, vt ſecundam lentem euoluamus, vt ante.

Scilicet duos caſus contemplabimur, alterum, quo $m = 10$, alterum, quo $m = \infty$.

Sit igitur primo $m = 10$ eritque $\lambda' = 1.93937$ et $\lambda' - 1 = 0.93937$ et $\tau.\sqrt{(\lambda' - 1)} = 0.85048$.

Quare

$$F = \frac{-\alpha}{0.1414 \pm 0.85048}.$$

$$G = \frac{-\alpha}{1.5827 + 0.85048}.$$

ſeu

$$F = \frac{-\alpha}{0.9919} = -1.0082.\,\alpha$$

$$G = \frac{-\alpha}{0.7322} = -1.3657.\,\alpha$$

Sit nunc $m = \infty$, erit

$$\lambda' = 1.9536$$

$$\tau \sqrt{(\lambda' - 1)} = 0.8569$$

hincque

$$F = \frac{-\alpha}{0.1414 \pm 0.8569}$$

$$G = \frac{-\alpha}{1.5827 \mp 0.8569}$$

$$F = \frac{-\alpha}{0.9983} = -1.0017.\,\alpha$$

$$G = \frac{-\alpha}{0.7258} = -1.3778.\,\alpha$$

Nunc pro multiplicatione quacunque m ſtatuatur

$$F = -\left(1.0017 + \frac{f}{m}\right)\alpha$$

$$G = -\left(1.3778 + \frac{g}{m}\right)\alpha$$

et ex priore caſu $m = 10$ colligitur

$$f = 0.065;\; g = -0.121$$

ita, vt ſit pro lente ſecunda

rad. fac. $\begin{cases} \text{anter.} = -\left(1.0017 + \frac{0.065}{m}\right)\alpha \\ \text{poſter.} = -\left(1.3778 - \frac{0.121}{m}\right)\alpha \end{cases}$

Hic

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Hic iam tuto ſumi poteſt $x = \frac{1}{4}\alpha = \frac{m}{50}$; hinc obtinetur $\alpha = \frac{8}{100} \cdot m$

Hinc ergo orietur ſequens

Teleſcopii primi generis Conſtructio:

I. Pro lente prima: Crown Glaſſ.

radius vtriusque faciei $= 0.0848. m.$ dig.

II. Pro lente ſecunda

rad. fac. $\begin{cases} \text{anter.} = (-0.08014m. - 0.0052) \text{ dig.} \\ \text{poſter.} = (-0.110224m. + 0.0097) \text{dig.} \end{cases}$

III. Pro lente tertia Crown Glaſſ.

radius vtriusque faciei $= 0.19784. m.$ dig.

IV. Pro lente quarta.

radius vtriusque faciei $= -0.19786.$ dig.

Tribus lentibus prioribus ſibi immediate iunctis ad interuallum $= (0,187)(m-1)$ dig. collocetur lens quarta, cui oculus immediate adplicatus cernet campum, cuius ſemidiameter erit $\frac{859}{m-1}$. min. prim.

Scholion.

182. Hic caſus inprimis eſt omni attentione dignus, quoniam pro quauis multiplicatione huius generis teleſcopia breviſſima ſuppeditat: ſi enim multiplicationem adeo centuplam deſideremus, longitudo vix ſuperabit $18\frac{1}{2}$ digitos. Haec igitur methodus,

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qua poſuimus $\mathfrak{B} = 1$ vtique mereretur, vt etiam ad alias vitri ſpecies ſeu vbi pro lentibus alia combinatio vitri coronarii et chryſtallini ſtatueretur, ſeorſim adplicaretur. Sed quia ea etiam ad problema noſtrum generale ſoluendum aeque felici ſucceſſu in vſum vocari poteſt eiusque beneficio inſignes difficultates ſupra commemoratae euaneſcunt, expediet ſequens problema generalius tractaſſe.

Problema 5.

183. Si teleſcopium ex quatuor lentibus ſit conſtruendum, duae priores vero lentes ita debeant eſſe comparatae, vt radii per eas transmiſſi iterum inter ſe fiant paralleli, regulas pro conſtructione deſcribere.

Solutio.

Cum igitur radii per ſecundam lentem refracti iterum fiant axi paralleli, erit $\beta = \infty$ ideoque $\frac{\beta}{b} = B = \infty$ et $\mathfrak{B} = 1$, erunt diſtantiae determinatrices

$$b = -\frac{\alpha}{k};\ \beta = -\frac{B\alpha}{k} = \infty$$

$$c = \frac{B\alpha}{kk'};\ \gamma = \frac{BC\alpha}{k.k'};\ d = -\frac{BC\alpha}{m}.$$

hicque iam notari oportet, vt diſtantia inter primam et ſecundam lentem $\beta + c$ fiat finita, debere ob $\beta = \infty$ eſſe $c = -\infty$, vnde fit $k' = 1$.

Quo

Quo autem rem clarius explicemus, ſtatuatur haec diſtantia $=\eta\alpha$, vt ſit $B\alpha\left(\frac{1}{kk'}-\frac{1}{k}\right)=\eta\alpha$ vnde ſit $k'=\frac{B}{B+\eta k}$, quae ob $B=\infty$ ſit $k'=1$; interim tamen conueniet, illam expreſſionem $k'=\frac{B}{B+\eta k}$ in vſum ſequentem notaſſe.

Deinde quia $c=\infty$, γ vero finita quantitas, erit $\frac{\gamma}{c}=C=0$, hincque etiam $\mathfrak{C}=\frac{C}{1+C}=C=0$; interim tamen productum B C debet eſſe finitum. Sit igitur $BC=\vartheta$, vt fiat $\gamma=\frac{\vartheta\alpha}{k}$ et $d=\frac{-\vartheta\alpha}{m}$; cum illa autem aequatione coniungi debet iſta, qua ſummo rigore eſt $C=\frac{\gamma}{c}=\frac{\vartheta}{B}$, hincque $\mathfrak{C}=\frac{\vartheta}{B+\vartheta}$. His notatis erunt interualla lentium

$$\alpha+b=\alpha\left(\frac{k-1}{k}\right)$$

$$\beta+c=\eta\alpha$$

$$\gamma+d=\left(\frac{1}{k}-\frac{1}{m}\right)\vartheta.\alpha$$

$$=\frac{m-k}{km}.\vartheta\alpha.$$

Vnde hae fractiones $\frac{\eta k}{k-1}$ et $\frac{m-k}{m(k-1)}\vartheta$ debent eſſe poſitiuae, ſeu $\frac{\eta}{k-1}>0$; $\frac{m-k}{k-1}.\vartheta>0$ ſeu $\frac{m-k}{\eta}\,\vartheta>0$. Iam inquiramus in valores litterarum π, π' et π'', ex tribus ſequentibus aequationibus definiendos

$$\text{I. } \mathfrak{B}\pi-\Phi=-k\Phi$$

$$\text{II. } \mathfrak{C}\pi'-\pi+\Phi=kk'\Phi$$

$$\text{III. } (m-1)\Phi=-\pi+\pi'-\pi''$$

 qua-

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quarum prima ſtatim dat ob $\mathfrak{B} = 1$

$$\pi = (1 - k)\Phi = -(k-1)\Phi$$

vnde vt hic valor campo augendo inſeruiat, π numerus negatiuus eſſe debet ideoque $k > 1$.

Secunda autem aequatio ob $\mathfrak{C} = 0$ et $K' = 1$. daret $-\pi + \Phi = k\Phi$, vnde pro π' nihil concludere liceret, quare pro $\mathfrak{C}$ valores illos exactiores ſcribi oportebit fietque

$$\frac{\theta}{B+\theta}\cdot \pi' - \pi + \Phi = \frac{Bk}{B+\eta k}\Phi$$

quae ob $\pi = -(k-1)\Phi$ abit in hanc

$$\frac{\theta}{B+\theta}\cdot \pi' + k\Phi = \frac{Bk}{B+\eta k}\Phi$$

ſeu $\frac{\theta}{B+\theta}\pi' = \frac{-\eta k^2}{B+\eta k}\cdot\Phi$

quae ergo ob $B = \infty$ dat

$$\pi' = \frac{-\eta k^2}{\theta}\cdot\Phi$$

quia autem conuenit ſumere $k > 1$ debet eſſe $\alpha > 0$ ideoque et $\eta > 0$, hic valor π' erit negatiuus, ſi fuerit $\vartheta > 0$; ſin autem $\vartheta < 0$, is erit poſitiuus, vbi autem meminiſſe oportet eſſe debere $(m-k)\vartheta > 0$.

Tertia denique aequatio abit in hanc formam:

$$(m-1)\Phi = +(k-1)\Phi - \frac{\eta k^2}{\theta}\Phi - \pi''$$

hincque $\pi'' = (k - m - \frac{\eta k^2}{\theta})\Phi$

ſiue $\pi'' = -(m - k + \frac{\eta k^2}{\theta})\Phi$

quae

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quae formula cum etiam inſeruiat campo definiendo, ſi capiatur $\pi'' = -\frac{1}{4}$; reperitur

$$\Phi = \frac{859.}{m - k + \frac{\eta k^2}{\theta}} \text{ minut.}$$

$$\Phi = \frac{859.\theta}{(m-k)\theta + \eta k^2}$$

quare curandum eſt, vt $\frac{\eta k^2}{\theta}$ quam minimum reddatur, quod facile praeſtatur faciendo interuallum ſecundae et tertiae lentis quam minimum adeoque euaneſcens, quo caſu erit $\Phi = \frac{859}{m-k}$ qui eo maior fit, quo maior ſumitur k. Nunc igitur aequationem pro margine colorato tollendo conſideremus, quae erit

$$0 = N\vartheta\pi'' - \frac{N'}{k}(\vartheta\pi'' - \pi)$$
$$+ \frac{N''}{k}(\pi'' - \pi')$$

quae ſubſtitutis valoribus dat

$$0 = -N((m-k)\vartheta + \eta k^2)$$
$$+ \frac{N'}{k}((m-k)\vartheta + \eta k^2 - k + 1)$$
$$- \frac{N''}{k}(m-k)$$

ex qua aequatione ϑ commode definiri poteſt reperieturque.

$$\vartheta = \frac{-N\eta k^2 + N'\eta k^2 - N'(k-1) - N''(m-k)}{(m-k)(Nk-N')}$$

quia autem conuenit η quam minimum aſſumere ac praeterea non neceſſe eſt, vt iſti aequationi ſummo rigore ſatisfiat, his terminis omiſſis habebimus.

$\vartheta =$

$$\vartheta = \frac{-N'(k-1)-N''(m-k)}{(m-k)(Nk-N')}$$

vnde fit

$$(m-k)\,\vartheta = \frac{-N'(k-1)-N''(m-k)}{Nk-N'}$$

quae quantitas cum debeat eſſe poſitiua, numerator autem manifeſto ſit negatiuus; etiam denominatorem negatiuum eſſe oportet ideoque $N' > Nk$. Quodſi ergo N' maximum habeat valorem ex vitro ſcilicet chryſtallino, N vero minimum ex vitro coronario, vt ſit $N = 7$ et $N' = 10$; numerus k non amplius noſtro arbitrio relinquitur, ſed ita capi debet, vt fiat $7k < 10$; et $k < \frac{10}{7}$ ſeu contineri debet intra limites 1 et $\frac{10}{7}$. Notetur hic, ſi caperetur $k = 1$, caſum praecedentem eſſe oriturum, neque campum hinc auctum iri; Sin autem capiatur $k = \frac{10}{7}$ foret $\vartheta = \infty$ et longitudo teleſcopii fieret infinita; vnde conueniet k propius vnitati; quam alteri limiti aſſumere. His probe perpenſis ſtatuamus $k = \frac{8}{7}$, $N = 7$: $N' = 10$. $N'' = 7$, quo ϑ obtineat valorem minorem. Vnde fiet $\vartheta = \frac{49m-46}{2(7m-8)}$ hincque $\frac{\eta k^2}{\vartheta}$ habebit hunc valorem $\frac{2\cdot 8^2(7m-8)}{7^2(49m-46)}\cdot\eta$ qui ſumto $m = \infty$ fit $= \frac{128}{343}\cdot\eta$ ex quo colligitur, ſi modo η non excedat $\frac{1}{10}$ campi diminutionem non fore ſenſibilem.

Denique pro ſemidiametro confuſionis ad nihilum redigendo ſatisfiat huic aequationi:

$$0 = \mu\lambda - \frac{\mu'\lambda'}{k} + \frac{\mu''\lambda''}{k\theta^3} - \frac{\mu'''\lambda'''}{m\theta^3}$$

ex

ex qua commodiſſime definiemus λ', qui erit ob $\mu' = \mu'' = \mu'''$

$$\lambda' = \frac{\mu}{\mu'}\left(k\lambda + \frac{\lambda''}{\theta^2} - \frac{k\lambda'''}{m\theta^3}\right)$$

ſicque hoc problema feliciter eſt ſolutum.

Coroll. I.

184. Diſtantiae ergo determinatrices ſingularum lentium erunt

Pro príma: ∞ et α cum λ

Pro ſecunda: $b = \frac{-\alpha}{k}$; et $\beta = \infty$ cum λ'

Pro tertia: $c = \infty$ et $\gamma = \frac{\theta\alpha}{k}$ cum λ''

Pro quarta: $d = \frac{-\theta\alpha}{m}$; $\delta = \infty$ cum λ'''.

vbi notandum, primam, tertiam et quartam ex vitro coronario, ſecundam ex chryſtallino eſſe parandam; tum vero fore interualla lentium

$$\alpha + b = \alpha\left(\frac{k-1}{k}\right) = \frac{1}{i}\alpha$$

$$\beta + c = \eta\alpha$$

de qua diſtantia notetur, eam ſtatui debere quam minimam; ac denique

$$\gamma + d = \frac{m-k}{km} . \vartheta\alpha$$

vnde tota longitudo prodit

$$= \alpha\left(\frac{k-1}{k} + \eta + \frac{m-k}{km} . \vartheta\right).$$

Coroll. 2.

185. Pro litteris autem k et ϑ hos valores ſtatuimus, $k = \frac{8}{7}$, $\vartheta = \frac{40m - 46}{2(7m - 6)}$, quae expreſſio cum adhuc m inuoluat, calculum non, vt ante, pro quauis multiplicatione in genere abſoluere licebit; interim tamen ſimili modo, quo ante vſi ſumus, poſtquam pro duabus tribusue multiplicationibus calculum abſoluerimus, interpolando formulas generaliores pro omni multiplicatione concludere poterimus.

Scholion.

186. Haec teleſcopia iis, quae modo ante deſcripſimus, ideo erunt praeferenda, quod in his nullae lentes ſibi immediate iunctae aſſumuntur, quippe quod in praxi locum habere nequit; tum vero etiam quod aliquod campi augmentum largiuntur. Ceterum haec teleſcopia aliquanto fiunt longiora, tam ob diſtantiam inter lentes primam et ſecundam, quam potiſſimum ob maiorem valorem ipſius ϑ, a quo interuallum tertiae et quartae lentis potiſſimum pendet. Interuallum autem medium $\eta\,\alpha$ hic merito negligimus. Quo tamen breuitati inſtrumenti quantum fieri licet, conſulamus, expediet ſine dubio, vt modo ante fecimus, tam primam et tertiam lentem, quam quartam vtrinque aequales formare, ita, vt ſit $\lambda = \lambda'' = \lambda''' = 1{,}60006$; tum vero erit $\mu = 0.9875$; $\mu' = 0.8724$. vnde harum lentium conſtructio ſtatim ſequitur.

Erit ſcilicet radius vtriusque faciei.

I. Pro lente prima

$= 1.06\, a.$

II. Pro lente tertia

$= 1.06. \frac{\theta a}{k}.$

III. Pro lente quarta

$= -1.06. \frac{\theta a}{m}.$

Nihil igitur aliud reſtat, niſi vt pro quibusdam multiplicationibus calculum expediamus; ac primo quidem conueniet, multiplicationem quandam exiguam $m = 5$ euoluere, vt pateat, quantum haec inueſtigatio in minimis teleſcopiis huius generis praeſtare poſſit; tum vero multiplicationem quandam maiorem veluti $m = 10$, indeque ſubito $m = \infty$ euoluamus, vt ex horum caſuum comparatione concluſionem pro quauis maiore multiplicatione formare queamus.

Exemplum I.

$m = 5.$

187. Teleſcopium pro multiplicatione $m = 5$ deſcribere.

Erit hoc caſu

$$\left.\begin{aligned} \vartheta &= \tfrac{199}{54} = 3.6852 \\ \text{Log. } \vartheta &= 0.5664593 \end{aligned}\right\} \text{ et Log. } k = 0.0579920$$

hincque $b = -\frac{7}{4} a$; $\beta = \infty$; $c = \infty$

 $\gamma =$

$$\gamma = 3,2246.\alpha$$
$$d = -0,7370.\alpha$$

hincque

$$\alpha + b = \tfrac{1}{1}\,\alpha;\ \beta + c = \eta\,\alpha = \text{minimo.}$$
$$\gamma + d = 2.4876.\alpha$$

ſicque longitudo tota teleſcopii erit $= 2.6126\,\alpha + \eta\alpha$.

Vnde tres lentes ex vitro coronario parandae ita ſe habebunt:

I. Pro prima lente

radius vtriusque faciei $= 1.06.\alpha$.

II. Pro tertia lente

radius vtriusque faciei $= 3.4181.\alpha$.

III. Pro quarta lente

radius vtriusque faciei $= -0.7812.\alpha$.

IV. Pro ſecunda lente. Flint Glaſſ.

ante omnia quaeri debet numerus λ'. ex formula

$$\lambda' = 1.\tfrac{600-6.\mu}{\mu'}\left(k + \tfrac{1}{\theta^3} - \tfrac{k}{m\theta^3}\right) \text{ vnde}$$
$$\lambda' = 2,0977;\ \text{ergo } \lambda' - 1 = 1,0977$$

hincque $\tau.\sqrt{(\lambda' - 1)} = 0.91936$.

Quare

Quare pro hac lente erit

$F = \frac{b}{0{,}1414 \pm 0{,}1536} = \frac{b}{1{,}0608}$

$G = \frac{b}{1{,}5827 \mp 0{,}91536} = \frac{b}{0{,}6635}$

$F = -0{,}8248.\, a;\ G = -1{,}3192.\, a$

Vnde fluit ſequens

Conſtructio Teleſcopii

I. Pro lente prima Crown Glaſſ.

radius vtriusque faciei $= + 1.06\, a$

Interuallum $= 0.125\, a.$

II. Pro lente ſecunda Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0.8248\, a \\ \text{poſter.} = -1.3192.\, a \end{cases}$

Interuallum minimum.

III. Pro lente tertia Crown Glaſſ

radius faciei vtriusque $= 3.4181.\, a$

Interuallum $= 2.4876.\, a.$

IV. Pro lente quarta Crown Glaſſ.

radius vtriusque faciei $= -0.7812.\, a.$

Lenti obiectiuae tribui poteſt apertura, cuius ſemidiameter $x = \frac{1}{4}\, a.$

Cum autem ob claritatem ſtatui debeat $x = \frac{m}{50}$. dig. $= \frac{1}{10}$ dig. vnde $a = \frac{2}{5}$. dig.

et teleſcopii longitudo $= 2.6126.\alpha + \eta\alpha$

et ſemidiameter campi $\Phi = 223$ min. $= 3^\circ\, 43'$.

Exempl. II.

188. Si multiplicatio $m = 10$ deſideretur, teleſcopium huius generis deſcribere.

Ob $m = 10$ erit $\vartheta = \frac{444}{124} = 3.5806$

Log. $\vartheta = 0.5539613$.

Log. $\frac{1}{\vartheta} = 9.4460386$.

vnde $b = -\frac{7}{8}\alpha = -0,875\,\alpha$

$\beta = \infty = c;\ \gamma = 3.1331.\alpha$

$d = -0.35806.\alpha$.

Nunc euoluatur numerus λ', qui reperitur

$\lambda' = 2.1049;\ \lambda' - 1 = 1.1049$

hinc $\tau.\sqrt{(\lambda' - 1)} = 0.92132$

Vnde radii facierum

$$F = \frac{b}{0,1414 \pm 0,9213} = \frac{b}{1,0627}$$

$$G = \frac{b}{1,5827 \mp 0,9213} = \frac{b}{0,6614}$$

ſeu $F = -0.8234\,\alpha$

$G = -1.3230\,\alpha$

Vnde colligitur ſequens

Con-

Conſtructio Teleſcopii

I. Pro prima lente Crown Glaſſ.

radius vtriusque faciei $= 1.06.\alpha$

Interuallum $= 0.125.\alpha$

II. Pro ſecunda lente Flint Glaſſ.

rad. fac. $\begin{cases} \text{anter.} = -0.8234.\alpha \\ \text{poſter.} = -1.3230.\alpha \end{cases}$

Interuallum minimum.

III. Pro lente tertia: Crown Glaſſ.

radius vtriusque faciei $= 3.3211.\alpha$

Interuallum $= 2.7751.\alpha$

IV. Pro lente quarta Crown Glaſſ.

radius vtriusque faciei $= -0.3796.\alpha$

Vnde fit tota longitudo $= 2.9001\,\alpha$. Lenti primae autem apertura tribui debet, cuius ſemidiameter $x = \frac{m}{50} = \frac{1}{4}\alpha = \frac{1}{5}$ dig. Vnde ſequitur $\alpha = \frac{4}{5}$ dig. ſiue maius. Campi autem viſi ſemidiameter erit $\Phi = 94$ minut. $= 1^\circ\,34'$.

Exempl. III.

189. Si multiplicatio m fuerit ∞, teleſcopium huius generis deſcribere.

Ob $m = \infty$ erit $\vartheta = 3,5$.

et

et $\text{Log.}\ \vartheta = 0.5440680$

$\text{Log}\ \frac{1}{\vartheta} = 9.4559319$

hincque $b = -0.875.\alpha;\ \beta = \infty = c$

$\gamma = 3.0625.\alpha;\ d = -3,5.\frac{\alpha}{m}.$

Pro lente autem ſecunda inuenimus

$\lambda' = 2.1120;\ \lambda' - 1 = 1,1120$

et $\tau.\sqrt{(\lambda' - 1)} = 0,9253.$

Ex quibus colligitur

$F = \frac{b}{0,1414 \pm 0,9253} = \frac{b}{1,0667}$

$G = \frac{b}{1,0827 \mp 0,9253} = \frac{b}{0,1574}$

$F = -0.8205.\alpha$

$G = -1.3309.\alpha$

Vnde colligitur ſequens

Conſtructio Teleſcopii

I. Pro prima lente Crown Glaſſ.

radius vtriusque faciei $= 1,06.\alpha$

Interuallum $= 0.125.\alpha$

II. Pro ſecunda lente Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0.8205.\alpha \\ \text{poſter.} = -1,3309,\alpha \end{cases}$

Interuallum minimum.

III. Pro

III. Pro tertia lente Crown Glaſſ.

radius vtriusque faciei $= 3.2462.\alpha$

Interuallum $= (3.0625 - \frac{3,5}{m})\alpha$

IV. Pro quarta lente Crown Glaſſ.

radius vtriusque faciei $= -3.710.\frac{\alpha}{m}$

Hincque longitudo teleſcopii erit $= (3,1875 - \frac{3,5}{m}).\alpha$

Lenti vero obiectiuae apertura tribuatur, cuius ſemidiameter $= \frac{1}{4}\alpha = \frac{m}{30}$ ita, vt capi poſſit

$\alpha = \frac{2}{15}.m.$ dig. $= \infty$.

Exempl. IV. generale.

190. Si multiplicatio fuerit quaecunque m, ſaltem denario maior, teleſcopium huius generis deſcribere.

Cum pro caſu $m = \infty$ inuenerimus $\vartheta = 3,5$ nunc in genere ponamus $\vartheta = 3,5 + \frac{e}{m}$ et quia pro $m = 10$ fuerat $\vartheta = 3,5806$, erit $e = 0.806$, ita vt ſit $\vartheta = 3,5 + \frac{0,806}{m}$ vnde diſtantiae ita ſe habebunt:

$b = -0.875.\alpha$; $\beta = \infty = c$

$\gamma = (3,0625 + \frac{0,7053}{m})\alpha$

$d = -(3,5 + \frac{0,8060}{m})\frac{\alpha}{m}$

Pro lente autem ſecunda ponatur

$F = -(0.8205 + \frac{f}{m})\alpha$

$G = -(1.3309 + \frac{g}{m})\alpha$

Comparatione igitur inſtituta cum caſu $m = 10$ erit $f = 0.0290$, $g = -0.0790$.

Conſtructio huius Teleſcopii

I. Pro prima lente. Crown Glaſſ.

radius vtriusque faciei $= 1.06.\alpha$

Interuallum $= 0.125.\alpha$.

II. Pro ſecunda lente. Flint Glaſſ.

rad. faciei $\begin{cases} \text{anter.} = -(0.8205 + \frac{0.0290}{m})\alpha \\ \text{poſter.} = -(1.3309 - \frac{0.0700}{m})\alpha \end{cases}$

Interuallum minimum.

III. Pro tertia lente. Crown Glaſſ.

radius vtriusque faciei $= +(3,2462 + \frac{0.7476}{m})\alpha$

Interuallum $= (3,0625 - \frac{2.7047}{m} - \frac{0.8060}{m^2})\alpha$.

IV. Pro quarta lente.

radius faciei vtriusque $= -(3,710 + \frac{0.8544}{m})\frac{\alpha}{m}$

ſicque tota longitudo erit

$= (3,1875 - \frac{2.7047}{m} - \frac{0.8060}{m^2})\alpha$

deinde lentis obiectiuae ſemidiameter aperturae debet eſſe $x = \frac{m}{50}$ dig. vnde α capi debebit $\alpha = \frac{2}{25}m$. dig. ſiue maius, campique viſi ſemidiameter

$$= \Phi = \frac{859}{m - \frac{8}{7}} \text{ min. prim.}$$

Scho-

Scholion.

191. En ergo inſignem multitudinem variorum primi generis teleſcopiorum, quae adhuc in infinitum multiplicari poſſent, ſi litteris B et C alios valores tribuere vel etiam pluribus lentibus vti vellemus. Verum huiusmodi inueſtigatio prorſus ſuperflua videtur, cum maior perfectionis gradus exſpectari nequeat ac plures lentes claritati ſemper obſint, neque etiam maior campus ſperari poſſit. Inprimis autem obſeruandum eſt in his teleſcopiis marginem coloratum aliter deſtrui non potuiſſe, niſi diuerſis vitri ſpeciebus adhibendis, ita, vt iam affirmare poſſimus, ex eadem vitri ſpecie huiusmodi teleſcopia confici non poſſe, quae non vitio marginis colorati laborent, cum tamen in ſequentibus generibus, lentibus ex vna vitri ſpecie factis talis margo feliciter tolli poſſit, etiamſi tunc ipſum diffuſionis ſpatium ad nihilum redigere non liceat. Haec reſtrictio etiam in cauſſa erat, quod campum apparentem vix notabiliter augere licuerat; ſin autem marginem coloratum negligere vellemus, campus haud mediocriter augeri poſſet. Tum enim in caſu vltimi problematis litterae k et ϑ manerent arbitrio noſtro relictae et cum ſemidiameter campi eſſet $\Phi = -\frac{\pi''}{m-k}$, poſito $\eta = 0$, videtur is ad lubitum augeri poſſe, dum tantum k parum ab m deficiens aſſumatur atque adeo ſumto $k = m$ in infinitum abiret; quod tamen nullo modo praeſtari poſſe experientia abunde teſtatur. Quare hoc dubium ſoluiſſe operae erit

pretium; ad quod tantum recordari oportet, litteris π, π' et π'' certum praeſcriptum eſſe terminum veluti $\frac{1}{4}$ quem transgredi nunquam debent; quare etſi hoc caſu valor $-\pi'' = \frac{1}{4}$ enormem magnitudinem pro Φ praebet, tamen hic etiam ad valorem ipſius π ſpectari conuenit, qui cum ante iam inuentus eſſet $\pi = -\Phi(k-1)$, ideoque $\pi = \frac{\pi''(k-1)}{m-k}$, maxime cauendum eſt, ne hinc prodeat $\pi > \pi''$. quamobrem litteram k iam non pro lubitu augere licebit, ſed eo usque tantum, quoad fiat $k-1 = m-k$, ſiue $k = \frac{m+1}{2}$, quae poſitio campum duplo maiorem, quam ante, produceret, ſcilicet $\Phi = \frac{-\pi''}{m-k} = \frac{-2\pi''}{m-1}$, quem ergo obtinere poſſemus, ſi modo marginem coloratum deſpiciamus. Tum autem pro eodem caſu vltimi problematis forent diſtantiae determinatrices $b = \frac{-2\alpha}{m+1}$; $\beta = \infty = c$ et $\gamma = \frac{+2\theta\alpha}{m+1}$ et $d = \frac{-\theta\alpha}{m}$; vnde fit poſtremum interuallum $\gamma + d = +\vartheta\alpha\left(\frac{2}{m+1} - \frac{1}{m}\right) = \frac{\theta\alpha(m-1)}{m(m+1)}$ vbi adhuc ϑ noſtro arbitrio permittitur, dummodo poſitiue capiatur; verum quia hoc modo margo coloratus praemagnus eſſet proditurus; huiusmodi teleſcopia nullo modo commendari poterunt atque hoc praeceptum etiam in poſterum obſeruabimus, nullaque alia teleſcopia exceptis tantum ſimpliciſſimis proferemus, niſi quae ſaltem a margine colorato ſint immunia, ſiquidem tota haec confuſio non vitari queat.

LIBRI

LIBRI SECVNDI,

DE

CONSTRVCTIONE TELESCOPIORVM

SECTIO SECVNDA.

DE

TELESCOPIIS SECVNDI GENERIS,

QVAE

LENTE OCULARI CONUEXA INSTRVCTA,
OBIECTA SITU INUERSO REPRAESENTANT.

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CAPVT I.

DE
TELESCOPIIS SIMPLICIORIBVS SECVNDI GENERIS, EX VNICA VITRI SPECIE PARATIS.

Praeceptum generale.

192.

Cum in hac ſectione obiectorum repraeſentatio ſemper futura ſit inuerſa, hic ante omnia monendum eſt, in omnibus formulis generalibus ſupra traditis litteram m, qua multiplicatio indicatur, vbique negatiue capi debere, ita, vt in illis formulis, quoties m occurrit, eius loco $-m$ ſcribi oporteat.

Pro-

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Problema I.

193. Simpliciſſimum huius generis teleſcopium ex duabus lentibus eademque vitri ſpecie conſtruere, quod obiecta ſecundum datam multiplicationem m aucta ſituque inuerſo repraeſentet.

Solutio.

Propoſita multiplicatione m formulae noſtrae generales ſtatim praebent hanc determinationem: $m = \frac{\alpha}{b}$ vbi manifeſtum eſt, α exprimere diſtantiam focalem lentis obiectiuae, b vero ocularis ob $\beta = \infty$. Cum igitur fractio $\frac{\alpha}{b}$ hic ſit poſitiua, ſimulque harum lentium diſtantia $\alpha + b$, vtramque diſtantiam α et b, poſitiuam eſſe oportet, ita, vt ambae lentes futurae

Tab. III. Tom. I. Fig. 13. ſint conuexae et imago realis in puncto F repraeſentetur, quod ſimul eſt focus communis vtriusque lentis. Tum vero campi apparentis ſemidiameter erit $\Phi = \frac{\pi}{m+1}$, qui autem non conſpicietur, niſi oculo in certo loco conſtituto cuius diſtantia poſt lentem ocularem eſt $O = \frac{\pi q}{m\Phi}$ denotante q diſtantiam focalem lentis ocularis, quam vidimus eſſe $= b$. Cum igitur ſit $\pi = (m+1)\Phi$; erit haec diſtantia $O = \frac{m+1}{m}. q$ ideoque tantillo maior, quam q. Vt iam obiecta dato claritatis gradu adpareant, quem vocauimus $= y$, ita, vt y ſit menſura ſemidiametro pupillae minor, oſtenſum eſt, aperturam lentis obiectiuae tantam eſſe debere, vt eius ſemidiameter ſit $x = my$ vnde iam intelli-

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telligitur, eius diſtantiam focalem p vel α certe minorem ſtatui non poſſe, quam $4x$. Videamus nunc etiam quomodo hoc teleſcopium ratione marginis colorati futurum ſit comparatum. Cum is prorſus tolli non poſſit, quia fieri nequit, vt ſit $0 = \frac{dn'}{n'-1} \cdot \frac{\pi}{m\Phi} = \frac{dn'}{n'-1} \cdot \frac{m+1}{m}$; multo minus haec confuſio penitus deſtrui poteſt, cum eſſe deberet $0 = \frac{dn}{n-1} \cdot (p+q)$, quia $p+q$ eſt diſtantia lentium. Eo magis autem in id eſt incumbendum, vt confuſio primae ſpeciei ab apertura pendens inſenſibilis reddatur, ſeu vt ſemidiameter huius confuſionis certum quendam limitem, quem littera k indicauimus, non ſuperet. Quare ex ſuperioribus colligetur haec conditio:

$$+\frac{m\mu x^3}{4p^3}\left(\lambda + \frac{\lambda'}{m}\right) < \frac{1}{4k^3}$$

$$\text{ſeu } \frac{x^3}{p^3}(\mu\lambda m + \mu\lambda') < \frac{1}{k^3}$$

vnde pro diſtantia focali lentis obiectiuae $p = \alpha$ hanc obtinemus conditionem; $p > kx\sqrt[3]{(\mu\lambda m + \mu\lambda')}$ et ob $x = my$, erit $p > kmy\sqrt[3]{(\mu\lambda m + \mu\lambda')}$ ſeu ad minimum p huic formulae aequalis capi poterit.

Coroll. 1.

194. Hinc ergo ſtatim apparet, quo maior requiratur multiplicatio, eo maiorem eſſe debere lentis obiectiuae diſtantiam focalem ideoque etiam longitudinem teleſcopii neque id in ratione tantum ſimplici,

ſed fere in ratione ſesquitriplicata multiplicationis, ſcilicet vt $m^{\frac{4}{3}}$, hincque iſta longitudo mox tanta euadit, vt neutiquam ſit verendum, ne quantitas p minor fiat, quam $4\,my$.

Coroll. 2.

195. Numerus μ ab indole vitri pendet, vnde ſequitur, quo minor is fuerit, eo magis longitudinem p imminui. Vidimus autem ſupra creſcente ratione refractionis n iſtum numerum μ diminui; ſed quia tum formula $\frac{dn}{n-1}$ creſcit, ideoque margo coloratus augetur, praeſtabit vitro vti communi.

Coroll. 3.

196. Hinc etiam intelligimus, quo maior gradus claritatis y deſideretur, eo magis quantitatem p augeri debere quod etiam vſu venit, ſi maior diſtinctio requiratur, quia tum litterae k maior valor tribui deberet.

Coroll. 4.

197. Ad longitudinem autem horum inſtrumentorum contrahendam plurimum intereſt, lentem obiectiuam ita conficere, vt fiat $\lambda = 1$, quippe qui huius litterae minimus eſt valor. Quare huic lenti eam formam tribui conueniet, quam ſupra in capite de lentibus obiectiuis deſcripſimus.

Co-

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Coroll. 5.

198. Circa lentem autem ocularem parum lucraremur ſi et $\lambda' = 1$ capere vellemus, quoniam in maioribus multiplicationibus hic terminus prae primo euaneſcit; quin potius huic lenti eiusmodi figuram tribui neceſſe eſt, quae maximae aperturae ſit capax; quoniam ab ea campus apparens potiſſimum pendet; quare haec ſanciatur regula, vt lens ocularis vtrinque aequaliter conuexa conficiatur, quoniam tum demum littera π valorem $\frac{1}{4}$ vel etiam maiorem accipere poteſt. Tum vero erit

I. Pro vitro coronario ſeu $n = 1.53$

$\lambda' = 1.60006$.

II. Pro vitro communi ſeu $n = 1.55$

$\lambda' = 1.62991$.

III. Pro vitro denique chryſtallino $n = 1.58$

$\lambda' = 1.67445$.

Scholion. 1.

199. Hugenius partim theoriae ſatis incompletae partim experimentis innixus diſtantiam focalem lentis obiectiuae quadrato multiplicationis proportionalem ſtatuit, cui tantum abeſt, vt aduerſari velim, vt potius in praxi eius praeſertim temporis aſſentiar, noſtra enim determinatio innititur huic rationi quod

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facies lentium ad figuram ſphaericam perfecte ſint formatae, quam ſi artifex exacte efficere poſſet, nullum eſt dubium, quin noſtra formula veritati ſit conſentanea, quod quidem nunc ſummorum artificum induſtriae concedendum videtur; ſed quando figura lentium a ſphaerica figura tantillum aberrat, notum eſt, vitium eo magis eſſe ſenſibile, quo maior fuerit diſtantia focalis lentis, cui propterea aliter occurri nequit, niſi diſtantiam focalem maiorem reddendo, quam ſecundum noſtram regulam. Num autem praeciſe ratio duplicata inde exſurgat, neutiquam affirmare licet, ſed prout quaeque lens feliciori ſucceſſu fuerit elaborata eo minor diſtantia focalis ſufficit eidem multiplicationi producendae, ſeu potius eadem lens maiori multiplicationi producendae erit apta, quod etſi perpetuo eſt obſeruandum, tamen hic aſſumo, lentibus non ſolum ſphaericas figuras, ſed etiam ſecundum datos radios tribui poſſe.

Scholion. 2.

200. His autem Hugenii obſeruationibus praecipue vtemur, ad gradus tam claritatis, quam diſtinctionis definiendos, quibus aſtronomi contenti eſſe ſolent; etiamſi cuique liberum relinquatur, ſiue maiorem ſiue minorem gradum eligere. Quod igitur primo ad gradum claritatis attinet, Hugenius lenti obiectiuae, cuius diſtantia focalis $= 20$ ped. ſiue 240 digit. aſſignat aperturam, cuius ſemidiameter $= 1.225$ digit.

digit. eamque ad multiplicationem $m = 89$ aptam iudicat; quam rationem etiam in reliquis lentibus obiectiuis obſeruat; quare cum hic ſit $x = 1.225$ dig. et $m = 89$ ob $x = my$, hinc colligimus $y = \frac{x}{m} = \frac{1.225}{89} = \frac{1}{73}$, quare cum ſupra paſſim aſſumſerimus $y = \frac{1}{35}$, multo maiorem claritatis gradum illis inſtrumentis conciliauimus eumque adeo duplo maiorem.

Quod dein ad gradum diſtinctionis attinet littera k contentum, in allegato Hugenii exemplo perpendamus eſſe $p = 240$ dig. $m = 89$ et $y = \frac{1}{73}$ ſumtoque $\mu = \frac{9}{10}$ et $\lambda = 1$. reiectoque altero termino in formula radicali hi valores in noſtra formula ſubſtituti dabunt

$$240 = 89 . \tfrac{1}{73} . k . \sqrt[3]{\tfrac{9}{10} . 89}$$

$$\text{ergo } k = \frac{73 . 240}{89 . \sqrt[3]{80}}$$

quae fractio euoluta dat $k = 45$. Quare cum ſupra paſſim ſumſerimus $k = \frac{1}{35}$, maiorem diſtinctionis gradum, quam hinc oritur, ſumus complexi. Cum igitur in noſtra formula ky occurrat, ſecundum Hugenium ſufficeret, ſtatuere $ky = \frac{45}{73} = \frac{5}{8}$ circiter ex quo patet, ſi ſtatuamus $ky = 1$ non ſolum claritatis, ſed et diſtinctionis maiorem gradum obtineri, ſimul vero longitudinem teleſcopii multo maiorem eſſe prodituram, quam ſi poneremus $ky = \frac{5}{8}$.

Scholion 3.

201. Quoniam vitrum chryſtallinum ad huiusmodi teleſcopia ineptum eſt iudicandum, ſiquidem margo coloratus augeretur, pro duabus vitri ſpeciebus, altera qua $n = 1.53$, altera qua $n = 1.55$ conſtructiones hic apponamus.

Conſtructio huiusmodi teleſcopii vtraque lente ex vitro coronario $n = 1.53$. parata

I. Pro lente obiectiua

radius faciei $\begin{cases} \text{anter.} = 0.6023. a \\ \text{poſter.} = 4.4131. a \end{cases}$

Interuallum $= \frac{m+1}{m}. a.$

II. Pro lente oculari

radius vtriusque faciei $1, 06. \frac{a}{m}.$

Diſtantia oculi ab hac lente

$= \frac{m+1}{m}. \frac{a}{m}.$

Semidiameter aperturae lentis obiectiuae $= m.y$;

lentis ocularis $= \frac{1}{4} \frac{a}{m}$

Semidiameter campi $\Phi = \frac{850}{m+1}$ minut.

ſumendo $a = k m y \sqrt[3]{(0.9875 (m + 1,60006))}$

Con-

Conſtructio huiusmodi teleſcopii vtraque lente ex vitro communi $n = 1.55$ parata.

I. Pro lente obiectiua

radius faciei $\begin{cases} \text{anter.} = 0.6145.\alpha \\ \text{poſter.} = 5.2438.\alpha \end{cases}$

Semidiameter aperturae $= my$

Interuallum $= \frac{m+1}{m}.\alpha.$

II. Pro lente oculari

Radius vtriusque faciei $= 1,10.\frac{\alpha}{m}.$

Semidiameter aperturae $= \frac{1}{4}\frac{\alpha}{m}.$

Diſtantia oculi $= \frac{m+1}{m}.\frac{\alpha}{m}.$

Semidiam. campi appar. $\Phi = \frac{850}{m+1}.$ min.

ſumendo $\alpha = kmy\sqrt[3]{0,9381\,(m+1,62991)}$

vbi quilibet gradum claritatis et diſtinctionis pro lubitu aſſumere poteſt.

Problema 2.

202. Si lens obiectiua fuerit duplicata eius generis, quod deſcripſimus, §. 65. conſtructionem teleſcopii deſcribere.

Solutio.

Hic omnia, quae in praecedente problemate de multiplicatione, campo apparente et loco oculi definimus,

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vimus, manent eadem; tantum in expreſſione pro ſemidiametro confuſionis inuenta numerus λ minorem adipiſcitur valorem, vltra partem quintam vnitatis imminutum; vnde diſtantia focalis lentis obiectiuae etiam minorem valorem habere poterit; id quod ſine dubio tanquam inſigne lucrum eſt ſpectandum, cum hoc modo longitudo inſtrumenti haud mediocriter contrahatur. Hic autem ad vitri ſpeciem, ex quo lentes parantur, inprimis eſt attendendum, quandoquidem numerus λ per eam definitur, vnde excluſo vitro chryſtallino ob rationes ante allegatas conſtructiones huiusmodi teleſcopiorum pro binis reliquis ſpeciebus hic exhibeamus:

Conſtructio huiusmodi teleſcopii, vtraque lente ex vitro coronario, pro quo $n = 1,53$ parata.

I. Pro lente obiectiua duplicata.

Lentis prioris radius faciei $\begin{cases} \text{anter.} = +1.2047.\alpha \\ \text{poſter.} = +8.8262.\alpha \end{cases}$

Lent. poſterioris radius faciei $\begin{cases} \text{anter.} = +0,6464.\alpha \\ \text{poſter.} = -1,6570.\alpha \end{cases}$

Semidiameter aperturae $x = my$

Interuallum vsque ad lentem ocularem $= \frac{m+1}{m}.\alpha$.

II. Pro lente oculari.

radius faciei vtriusque $= 1,06.\frac{\alpha}{m}$

Semidiameter aperturae $= \frac{1}{4}.\frac{\alpha}{m}$

Diſtantia oculi poſt hanc lentem $O = \frac{m+1}{m}.\frac{\alpha}{m}$.

Hic

Hic ſcilicet ipſa lens obiectiua duplicata vt ſimplex ſpectatur, cuius diſtantia focalis ſit $= \alpha$, quae iam ita capi debet, vt fiat

$$\alpha \overset{>}{=} k\, m y \sqrt[3]{0,9875\,(0,1951.\, m + 1,60006)}$$

Semidiameter vero campi apparentis eſt, vt ante, $\Phi = \frac{859}{m+1}$.

Conſtructio huiusmodi teleſcopii, vtraque lente ex vitro communi, $n = 1,55$, parata.

I. Pro lente obiectiua duplicata.

Lentis prioris radius faciei $\begin{cases} \text{anter.} = 1,2289.\,\alpha \\ \text{poſter.} = 10,4876.\,\alpha \end{cases}$

Lentis poſter. radius faciei $\begin{cases} \text{anter.} = 0,6527.\,\alpha \\ \text{poſter.} = -1,6053.\,\alpha \end{cases}$

Semidiameter aperturae $x = my$

Interuallum vsque ad lentem ocularem $= \frac{m+1}{m}.\,\alpha$.

II. Pro lente oculari

radius faciei vtriusque $= 1,10.\,\frac{\alpha}{m}$

Semidiameter aperturae $= \frac{1}{4}.\,\frac{\alpha}{m}$

Diſtantia oculi poſt hanc lentem $O = \frac{m+1}{m}.\,\frac{\alpha}{m}$. vbi cernetur campus, cuius ſemidiameter $= \frac{85}{m+1}$ minut.

At diſtantia focalis ipſius lentis obiectiuae duplicatae ita capi debet, vt ſit

$$\alpha \overset{>}{=} k\, m y \sqrt[3]{0.9381\,(0,1918.\, m + 1,6299)}.$$

 Corol-

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Corollarium 1.

203. Si ergo multiplicatio tanta ſit, vt in valore ipſius α poſtremus terminus prae altero euaneſcat, hoc caſu diſtantia α minor erit, quam praecedente, in ratione circiter $\sqrt[3]{\frac{1}{5}} : 1$. vel $1 : \sqrt[3]{5}$ hoc eſt fere vt $10 : 17$.

Coroll. 2.

204. Cum iſtae lentes duplicatae ex principio minimi ſint deductae, eo magis ſunt ad praxin accommodatae, cum metuendum non ſit, vt exigui errores ab artifice commiſſi effectum perturbent, quod maxime eſſet metuendum, ſi reliquas lentes compoſitas, quae quidem perfectae ſunt vocatae, loco obiectiuae ſubſtituere vellemus.

Scholion.

205. Quo clarius appareat, quantum lucrum hinc ſit exſpectandum, accommodemus ambos caſus ad datam multiplicationem, puta, $m = 100$, vbi quidem ſolum vitrum commune conſideremus. Si igitur I.) lente obiectiua ſimplici vtamur, diſtantia focalis α ita accipi debet, vt ſit

$$\alpha = 100 . ky \sqrt[3]{(0,9381 . 101,6299.)}$$

$$\alpha = 100\, ky . 4,5684$$

vnde

vnde ſi cum Hugenio capiatur

$ky = \frac{5}{8}$ dig. prodit

$\alpha = 285\frac{1}{4}$ dig. $=$ 23 ped. $9\frac{1}{4}$ dig.

ſi autem II) vtamur lente obiectiua duplicata, habebimus

$$\alpha = 100\, k y \sqrt[3]{0.9381\,(20,8099)}$$

$$\alpha = 100 . k y . 2,6926;$$

ſumtoque iterum $ky = \frac{5}{8}$ dig. erit

$\alpha = 168\frac{1}{3}$ dig. $=$ 14 ped. $\frac{1}{3}$ dig.

haec certe contractio ante hac maximi momenti foret viſa; nunc autem cum multo adhuc breuiora teleſcopia deſideremus, non admodum notatu digna videbitur, quod etiam eueniet in caſu ſequentis problematis, vbi lentem obiectiuam triplicatam faciemus.

Problema 3.

206. Si lens obiectiua fuerit triplicata, quam §. 66. deſcripſimus, teleſcopii conſtructionem deſcribere.

Solutio.

Omnia manent, vt ante, niſi quod pro hac lente triplicata futurum ſit $\lambda = \frac{3-8\nu}{27}$; vnde conſiderando tantum vitrum commune, pro quo $n = 1.55$ lentis huius obiectiuae diſtantia focalis α ita definiri debet,

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vt ſit $\alpha = k\,m\,y\,\sqrt[3]{0,9381\,(0,0422.\,m + 1.\,6299)}$ hinc igitur ſequenti modo talia teleſcopia erunt conſtruenda:

Conſtructio huiusmodi teleſcopii, vtraque lente ex vitro communi, pro quo $n = 1,55$ parata.

I. Pro lente obiectiua triplicata.

Lentis primae radius faciei $\begin{cases} \text{anter.} = 1.\,8433.\,\alpha \\ \text{poſter.} = 15.\,7315.\,\alpha \end{cases}$

Lentis ſecundae radius faciei $\begin{cases} \text{anter.} = 0.\,9790.\,\alpha \\ \text{poſter.} = -\,2.\,4079.\,\alpha \end{cases}$

Lentis tertiae radius faciei $\begin{cases} \text{anter.} = +\,13.\,5024.\,\alpha \\ \text{poſter.} = -\,8.\,0481.\,\alpha \end{cases}$

Eius aperturae ſemidiameter $x = my$. Interuallum usque ad lentem ocularem $= \frac{m+1}{m}.\,\alpha$.

II. Pro lente oculari

radius faciei vtriusque $= 1,10.\,\frac{\alpha}{m}$

eius aperturae ſemidiameter $= \frac{1}{4}.\,\frac{\alpha}{m}$

Diſtantia oculi $= \frac{m+1}{m}.\,\frac{\alpha}{m}$

campique apparentis ſemidiameter $\Phi = \frac{850}{m+1}$ minut.

Ipſa autem lentis obiectiuae diſtantia focalis tanta accipi debet, vt ſit

$$\alpha = k\,m\,y\,\sqrt[3]{0,9381\,(0,0422\,m + 1.\,6299)}$$

Co-

Coroll. 1.

207. Si ergo multiplicatio ſtatuatur $m = 100$, capi poterit

$$\alpha = 100\, ky \sqrt[3]{0,9381\,(5,8499)}$$

ſiue $\alpha = 100.\, ky.\, 1,7639$

ſumtoque $ky = \frac{5}{8}$ dig.

$$\alpha = 110\tfrac{1}{4} \text{ dig.} = 9 \text{ ped. } 2\tfrac{1}{4} \text{ dig.}$$

ſicque longitudo totius teleſcopii usque ad oculum prodibit

$$\left(\frac{m+1}{m}\right)^2.\, \alpha = 117,\tfrac{1}{2} \text{ dig.} = 9. \text{ ped. } 4\tfrac{1}{2} \text{ dig.}$$

Coroll. 2.

208. Quod ad gradum claritatis y attinet, quoniam hic plures ſunt lentes per quas radiis eſt transeundum, eorumque ideo maior iactura metuenda etiamſi maiorem claritatem non requiramus, quam Hugenius; tamen ipſi y maior valor tribui debet, quam $\frac{1}{72}$; quare retento valore k longitudo teleſcopii maior prodibit.

Scholion.

209. Haec vltima cautela maximi eſt momenti et ſemper probe obſeruanda quoties maiore lentium numero vtemur, atque hac occaſione haud abs re erit eorum teleſcopiorum ex Anglia allatorum,

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quae nocturna ſunt appellata, mentionem facere; circa quae primum obſeruo, eorum vſum in ſummis tenebris plane fore nullum ſed tantum tempore crepuſculi vel lucente Luna ea adhiberi ſolere ad obiecta non nimis longinqua ſpectanda. Totum autem myſterium quod in his teleſcopiis plerique quaeſiuerunt, huc redit, vt iis ſummus claritatis gradus concilietur, ſeu vt litterae y ſemidiameter ipſius pupillae tribuatur, ſiue circiter ſtatuatur $y = \frac{1}{15}$ dig. ſiquidem tum claritas viſa tricies ſexies maior ſentietur, quam ſi ſumeretur $y = \frac{1}{75}$. Quare ne haec teleſcopia nimis fiant longa multo minori multiplicatione nos contentos eſſe oportet. Ad hunc autem ſcopum multiplicatio $m = 10$ plus quam ſufficiens eſſe ſolet. Si enim noctu obiecta longinqua quaſi nobis decuplo eſſent propiora eaque eodem claritatis gradu aſpicere licebit atque nudis oculis, plus certe deſiderari non poterit.

Problema 4.

210. Si denique lens obiectiua fuerit quadruplicata, ſecundum principia §. 154. libro ſuperiore tradita constructa, teleſcopii constructionem deſcribere.

Solutio.

Hic denuo omnia manent, vt ante, ſed quod hic inprimis notatu dignum occurrit, eſt quod in formula pro diſtantia focali a reſultante ſcilicet

$a =$

$$\alpha = k m y \sqrt[3]{\mu\left(\frac{1-5\nu}{16} . m + \lambda''\right)}$$

valor numeri λ prodeat negatiuus ideoque certo caſu tota confuſio euaneſcere queat; qui caſus maxime meretur, vt omni diligentia euoluatur. Sumamus igitur omnes iſtas lentes ex vitro communi pro quo $n = 1,55$ eſſe confectas lentemque ocularem vtrinque vt hactenus, aeque conuexam formari atque habebimus $\lambda = \frac{1-\nu}{10} = -0,010216$ et $\lambda'' = 1,6299$ vnde intelligitur ſemidiametrum confuſionis prorſus in nihilum abire, ſi capiatur

$$m = \frac{1,6299}{0,010216} = 159\tfrac{1}{2}.$$

Pro hoc ergo caſu quantitas α non amplius ex hac formula ſed vnice ex apertura, quam gradus claritatis poſtulat, determinabitur; ſi enim pro gradu claritatis in genere ſumamus y, ſemidiameter aperturae debet eſſe $= my$, vnde diſtantia α tanta accipi debet, vt pro radiis ſingularum facierum tantam aperturam recipere poſſit. Quare ſi α etiam nunc vt quantitatem indefinitam ſpectemus, conſtructio Teleſcopii ita ſe habebit.

Conſtructio huiusmodi Teleſcopii lentibus ex vitro communi paratis.

I. Pro lente obiectiua quadruplicata.

Lentis Primae rad. fac. $\begin{cases} \text{anter.} = 2.4580.\,\alpha \\ \text{poſter.} = 20.975.\,\alpha \end{cases}$

Secun-

Secundae rad. fac. $\begin{cases} \text{anter.} = 1.305.\alpha \\ \text{poſter.} = -3.2108.\alpha \end{cases}$

Tertiae rad. fac. $\begin{cases} \text{anter.} = 0.8887.\alpha \\ \text{poſter.} = -1.4917.\alpha \end{cases}$

Quartae rad. fac. $\begin{cases} \text{anter.} = 0.6733.\alpha \\ \text{poſter.} = -0.9708.\alpha \end{cases}$

Semidiameter aperturae $x = my$.

Interuallum vsque ad lentem ocularem $= \frac{m+1}{m}.\alpha$.

II. Pro lente oculari.

Radius vtriusque faciei $= 1,10.\frac{\alpha}{m}$.

Semidiameter aperturae $= \frac{1}{4}.\frac{\alpha}{m}$.

Diſtantia oculi $= \frac{m+1}{m}.\frac{\alpha}{m}$.

Campique viſi Semidiam. $= \frac{850}{m+1}$ min.

Hic autem in genere capi deberet

$$\alpha \geqq kmy\sqrt[3]{0.9381(-0.010216.m+1.6299)}$$

niſi valor hinc prodiens minor fuerit, quam vt praeſcripta apertura $x = my$ locum habere poſſit, id quod potiſſimum pro $m = 159\frac{1}{2}$ eueniet.

Pro quo radii facierum modo exhibiti perpendi debent, inter quos minimus cum ſit $0,6733.\alpha$, huius pars quarta $0,1683.\alpha$, ſeu fere $\frac{1}{6}\alpha$ determinabit ſemidiametrum aperturae, qui cum ob $m = 159\frac{1}{2}$ ſit

159

$159\frac{1}{3}y$ capi debebit $\alpha > 6 . 159\frac{1}{3}y$ ſeu $\alpha > 957 . y$ ſi igitur ſumamus $y = \frac{1}{50}$ dig. capi debebit $\alpha > 19\frac{7}{50}$ dig. quocirca ſtatuamus $\alpha = 20$. dig. Sumtaque multiplicatione $m = 160$ habebimus hanc ſpecialiſſimam conſtructionem.

Conſtructio Teleſcopii pro multiplicatione $m = 160$, lentibus e vitro communi $n = 1, 55$ confectis.

I. Pro lente obiectiua quadruplicata.

Lentis

Primae rad. fac. { anter. $= + 49, 16$ dig.
poſter. $= + 419, 50$ dig.

Secundae rad. fac. { anter. $= 26, 10$. dig.
poſter. $= - 64, 21$. dig.

Tertiae rad. fac. { anter. $= 17, 77$. dig.
poſter. $= - 29, 83$. dig.

Quartae rad. fac. { anter. $= 13, 47$ dig.
poſter. $= - 19, 42$ dig.

Eius aperturae ſemidiameter $x = my = 3, 2$ dig.
Interuallum vſque ad lentem ocularem $= 20\frac{1}{4}$ dig.

II. Pro lente oculari.

Radius vtriusque faciei $= 0, 1375$. dig.
eiusque ſemidiameter aperturae $= \frac{1}{32}$ dig.
diſtantia oculi $= 0, 1258$ dig.
ita vt fit tota teleſcopii longitudo $= 20\frac{1}{4}$ dig.
campique viſi ſemidiameter $= 5' 21''$.

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Coroll. 1.

211. Si multiplicationem minorem ſtatuiſſemus, longitudo teleſcopii maior prodiiſſet. Si enim ſtatuamus $m = 50$ litteraeque k etiam valorem 50 tribuamus, prodiret $\alpha = 50 \sqrt[3]{0,9381 \cdot 1,1189}$. ſeu $\alpha > 50,81$ ideoque plusquam duplo maior, quam caſu $m = 160$ quod certe ingens eſt paradoxon.

Coroll. 2.

212. Si artifex in conſtructione lentis obiectiuae tantillum aberret, eius error valorem numeri λ tantum pauliſper augebit, quia ille valor $\lambda = -0,010216$ omnium eſt minimus, ſi enim ob hos errores λ. particula $\frac{1}{1000}$ augeatur, prodit $\lambda = -0.000784$ ita, vt tum iſta lens quadruplicata ad maiorem multiplicationem producendam ſit apta; quod paradoxon priori non cedit.

Scholion 1.

213. Neque hic neque in praecedentibus definiuimus cuiusmodi menſuram digitorum intelligamus. An ſint Pariſini an Londinenſes, an Rhenani etc. Verum conſultum potius eſt, hanc menſuram prorſus indeterminatam relinquere. Quodſi enim cauſſam dubitandi habeamus, lentes ſecundum regulas praeſcriptas accurate eſſe elaboratas, maxime e re erit, maiorem menſuram pro digitis adhibere. Sin autem de exſecutione plane ſimus certi, menſura digitorum minore tuto

tuto vti poterimus. Semper autem praxi conſulendo vtile erit, maiorem digitorum menſuram adhibere; atque adeo ipſa ratio, quae nos ad digitorum menſuram perduxit, hoc ſuadet; haec enim ratio ex apertura pupillae nobis eſt nata, quam in partibus digiti expreſſimus. Cum igitur ipſa pupilla tantopere ſit mutabilis, vt nihil plane certi de ea ſtatui poſſit, manifeſtum eſt, tantum abeſſe, vt nobis certa quaedam menſura ſit praeſcripta, vt nobis potius liberum ſit, eam ſiue augendo ſiue minuendo notabiliter immutare.

Scholion 2.

214. Hactenus oſtendimus, quemadmodum lentibus compoſitis loco obiectiuae adhibendis haec teleſcopia non mediocriter contrahi queant. Verum hoc modo nullum plane augmentum campo apparenti inducitur. Iam dudum autem eſt obſeruatum, campum quoque apparentem non mediocriter augeri poſſe, ſi etiam lens ocularis ſiue duplicetur, ſiue adeo triplicetur. Cum enim campus apparens inprimis ab apertura lentis ocularis pendeat, quam ob cauſſam etiam huic lenti figuram vtrinque aequalem tribuimus, vt maioris aperturae capax redderetur: euidens eſt, ſi hanc lentem ita inſtruere liceret, vt adhuc maiorem aperturam recipere poſſet; campum apparentem in eadem ratione auctum iri. Quo hoc clarius perſpiciatur, ponamus lentis ocularis diſtantiam focalem eſſe vnius digiti, ita, vt aperturam admittat, cuius ſemidia-

diameter $= \frac{1}{4}$ dig. Iam ſatis manifeſtum eſt, ſi eius loco binae lentes inter ſe iunctae quarum vtriusque diſtantia focalis $= 2$ dig. ſubſtituantur; tum iſtius lentis compoſitae diſtantiam focalem quoque fore vnius digiti, ſed hanc lentem compoſitam duplo maiorem aperturam eſſe admiſſuram, ſiquidem vtraque faciebus inter ſe aequalibus conſtet, ideoque aperturam admittat cuius ſemidiameter dimidii digiti atque hoc modo campus apparens duplicabitur. Simili modo ſi loco eius lentis ocularis ſimplicis ſubſtituantur ternae lentes, quarum ſingularum diſtantia focalis ſit trium digitorum, idem effectus ratione multiplicationis obtinebitur, ſed quia aperturam triplo maiorem admittunt, campus triplicabitur. Haec autem omnino digna ſunt, vt adcuratius ex noſtris principiis explicentur atque inprimis influxum huiusmodi lentium compoſitarum, quo confuſionem afficiunt, determinemus.

Problema 5.

215. Si lens ocularis duplicetur, vt ſemidiameter campi apparentis duplo maiorem valorem nanciſcatur, conſtructionem huiusmodi teleſcopii deſcribere.

Solutio.

Cum hic teleſcopium reuera tribus conſtet lentibus, quarum binae poſteriores ſibi immediate ſunt iunctae; haec inueſtigatio ex caſu trium lentium eſt

repe-

repetenda. Primo igitur pro multiplicatione habebimus $m = -\frac{\alpha}{b}\cdot\frac{\beta}{c}$ vbi cum eſſe debeat interuallum $\beta + c = 0$, erit $c = -\beta$ ideoque $\frac{\beta}{c} = -1$ vnde fit, vt hactenus, $m = \frac{\alpha}{b}$ ſeu $b = \frac{\alpha}{m}$; tum vero poſuimus $\beta = \mathrm{B}\, b = \frac{\mathrm{B}\alpha}{m}$ ideoque etiam $c = -\frac{\mathrm{B}\alpha}{m}$, quae cum ſit diſtantia focalis poſtremae lentis ob $\gamma = \infty$ ſi ſecunda lens ipſi iuncta parem haberet diſtantiam focalem foret $\frac{b\beta}{b+\beta} = c$, ſiue $\frac{\mathrm{B}\alpha}{m(1+\mathrm{B})} = \frac{-\mathrm{B}\alpha}{m}$, hincque $\mathrm{B} = -2$ ſed praeſtat haec ex noſtris principiis deducere; quia enim campi apparentis ſemidiameter nunc eſt $\Phi = \frac{\pi - \pi'}{m+1}$ vt hic duplo maior fiat, quam caſu praecedente debet eſſe $-\pi = \pi'$, vt fiat $\Phi = \frac{2\pi}{m+1}$. Ex principiis autem ſuperioribus colligimus $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{\alpha}{b} = m$ vnde fit $\mathfrak{B}\pi = (m+1)\Phi$, ideoque $\mathfrak{B} = 2$ hincque $\mathrm{B} = -2$ ita, vt poſtremae lentes fiant inter ſe aequales. Hoc autem valore inuento pro ſemidiametro confuſionis habebimus

$$\frac{m x^3}{4 p^3}\cdot \mu \left(\lambda + \frac{q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{\mathrm{B}}\right) - \frac{\lambda''}{\mathrm{B}^3 . m}\right)$$

vbi eſt $p = \alpha$, $q = \mathfrak{B}\, b = \frac{2\alpha}{m}$ ita, vt haec expreſſio abeat in iſtam

$$\frac{\mu m x^3}{4 \alpha^3}\left(\lambda + \frac{1}{2m}\left(\frac{\lambda'}{4} - \frac{\nu'}{2}\right) + \frac{\lambda''}{8m}\right).$$

Nunc autem probe notandum eſt, has duas lentes poſteriores aſſumtam aperturam vt fiat $\pi = \frac{1}{4}$ admittere non poſſe niſi vtraque ſibi vtrinque reddatur aequalis. Ex qua conditione ſi quidem vitro com-

 muni

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muni vtamur, pro quo $n = 1,55$, pro lente tertia erit, vti vidimus, $\lambda'' = 1.6299$. Quemnam autem valorem numerus λ' ſit habiturus, ex ſupra allatis definire poterimus, cum ſit $\sqrt{\lambda' - 1} = \frac{\sigma-\rho}{2\tau} \cdot \frac{b-\beta}{b+\beta} = \frac{3(\sigma-\rho)}{2\tau}$ vnde fit $\lambda' = 1 + \frac{(\sigma-\rho)^2 \cdot 9}{4\tau^2}$ quare cum fuerit $\lambda'' = 1 + \frac{(\sigma-\rho)^2}{4\tau^2} = 1.6299$ erit $\frac{(\sigma-\rho)^2}{4\tau^2} = 0.6299$ ideoque $\lambda' = 6,6691$. ex quo obtinemus $\frac{\lambda'}{4} - \frac{\nu}{8} = 1,5509$ hincque confuſionis pars ex ſecunda lente orta fit $\frac{0,7754}{m}$ dum pars ex tertia lente orta eſt $\frac{0,2037}{m}$ ſicque tota noſtra lens ocularis duplicata producet in expreſſione confuſionis partem $= \frac{0,9791}{m}$. Poſito igitur illo ſemidiametro $= \frac{1}{4k^3}$ colligemus diſtantiam focalem lentis obiectiuae

$$\alpha = k m y \sqrt[3]{0,9381 (\lambda m + 0,9791)}$$

ob $x = my$, vbi λ indefinitum relinquo, vt etiam lens obiectiua pro lubitu ſiue ſimplex ſiue duplicata ſiue triplicata ſiue etiam quadruplicata aſſumi queat. Binae autem lentes poſteriores inter ſe aequales fient et vtrinque aeque conuexae, radio conuexitatis exiſtente $= \frac{2.20.\alpha}{m}$. Oculi vero diſtantia poſt hanc lentem reperitur $O = \frac{-\pi' r}{m\Phi} = \frac{\pi r}{m\Phi}$; quia nunc eſt $r = \frac{2\alpha}{m}$ et $\frac{\pi}{\Phi} = \frac{m+1}{2}$ erit $O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$ prorſus vt ante; tum autem campi apparentis ſemidiameter erit $= \frac{1718}{m+1}$ minut.

Co-

Coroll. 1.

216. Hinc ergo patet, ſi lens ocularis hac ratione duplicetur, eius effectum in confuſione augenda minorem eſſe futurum, quam ſi haec lens eſſet ſimplex.

Coroll. 2.

217. Operae pretium erit pro hoc caſu in marginem coloratum inquirere pro quo diuiſione per $\frac{dn}{n-1}$ facta haec in ſuperioribus occurrit aequatio

$$0 = \frac{\pi b}{\Phi p} - \frac{\pi'}{m\Phi} = \frac{\pi}{\Phi} \cdot \frac{1}{m}$$

cum nunc ſit $\frac{\pi}{\Phi} = \frac{m+1}{2}$ haec quantitas, quae euaneſcere deberet, ſit $\frac{m+1}{m}$ prorſus vt ante inuenimus pro lente oculari ſimplici, ita, vt hinc pro margine colorato nihil amplius ſit metuendum.

Coroll. 3.

218. Omnes igitur formulae ſupra allatae pro conſtructione teleſcopiorum ſiue lens obiectiua fuerit ſimplex ſiue multiplicata, etiam hic locum obtinere poſſunt, ſi modo loco lentis ocularis ſimplicis huiusmodi lens duplicata ſubſtituatur, cuius ſingulae facies ſecundum radium duplo maiorem ſunt elaborandae; tum vero etiam in valore diſtantiae α poſt ſignum radicale loco numeri 1, 6299 ſcribatur hic numerus 0, 9791 atque tum campi apparentis ſemidiameter duplo euadet maior. Vix autem opus eſt, in formula

la pro α iſtam correctionem facere, quia tantum de limite ſermo eſt, infra quem α accipi non oportet.

Scholion.

219. Hic autem inprimis conſiderari meretur caſus, quo lens obiectiua eſt quadruplicata ſiue $\lambda = -0,010216$ et multiplicatio tanta accipitur, vt confuſio penitus euaneſcat, quod fit ſi fuerit $m = \frac{0,791}{0,010218} = 95 +$ [illegible], quare capi poteſt $m = 96$ et ſi pro gradu claritatis capiatur $y = \frac{1}{48}$ dig. ſemidiameter aperturae lentis obiectiuae debebit eſſe $= my = 2$ vnde α facile definitur; ſupra enim vidimus, hanc lentem quadruplicatam maiorem aperturam non admittere, quam cuius ſemidiameter ſit $\frac{1}{6}\alpha$, vnde poſito $\frac{1}{6}\alpha = 2$ dig. fiet $\alpha = 12$ dig. ex quo ſequens habebitur conſtructio.

Conſtructio Teleſcopii pro multiplicatione $m = 96$, lentibus ex vitro communi, pro quo $n = 1,55$ confectis.

I. Pro lente obiectiua quadruplicata.

Lentis

Primae rad. fac. $\begin{cases} \text{anter.} = 29,50 \text{ dig.} \\ \text{poſter.} = 251,70. \text{ dig.} \end{cases}$

Secundae rad. fac. $\begin{cases} \text{anter.} = 15,66. \text{ dig.} \\ \text{poſter.} = -38,53. \text{ dig.} \end{cases}$

Ter-

Tertiae rad. fac. $\begin{cases} \text{anter.} = 10, 66.\ \text{dig.} \\ \text{poſter.} = -17, 90.\ \text{dig.} \end{cases}$

Quartae rad. fac. $\begin{cases} \text{anter.} = 8, 08.\ \text{dig.} \\ \text{poſter.} = -11, 65.\ \text{dig.} \end{cases}$

Eius aperturae ſemidiameter $= 2$ dig.

Interuallum vsque ad lentem ocularem $= 12\frac{1}{4}$. dig.

II. Pro oculari duplicata

lentis vtriusque radius faciei vtriusque $= 0, 275$ dig.

Eius aperturae ſemidiameter $= \frac{1}{18}$ dig.

Diſtantia oculi $= 0. 126$ dig.

ita, vt ſit longitudo tota $= 12, 251$. dig.

campi autem apparentis ſemidiameter $= \frac{1718}{97}$ minut $= 17$ min. 43 ſec.

Problema 6.

220. Si lens ocularis fuerit triplicata, vt ſemidiameter campi reddatur triplo maior, teleſcopii conſtructionem deſcribere.

Solutio.

Quia hic quatuor lentes ſunt conſiderandae formula pro multiplicatione erit $m = \frac{\alpha}{b}.\frac{\beta}{c}.\frac{\gamma}{d}$ et quia ternae poſteriores ſibi immediate iunguntur, fiet $\beta + c = 0$; et $\gamma + d = 0$, vnde ſequentes prodeunt

 deter-

determinationes $b = \frac{\alpha}{m}$; $\beta = \mathrm{B}\,b = \frac{\mathrm{B}\alpha}{m}$, $c = -\frac{\mathrm{B}\alpha}{m}$; $\gamma = \mathrm{C}\,c = -\frac{\mathrm{BC}\alpha}{m}$ et $d = \frac{\mathrm{BC}\alpha}{m}$; formula autem pro campo apparente eſt $\Phi = \frac{\pi - \pi' + \pi''}{m+1}$, qui vt triplo fiat maior, quam ſupra, ſtatui debet $\pi' = -\pi$, et $\pi'' = \pi$; tum enim erit $\Phi = \frac{3\pi}{m+1}$, ita, vt ſit

$$\pi = -\pi' = \pi'' = \frac{m+1}{3}.\,\Phi$$

Pro his autem litteris formulae noſtrae ſunt

$$\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{\alpha}{b} = m$$

$$\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{\mathrm{B}\alpha}{c} = -m$$

vbi ſubſtitutis valoribus ipſius π et π' habebitur

$$\frac{\mathfrak{B}(m+1)}{3} - 1 = m \text{ et } \mathfrak{B} = 3 \text{ hincque } \mathrm{B} = -\frac{3}{2}$$

Deinde $+\frac{1}{3}\mathfrak{C} + \frac{1}{3} = 1$, et $\mathfrak{C} = 2$ hincque $\mathrm{C} = -2$, ſicque trium lentium poſtremarum diſtantiae focales erunt

IIdae. $\mathfrak{B}\,b = \frac{3\alpha}{m}$;

IIItiae. $\mathfrak{C}\,c = \frac{3\alpha}{m}$;

IVtae. $d = \frac{3\alpha}{m}$;

ita, vt hae tres lentes fiant inter ſe aequales; diſtantiae vero determinatrices erunt

$$b = \frac{\alpha}{m};\ \beta = -\frac{3}{2}.\ b = -\frac{3\alpha}{2m}$$

$c =$

$c = \frac{3\alpha}{2m}$; $\gamma = -2c = -\frac{3\alpha}{m}$

$d = \frac{3\alpha}{m}$.

Subſtituamus hos valores in formula pro ſemidiametro confuſionis, quae fiet

$$\frac{\mu m x^3}{4 p^3} \left\{ \begin{array}{l} \lambda + \frac{q}{\mathfrak{B}\cdot p}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) \\ + \frac{r}{B^4 \mathfrak{C}^2 \cdot p}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) + \frac{\lambda'''}{B \cdot C^4 \cdot n}. \end{array} \right.$$

quae ob $p = \alpha$, $q = \frac{3\alpha}{m} = r$ abit in hanc formam:

$$\frac{\mu m x^3}{4 \cdot \alpha^3} \left\{ \begin{array}{l} \lambda + \frac{1}{3m}\left(\frac{\lambda'}{9} - \frac{2\nu}{3}\right) \\ + \frac{4}{27 \cdot m}\left(\frac{\lambda''}{4} - \frac{\nu}{2}\right) + \frac{\lambda'''}{27 \cdot m} \end{array} \right.$$

vbi pro λ', λ'', λ''' numeri idonei ſunt quaerendi. Quia autem volumus, vt quaeuis harum lentium maximam admittat aperturam, quod fit, ſi litteris π, π', π'' valor $= \frac{1}{4}$ tribui poſſit, neceſſe eſt, vt quaelibet earum ſit vtrinque aequaliter conuexa, id quod eueniet, ſi ſtatuatur

$$\sqrt{(\lambda''' - 1)} = \frac{\sigma - \rho}{2\tau}; \quad \sqrt{(\lambda'' - 1)} = \frac{\sigma - \rho}{2\tau} \cdot \frac{c - \gamma}{c + \gamma}$$

et $\sqrt{(\lambda' - 1)} = \frac{\sigma - \rho}{2\tau} \cdot \frac{b - \beta}{b + \beta}$ ideoque

$$\sqrt{(\lambda'' - 1)} = + \frac{3}{1} \cdot \frac{\sigma - \rho}{2\tau}$$

$$\sqrt{(\lambda' - 1)} = \frac{3}{1} \cdot \frac{\sigma - \rho}{2\tau}$$

Cum igitur ſit, vt ſupra eſt oſtenſum,

$$\lambda''' = 1 + \left(\frac{\sigma - \rho}{2\tau}\right)^2 = 1,6299$$

erit $\left(\frac{\sigma - \rho}{2\tau}\right)^2 = 0,6299$, ex quo valore colligimus

 $\lambda'' =$

$$\lambda'' = 1 + 9.(0,6299) = 6,6691;$$

$$\text{et } \lambda' = 1 + 25\,(0,6299)$$

$$\text{ſeu } \lambda' = 16,7477.$$

Cum iam pro vitri ſpecie propoſita pro qua $n = 1,55$, ſit $\mu = 0.9381$ et $\nu = 0,2326$, nunc poterimus partem aſſignare, quam haec lens ocularis triplicata in formulam pro confuſione infert, quippe in quo cardo rei verſatur. Reperietur autem

$$\tfrac{\lambda'}{9} - \tfrac{2\nu}{3} = 1,7058;\ \text{et}$$

$$\tfrac{1}{3m}\left(\tfrac{\lambda'}{9} - \tfrac{2\nu}{3}\right) = \tfrac{0.5686}{m}$$

$$\text{et } \tfrac{\lambda''}{4} - \tfrac{\nu}{5} = 1,5509$$

$$\text{et totus terminus} = \tfrac{0.2209}{m}$$

$$\text{et } \tfrac{\lambda'''}{27.m} = \tfrac{0.0803}{m}$$

vnde pars a tota lente oculari orta erit $= \frac{0.8586}{m}$

ita, vt ſit tota expreſſio

$$\tfrac{\mu x^3}{4a^3}\,(\lambda m + 0.8586)$$

ſumto igitur $x = my$ poſitaque hac formula $= \frac{1}{4k^3}$, determinabimus lentis obiectiuae diſtantiam focalem $= \alpha$, vt ſit

$$\alpha = kmy\sqrt[3]{0.9381\,(\lambda m + 0,8586)}$$

ſiue maius.

Inter-

Interuallum porro inter lentem obiectiuam et ocularem eſt

$$a + b = \frac{m+1}{m} \cdot \alpha$$

Et cum tres lentes ocularem conſtituentes ſint inter ſe aequales et vtrinque aequaliter conuexae ob cuiuslibet diſtantiam focalem $= \frac{3\alpha}{m}$, radius ſingularum facierum erit $= 3,30 \frac{\alpha}{m}$, ipſius huius lentis triplicatae diſtantia focali exiſtente $= \frac{\alpha}{m}$ et ſemidiametro aperturae $= \frac{3}{4} \cdot \frac{\alpha}{m}$. Pro diſtantia oculi autem poſt hanc lentem reperitur $O = \frac{\pi'' s}{m \Phi}$, quae ob $\pi'' = \frac{m+1}{3} \Phi$ et $s = \frac{3\alpha}{m}$ fiet $O = \frac{m+1}{m} \cdot \frac{\alpha}{m}$ prorſus, vt ante, at campi apparentis ſemidiameter erit

$$\Phi = \frac{3 \cdot 859}{m+1} \text{ minut} = \frac{2577}{m+1} \text{ min.}$$

Coroll. 1.

221. Circa lentem obiectiuam hic nihil definiuimus, et ea pro lubitu ſiue ſimplex, ſiue duplicata, ſiue triplicata, ſiue etiam quadruplicata ſtatui poteſt, atque etiam regulae conſtructionis manent eaedem, vt ante; dummodo quantitas α ex formula hic data definiatur.

Coroll. 2.

222. Eodem etiam modo, quo ante, oſtendi poteſt, haec teleſcopia non magis margini colorato eſſe obnoxia, quam praecedentia, neque enim duabus len-

 tibus,

tibus, ad quem caſum omnia haec teleſcopia referre licet, margo coloratus tolli poteſt.

Scholion.

223. Simili modo etiam lens ocularis quadruplicari poſſet, ita, vt ſemidiameter campi quadruplo maior redderetur, at hanc inueſtigationem non vlterius proſequor, quoniam ſi plures lentes adhibere velimus, iis inſuper alia commoda teleſcopiis induci poſſunt, quemadmodum in ſequentibus docebimus. Hic ſcilicet tantum ſimpliciſſimam horum teleſcopiorum ſpeciem ſumus contemplati, quae non niſi duabus lentibus, altera obiectiua, altera oculari, conſtare eſt cenſenda, etiamſi pro vtraque lentibus compoſitis vti liceat; quin etiam ambas has lentes ex eadem vitri ſpecie factas aſſumſimus, atque etiam in ſequente capite vnicam vitri ſpeciem adhibebimus vt intelligatur, ad quemnam perfectionis gradum haec teleſcopia euehi queant, ante quam vitri ſpecies diuerſas in ſubſidium vocemus. Probe enim diſtinguendae ſunt eae perfectiones, quae vnica vitri ſpecie obtineri poſſunt, ab iis, quae diuerſas ſpecies poſtulant; quo pacto iſta tractatio magis perſpicua reddetur. Hic autem adhuc meminiſſe oportet, qua ratione haec inſtrumenta ab alio inſigni incommodo liberare conueniat, quod in eo conſiſtit, quod ſaepenumero etiam radii peregrini, qui ſcilicet non ab obiecto ſpectando ſunt profecti, in tubum intrent atque viſionem non mediocriter perturbent.

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bent. Quemadmodum igitur tales radii peregrini arceri debeant, in ſequenti problemate oſtendemus.

Problema 7.

224. Conſtructis his lentibus ac tubo inſertis, radios peregrinos, qui per lentem obiectiuam in tubum ingrediuntur arcere, ne in oculum incidant et viſionem turbent.

Solutio.

Hunc in finem quandoque ſolet tubus aliquantillum diuergens lenti obiectiuae praefigi, vt radii a lateribus aduenientes intercipiantur; ſimul vero haec diuergentia tanta eſſe debet, vt radiorum ab obiecto verſus lentem obiectiuam emiſſorum nulli excludantur; id quod fit, ſi divergentia ſemidiametro campi fiat aequalis. Interim tamen hoc modo non omnes radii alieni ab introitu in obiectiuam arcentur; quare ne iis parietes tubi intus illuminentur; neceſſe eſt, vt tubi interna ſuperficies vbique colore nigro obducatur, quod etiam de tubo praefixo eſt intelligendum. Neque tamen hoc prorſus ſufficit, cum etiam color nigerrimus cuiuspiam illuminationis ſit capax atque ob hanc cauſſam diaphragmata ſeu ſepta his tubis inſeri ſolent, pertuſa foraminibus, quae maiora eſſe non debent, quam tranſitus radiorum ad viſionem neceſſariorum poſtulat, id quod commodiſſime fiet, in ipſo loco imaginis $F\zeta$, vbi omnes iſti radii in ſpatium arctiſſimum ſunt redacti. In hoc ergo loco huiusmodi diaphragma ſeu orbis circularis pariter nigerrimus

mus conſtituatur, cuius foramen praeciſe ſit aequale magnitudini imaginis, quam oculo cernere licet, hocque modo radiis peregrinis omnis acceſſus ad lentem ocularem praecludetur, et ſi qui forte eo pertingant non ita refringentur, vt in oculum ingredi poſſint.

Corollarium.

225. Ad quantitatem huius foraminis definiendam conſideretur ſemidiameter campi apparentis Φ et cum ſemidiameter imaginis $F\zeta$ ſit $= \alpha\Phi$ hic ſimul capiatur pro ſemidiametro foraminis.

Coroll. 2.

226. Quo maior ergo fuerit campus apparens, eo maiore opus erit foramine, quo diaphragma pertundatur. Ita in exemplo vltimo §. 219. allato cum ſit $\alpha = 12$ dig. et $\Phi = 17$ min. 43 ſec. ſeu in partibus radii $\Phi = \frac{1}{2.97}$ ſemidiameter iſtius foraminis debet eſſe $\frac{6}{97}$ dig. ſiue circiter $\frac{1}{16}$ dig. ita, vt eius diameter adaequet $\frac{1}{8}$ dig.

Scholion.

227. In tubis aſtronomicis ad hoc genus referendis hoc ipſum diaphragma etiam micrometro ſiue filis tenuiſſimis per hoc ſpatium diſpoſitis, inſtrui ſolet, quae cum in ipſo loco imaginis ſint extenſa, cum ea ſe quaſi confundunt et oculo aeque diſtincte atque ipſa imago repraeſentabuntur; vnde aſtronomi veram quantitatem obiecti diſtantiamque eius partium dijudicare ſolent.

SECTIO-

SECTIONIS SECVNDAE.

CAPVT II.

DE

VLTERIORI HORUM TELESCOPIORUM PERFECTIONE, QUAM QUIDEM VNICAM VITRI SPECIEM ADHIBENDO ASSEQUI LICET.

Problema I.

228.

Si inter lentem obiectiuam et ocularem in ipſo loco imaginis noua lens conſtituatur; inquirere in commoda, quae eius ope teleſcopio conciliare licet.

Solutio.

Quia igitur caſum trium lentium habemus multiplicatio m ſtatim praebet $m = -\frac{\alpha}{b} \cdot \frac{\beta}{c}$ vbi cum eſſe debeat interuallum inter lentem primam et ſecundam $= \alpha$, fit $b = 0$, ideoque $\beta = \mathrm{B}\,b = 0$ niſi forte $\mathrm{B} = \infty$. Quo autem hinc valorem ipſius B definire queamus, eius diſtantiam focalem in computum intro-

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ducamus, quae ſit $= q$, ita, vt iam habeamus $q = \frac{b\beta}{b+\beta}$, ex qua aequatione colligemus $\beta = \frac{bq}{b-q} = 0$, vnde valorem litterae B conſequimur, ſcilicet $B = \frac{\beta}{b} = -1$ hincque $\mathfrak{B} = \infty$. Quoniam igitur tam b, quam $\beta = 0$, ita tamen, vt ſit $\frac{\beta}{b} = -1$ erit $m = \frac{\alpha}{c}$ ideoque $c = \frac{\alpha}{m}$, vbi c denotat diſtantiam focalem lentis ocularis. His notatis ſemidiameter confuſionis erit

$$\frac{\mu m x^3}{4 p^3} \left(\lambda + 0 + \frac{\lambda''}{m}\right)$$

ita, vt lens media nihil plane ad hanc confuſionem conferat, perindeque ſit, quaecunque figura huic lenti tribuatur. Deinde pro campo apparente habebimus eius ſemidiametrum $\Phi = \frac{\pi - \pi'}{m+1}$ vbi valor ipſius π per hanc formulam definitur $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{\alpha}{b}$ ex qua vt aliquid concludi poſſit, loco b introducamus diſtantiam focalem ſecundae lentis q et cum ſit $q = \mathfrak{B}\, b$ erit $b = \frac{q}{\mathfrak{B}}$ qui valor nobis praebet hanc aequationem: $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{\alpha \mathfrak{B}}{q}$; ſiue ob $\mathfrak{B} = \infty$, $\frac{\pi}{\Phi} = \frac{\alpha}{q}$ ſeu $\pi = \frac{\alpha\Phi}{q}$, vbi tantum eſt animaduertendum, valorem π quadrantem vnitatis ſuperare non debere. Hoc autem valore π admiſſo, pro campo apparente erit $\Phi = \frac{-\pi' q}{(m+1)q-\alpha}$ hincque $\pi = \frac{-\alpha\pi'}{(m+1)q-\alpha}$ quare ſi ponamus $-\pi' = \frac{1}{4}$, etiam $\pi = \frac{+\frac{1}{4}\alpha}{(m+1)q-\alpha}$ maior quam $\frac{1}{4}$ eſſe nequit; ſi igitur quoque ſumamus $\pi = \frac{1}{4}$, nouam hanc nanciſcimur determinationem

$$1 = \frac{\alpha}{(m+1)q-\alpha} \text{ ſiue } (m+1)q = 2\alpha \text{ et } q = \frac{2\alpha}{m+1}.$$

Sin autem in formula $\pi = \frac{-\alpha\pi'}{(m+1)q-\alpha}$ fractio $\frac{\alpha}{(m+1)q-\alpha}$ maior eſſet vnitate, tum pro $-\pi'$ minorem valorem, quam $\frac{1}{4}$ ſcribi oporteret, vt prodiret $\pi = \frac{1}{4}$; tum autem campus apparens minor eſſet proditurus, quam ſi etiam $-\pi'$ eſſet $\frac{1}{4}$. Vnde concludimus ſiue haec fractio $\frac{\alpha}{(m+1)q-\alpha}$ maior ſit vnitate, ſiue minor, vtroque caſu fore $\Phi < \frac{\frac{1}{4}+\frac{1}{4}}{m+1}$ ac ſolo caſu $\frac{\alpha}{(m+1)q-\alpha} = 1$ fieri poſſe $\Phi = \frac{1}{2(m+1)}$, qui valor duplo maior eſt, quam caſu duarum lentium ſimplicium. Interim tamen de quantitate q nihil adhuc definiamus, ſed potius videamus, num hoc modo margo coloratus deſtrui poſſit, quod eueniet, ſi fuerit $0 = \frac{\pi b}{\Phi p} - \frac{\pi'}{m\Phi}$ ſiue $0 = 0 + \frac{(m+1)q-\alpha}{mq}$ ex qua ſequitur $q = \frac{\alpha}{m+1}$, vnde patet quantitatem q vtique ita aſſumi poſſe, vt margo coloratus penitus deſtruatur, quae determinatio praecedenti longe eſt anteferenda. Poſito igitur $q = \frac{\alpha}{m+1}$, pro campo apparente foret $\pi = \infty . \pi'$ ſeu $\pi' = \frac{\pi}{\infty}$, quare cum π maius quam $\frac{1}{4}$ capi non poſſit, fiet $\pi' = 0$, ita, vt hoc caſu lens ocularis nihil plane ad campum conferat quippe qui vnice a lente media pendebit, eritque $\Phi = \frac{1}{4(m+1)}$ ſeu $\Phi = \frac{859}{m+1}$ minut. tum vero pro loco oculi prodibit eius diſtantia a lente oculari $O = \frac{\pi' . r}{m\Phi} = 0$ ſeu oculum lenti oculari imme-

 diate

diate adplicari oportet. Conſtructio ergo huiusmodi teleſcopii ita ſe habebit.

Primo diſtantia focalis α ita eſt definienda, vt ſit $\alpha = kmy \sqrt[3]{\mu (\lambda m + \lambda'')}$ ſumto ſcilicet $x = my$, et λ ex forma lentis obiectiuae, quaecunque fuerit ſiue ſimplex ſiue multiplicata, definitur, vt in capite praecedente eſt expoſitum. Circa lentem autem ſecundam tenendum eſt, quia ab ea totus campus pendet, eam vtrinque aeque conuexam formari debere, vt ſtatui poſſit $\pi = \frac{1}{4}$, quare cum pro ea ſit $q = \frac{\alpha}{m+1}$ radius vtriusque faciei erit $= 1,10. \frac{\alpha}{m+1}$; pro tertia autem lente oculari quoniam eius apertura plane non in calculum ingreditur, perinde eſt, quaenam ipſi figura tribuatur, dummodo minimam aperturam recipere poſſit, quae ſaltim pupillae ſit aequalis. Conueniet igitur ſtatui $\lambda'' = 1$, vt diſtantia α minor capi poſſit, eiusque figura ſecundum praecepta ſupra data elaborari poterit.

Coroll. 1.

229. Mirum videbitur, quod media lens in ipſo loco imaginis conſtituta nihil plane ad confuſionem conferat, cum tamen naturam teleſcopii tantopere immutet, vt oculum adeo lenti oculari immediate adplicari oporteat eiusque ope margo coloratus deſtrui poſſit. Quod eo magis adhuc eſt mirandum, quod haec lens nihil plane in imagine neque in eius loco vel quantitate immutet.

Co-

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Coroll. 2.

230. In ipſa igitur hac lente media diaphragma ante memoratum conſtitui debebit, cuius foramen ipſi huius lentis aperturae aequale eſt capiendum, quin etiam ſuper hac ipſa lente micrometrum ſtatui poterit tenuiſſimis ſcilicet lineis ſuper eius ſuperficie ducendis.

Coroll. 3.

231. Videmus porro hanc lentem mediam tantillo minorem eſſe debere, quam lentem ocularem, cum eius diſtantia focalis ſit $q = \frac{\alpha}{m+1}$, huius vero $= \frac{\alpha}{m}$; nihiloque minus campum apparentem manere eundem ac ſi ſimplici lente oculari, vt ante, vteremur.

Scholion 1.

232. Introductio huius lentis in ipſo loco imaginis collocandae ideo eſt maximi momenti, quod margini colorato penitus tollendo inſeruiat. Vſus autem huiusmodi lentis Aſtronomis ob aliam rationem iam dudum innotuit, ſiquidem hoc modo campum apparentem auxerunt ſimul autem ingens huius lentis incommodum obſeruarunt, in eo conſtans, quod cum lentis huius quaſi ſubſtantia ſe cum imagine permiſceat omnes vel minimae inaequalitates vitri veluti bullulae vel ſtriae a politura relictae cum imagine ipſa vniantur oculoque in pari ratione multipli-

catae repraeſententur quod certe incommodum eo magis eſt vitandum, quod vix eiusmodi vitri fruſta reperire liceat, quae nullis plane inaequalitatibus ſint obnoxia. Interim tamen haud difficile erit, has vitri inaequalitates ab ipſo obiecto diſtinguere tubum quodammodo conuertendo; tum enim mox apparebit, quid ad obiectum pertineat quidue ad lentem. Iſtud autem incommodum tantum locum habet, quando lens in ipſo imaginis loco collocatur; ſimulatque ea tantillum inde remouetur, illud mox inſenſibile euadit. Ceterum hanc inueſtigationem ab hoc caſu ſum exorſus, quod lens in loco imaginis conſtituta terminum quaſi conſtituat lentium, quae vel propius ad obiectiuam vel ad ocularem collocabuntur; quas ideo diſtingui conuenit, quod illae magis ad obiectiuam, hae vero magis ad ocularem ſint referendae, quemadmodum etiam his quotquot fuerint, commune nomen lentium ocularium tribui ſolet, quae appellatio illis lentibus, quae obiectiuae ſunt propiores, vtiquam certe conueniet.

Scholion 2.

233. Si marginem coloratum non tantopere reformidemus, vt velimus tam inſigne campi apparentis augmentum repudiare; caſus in ſolutione memoratus omnem attentionem meretur. Ponamus igitur, vt ibi animaduertimus, $q = \frac{2\alpha}{m+1}$ vt ſtatui poſſit $\pi = -\pi' = \frac{1}{4}$ et campi apparentis ſemidiameter erit

$\Phi =$

$\Phi = \frac{1}{2(m+1)}$ ſiue $\Phi = \frac{1719}{m+1}$ min. atque tam lentem ſecundam, quam tertiam vtrinque fieri oportebit aeque conuexam; hac facta poſitione pro margine colorato tollendo aequatio fiet $0 = \frac{m+1}{2m}$, quae cum duplo ſit minor, quam ea, quae capite praecedente debebat ad nihilum redigi hic iſtud lucrum adipiſcimur, vt margo coloratus, dum penitus tolli nequit, duplo tamen minor fiat ita, vt vix ſenſibilis euadat; quod ſi ergo vitro communi, pro quo $n = 1,55$ vtamur, limes diſtantiae focalis lentis obiectiuae erit

$$\alpha > k m y \sqrt[3]{0,9381 (\lambda m + 1,6299)}$$

et pro loco oculi reperitur diſtantia $O = \frac{-\pi' r}{m \Phi}$, quae ob $\frac{-\pi'}{\Phi} = \frac{(m+1)q - \alpha}{q} = \frac{m+1}{2}$ et $r = \frac{\alpha}{m}$ abit in hanc $O = \frac{m+1}{2m} \cdot \frac{\alpha}{m}$, ita, vt iam oculus duplo propius lenti oculari admoueri debeat, quam caſu praecedentis capitis. Diſtantia autem huius lentis ab obiectiua eſt, vt ibi $= \frac{m+1}{m} \cdot \alpha$. Vnde ſequens oritur conſtructio:

Conſtructio Teleſcopii ex tribus lentibus compoſiti ex eadem vitri ſpecie formatis, pro qua $n = 1,55$.

I. Lens obiectiua pro lubitu ſiue ſimplex, exiſtente $\lambda = 1$; ſiue duplicata, pro $\lambda = 0.1918$; ſiue triplicata, pro $\lambda = 0.0422$ ſiue denique quadruplicata pro $\lambda = -0,0102$ eligatur, ita, vt in capite praecedente ex diſtantia focali α determinetur.

Iſtius

Iſtius lentis ſemidiameter aperturae eſto $x = my$; interuallum vsque ad ſecundam lentem $= \alpha$.

II. Lentis ſecundae radius vtriusque faciei $= 1,10.\frac{2\alpha}{m+1}$. Eius aperturae ſemidiameter $= \frac{\alpha}{2(m+1)}$. Interuallum ad lentem ocularem $= \frac{\alpha}{m}$.

III. Lentis ocularis radius faciei vtriusque $= 1,10.\frac{\alpha}{m}$. Eius ſemidiameter aperturae $= \frac{1}{4}.\frac{\alpha}{m}$. Pro loco oculi eius diſtantia ab oculari $O = \frac{m+1}{2m}.\frac{\alpha}{m}$. Campi vero viſi ſemidiameter $= \frac{1781}{m+1}$ minut.

et vt iam monitum quantitas α ita eſt definienda, vt ſit

$$\alpha > kmy\sqrt[3]{0,9381(\lambda m + 1,6299)}$$

niſi forte hic valor minor prodeat quam vt apertura praeſcripta locum habere poſſit; quo caſu ſemper diſtantia focalis ex apertura definiri debet, vt hactenus fecimus.

Problema 2.

234. Inter lentem obiectiuam et imaginem realem eiusmodi lentem conſtituere, qua omnis confuſio ab apertura lentium oriunda deſtruatur, ſimulque margo coloratus, ſi fieri queat, tollatur.

Solutio.

Cum hic iterum tres lentes in computum ſint ducendae, formula pro multiplicatione dabit $m = -\frac{\alpha}{b}.\frac{\beta}{c}$ vbi

vbi cum inter primam et ſecundam lentem non detur imago realis, ſed ea inter ſecundam et tertiam cadat, fractio $\frac{\alpha}{b}$ erit negatiua at fractio $\frac{\beta}{c}$ erit poſitiua. Ponamus ergo $\frac{\alpha}{b} = -k$ eritque $\frac{\beta}{c} = \frac{m}{k}$, vnde colligimus $b = -\frac{\alpha}{k}$; $\beta = \mathrm{B}\,b = \frac{-\mathrm{B}\alpha}{k}$ et $c = \frac{k\beta}{m} = \frac{-\mathrm{B}\alpha}{m}$. Interualla autem, quae debent eſſe poſitiua, erunt $\alpha + b = \frac{k-1}{k}\alpha$ ita, vt $(k-1)\alpha$ debeat eſſe poſitiuum, et $\beta + c = -\mathrm{B}\,\alpha\left(\frac{1}{k} + \frac{1}{m}\right)$ ſicque $\mathrm{B}\,\alpha$ debet eſſe negatiuum hincque etiam $\frac{\mathrm{B}}{k-1} < 0$. His notatis conſideremus formulas generales $\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{\alpha}{b} = -k$, ideoque $\pi = \frac{(1-k)\Phi}{\mathfrak{B}}$. Tum vero eſt $\Phi = \frac{\pi - \pi'}{m+1}$; vnde patet, vt valor π aliquid conferat ad campum augendum, debere eſſe $\pi > 0$ ſeu $\frac{1-k}{\mathfrak{B}} > 0$; at quia $\frac{\mathrm{B}}{k-1} < 0$ erit $-\frac{\mathrm{B}}{\mathfrak{B}} < 0$; ideoque $\frac{\mathrm{B}}{\mathfrak{B}} > 0$; hoc ſcilicet requiritur, ſi campum augere velimus. Nunc conſideremus aequationem pro margine colorato tollendo $0 = \frac{\pi b}{\Phi p} - \frac{\pi'}{m\Phi}$ quae ob $p = \alpha$, $b = -\frac{\alpha}{k}$; $\frac{\pi}{\Phi} = \frac{1-k}{\mathfrak{B}}$ et $\frac{\pi'}{\Phi} = \frac{1-k}{\mathfrak{B}} - m - 1$ abit in hanc

$$0 = -\frac{(1-k)}{\mathfrak{B}}\left(\frac{1}{k} + \frac{1}{m}\right) + \frac{(m+1)}{m}$$

vnde inuenimus

$$\mathfrak{B} = \frac{(1-k)(m+k)}{k(m+1)} \text{ ideoque } \mathrm{B} = \frac{(1-k)(m+k)}{2km - m + k^2}$$

Ex his autem valoribus fit $\frac{\mathrm{B}}{\mathfrak{B}} = \frac{k(m+1)}{2km - m + k^2}$; vnde patet, vt etiam ſecunda lens campum augeat, eſſe debere $2km - m + k^2 > 0$ ad quod requiritur, vt ſit

$k > \sqrt{(m^2 + m)} - m$ ſiue $k > \frac{1}{2}$; cum igitur eſſe debeat $(k - 1)\alpha$ poſitiuum, duo hic caſus ſunt conſtitu.ndi:

I. quo $\alpha > 0$; tum eſſe debet $k > 1$, vnde fit

$$\mathfrak{B} = \frac{-(k-1)(m+k)}{k(m+1)}, \text{ et } B = \frac{-(k-1)(m+k)}{2km - m + k^2}$$

Nunc igitur erit $\pi = \frac{-(k-1)\Phi}{\mathfrak{B}} = \frac{+k(m+1)\Phi}{m+k}$

et $\pi' = \frac{-m(m+1)\Phi}{m+k}$ et $\frac{\pi}{\pi'} = \frac{-k}{m}$ vnde patet, ſi ponatur $-\pi' = \frac{1}{4}$ fore $\pi = + \frac{1}{4} \cdot \frac{k}{m}$. Ambae ergo fractiones π et π' non aequales ſumi poterunt, niſi ſit $k = m$, quo caſu ſtatui poterit $\pi = \frac{1}{4}$ et $-\pi' = \frac{1}{4}$, ita, vt campus fiat maximus. Tum autem erit $b = -\frac{\alpha}{m}$ et $\beta = c = \frac{2(m-1)\alpha}{m(3m-1)}$ ob $\mathfrak{B} = -\frac{2(m-1)}{m+1}$ et $B = -\frac{2(m-1)}{3m-1}$; ita, vt nunc ſit diſtantia focalis lentis ſecundae $\mathfrak{B}b = \frac{2(m-1)\alpha}{m(m+1)}$ et lentis tertiae $= c = \frac{2(m-1)\alpha}{m(3m-1)}$.

II. Sin autem ſit $\alpha < 0$, debet eſſe $k < 1$ et tamen $k > \frac{1}{2}$ et litterae $\mathfrak{B}$ et B fiunt poſitiuae. Hincque habebitur $\frac{\pi}{\Phi} = \frac{k(m+1)}{m+k}$ et $\frac{\pi'}{\Phi} = \frac{k(m+1)}{m+k} - m - 1 = \frac{-m(m+1)}{m+k}$

ideoque $\frac{\pi}{\pi'} = \frac{-k}{m}$ ita, vt ob $k < 1$ littera π multo minor ſit, quam $-\pi'$; ideoque campus apparens hoc caſu vix vllum accipiat augmentum.

Nunc denique id, in quo cardo rei verſatur, perpendamus, formulam ſcilicet pro ſemidiametro confuſionis, quae eſt:

$\mu m x$

$$\frac{\mu m x^3}{4\alpha^3}\left(\lambda - \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{\lambda''}{B^3.m}\right)$$

quae vt ad nihilum redigi queat, neceſſe eſt, vt $\mathfrak{B}$ ſit quantitas poſitiua vnde caſus prior ante memoratus locum habere nequit, ex quo neceſſe eſt, vt ſit $k < 1$, ideoque etiam $\alpha < 0$ et $B > 0$; ex quo ſequitur capi debere $k > \frac{1}{2}$, ita, vt k intra limites $\frac{1}{2}$ et 1 contineri debeat; quare cum hoc caſu ſit $\frac{B}{\mathfrak{B}} > 0$, campus quoque augmentum quoddam accipiet, propterea quod ſit $\pi : -\pi' = k : m$ quod autem vix erit ſenſibile. Si itaque ſtatuatur ſemidiameter confuſionis $= 0$, habebitur $\lambda = \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{\lambda''}{B^3.m}$ vbi notandum eſt, litteras λ' et λ'' vnitate minores eſſe non poſſe.

Quo reſolutio huius aequationis clarius perſpiciatur, primum obſeruo, ſumi non poſſe $k = 1$; tum quia duae priores lentes fierent contiguae, tum vero quod prodiret $\mathfrak{B} = 0$ et $B = 0$; ſin autem poneretur $k = \frac{1}{2}$, fieret quidem $\mathfrak{B} = \frac{2m+1}{2(m+1)}$ et $B = 2m + 1$; vnde noſtrae diſtantiae erunt

$$b = -2\alpha;\ \beta = -2(2m+1)\alpha$$
$$c = \frac{-(2m+1)\alpha}{m}$$

ideoque interualla $\alpha + b = -\alpha$

et $\beta + c = -(2m+1)\alpha\left(2 + \frac{1}{m}\right) = -\frac{(2m+1)^2}{m}.\alpha$ quod poſterius in enormem longitudinem excreſceret, niſi $-\alpha$ perexiguum caperetur, quod autem fieri nequit, quia eius apertura ob claritatem per ſe definitur;

tur; ex quo manifeſtum eſt numerum k intra limites 1 et $\frac{1}{2}$ accipi debere.

Coroll. 1.

235. Hoc ergo modo duplicem perfectionem his teleſcopiis conciliare licet, alteram, qua margo coloratus prorſus deſtruitur; alteram vero, qua confuſio ab apertura oriunda ad nihilum redigitur. Neque vero campo apparenti vllum augmentum ſenſibile addi poteſt.

Coroll. 2.

236. Quod ad lentium harum aperturas attinet, pro prima quidem erit ſemidiameter $x = my$; pro ſecunda autem $\pi q + \frac{qx}{\mathfrak{B}p}$ (§. 23) ſiue

$$= \frac{(1-k)(m+k)\pi . \alpha}{k^2(m+1)} + \frac{x}{k}$$

quia autem eſt $\pi = -\frac{k}{m}. \pi'$ capique poteſt $\pi' = -\frac{1}{4}$, ſiquidem lens ocularis fiat vtrinque aequaliter conuexa, erit $\pi = \frac{k}{4m}$, ideoque ſemidiameter aperturae ſecundae lentis

$$= \frac{(1-k)(m+k)\alpha}{4mk(m+1)} + \frac{x}{k}$$

cuius pars prior prae poſteriore quaſi euaneſcit, ita, vt ſufficiat hunc ſemidiametrum ſtatuiſſe $= \frac{x}{k}$, qui vtique maior eſt, quam x ob $k < 1$.

Lens

Lens autem ocularis vtrinque aequaliter conuexa eſſe debet, vnde cum eius diſtantia focalis ſit

$$c = \frac{-(1-k)(m+k)x}{m(2km-m+k^2)}$$

eius pars quarta dabit ſemidiametrum aperturae.

Coroll. 3.

237. Quod autem ad locum oculi attinet poſt lentem ocularem, eius diſtantia reperitur $O = -\frac{\pi' \cdot r}{m\Phi}$ quia autem eſt

$$-\frac{\pi'}{\Phi} = \frac{m(m+1)}{m+k} \text{ et } r = \frac{-(1-k)(m+k)x}{m(2km-m+k^2)}$$

erit $O = \frac{-(1-k)(m+1)x}{m(2km-m+k^2)}$

quae eſt quantitas poſitiua.

Scholion.

238. Labor certe eſſet maxime operoſus ſi hos valores pro B et 𝔅 inuentos vellemus in vltima aequatione ſubſtituere indeque numeros λ et λ′ inueſtigare atque adeo coacti eſſemus pro quauis multiplicatione calculum de nouo ſuſcipere, cui incommodo medela eſt quaerenda. Perpendamus igitur iſtos tam complicatos valores pro 𝔅 et B ex aequatione pro margine tollendo eſſe erutos vt ſcilicet illi aequationi ſummo rigore ſatisfieret; quoniam autem ſuperfluum eſt, hanc aequationem perfectiſſime adimplere, propterea quod locus oculi ob aperturam pupillae haud me-

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diocrem latitudinem patitur; exiguaque eius mutatione margo coloratus, ſi quis forte obſeruatur, facillime euitabitur; ſufficiet ei quam proxime ſatisfeciſſe, quare cum ſemper m denotet numerum ſatis magnum, k autem ſit vnitate minor, prae m facile licebit k negligere et m quaſi infinitum ſpectare; vnde nanciſcemur hos valores

$$\mathfrak{B} = \frac{(1-k)}{k}, \quad B = \frac{1-k}{2k-1}$$

quibus itaque in euolutione noſtri problematis vtemur; ex iis autem noſtra elementa ita ſimplicius exprimentur:

$$b = -\frac{a}{k}; \ \beta = \frac{-(1-k)\alpha}{k(2k-1)} \ \text{et} \ c = \frac{-(1-k)\alpha}{m(2k-1)};$$

hinc interualla

$$a + b = \frac{-(1-k)a}{k} \ \text{et}$$

$$\beta + c = \frac{-(1-k)}{2k-1}\left(\frac{1}{k} + \frac{1}{m}\right)\alpha$$

$$= \frac{-(1-k)(k+m)\alpha}{(2k-1)km}$$

et pro oculi loco

$$O = \frac{-(m+1)(1-k)}{m(2k-1)} \cdot \frac{\alpha}{m}.$$

Lentium autem harum diſtantiae focales erunt

$$\text{I}^{\circ}.\ p = a; \quad \text{II}^{\circ}.\ q = \frac{-(1-k)}{k^2} \cdot a$$

$$\text{III}.\ r = c = \frac{-(1-k)\alpha}{m(2k-1)}$$

earum-

earumque aperturae ſemidiametri

Imae. $x = my$. IIdae. $\frac{x}{k} = \frac{my}{k}$.

IIItiae. $= \frac{1}{4} r = \frac{-(1-k)x}{4m(2k-1)}$.

Campi denique apparentis ſemidiameter erit

$$\Phi = \frac{\frac{1}{4}\left(1 + \frac{k}{m}\right)}{m + 1}$$

ſiue $\Phi = 859 \left(\frac{m+k}{m(m+1)}\right)$ min.

Nunc autem aequatio adhuc reſoluenda erit

$$\lambda = \frac{1}{1-k}\left(\frac{\lambda' . k^2}{(1-k)^2} + \frac{\nu(2k-1)}{1-k}\right) + \frac{\lambda''(2k-1)^3}{(1-k)^3 . m}$$

ſeu

$$\lambda = \frac{1}{(1-k)^3}\left(\lambda' k^2 + \nu(1-k)(2k-1) + \frac{\lambda''(2k-1)^3}{m}\right)$$

Nihil aliud igitur ſupereſt, niſi vt pro quibusdam valoribus ipſius k hanc aequationem reſoluamus, vbi notandum eſt, λ'' poni debere $= 1.6299$ ſiquidem vitro communi, pro quo eſt $n = 1,55$ vti velimus; quo caſu etiam eſt $\nu = 0.2326$.

Exemplum I.

239. Statuamus $k = \frac{3}{4}$ vt intra limites ſuos 1 et $\frac{1}{2}$ medium teneat et aequatio noſtra reſoluenda induet hanc formam:

$$\lambda = 64\left(\frac{9\lambda'}{16} + \frac{1}{8}\nu + \frac{\lambda''}{8m}\right)$$

ſiue $\lambda = 36\lambda' + 8\nu + \frac{8\lambda''}{m}$

$\lambda =$

et pro aperturis lentium ſemidiameter primae $= x$, ſecundae $= \frac{5}{3}x$ et tertiae $= -\frac{\alpha}{2m}$ oculique poſt lentem diſtantia $O = -\frac{2(m+1)}{m} \cdot \frac{\alpha}{m}$.

Scholion.

241. Si huiusmodi caſus pro variis multiplicationibus euoluere vellemus, ex ſuperioribus intelligitur, duos tantum caſus ſufficere poſſe; vt inde formulae generales pro quauis multiplicatione elici queant, dum ſcilicet altero, pro m numerus modice magnus, veluti 20, aſſumatur, altero vero numerus quaſi infinitus; quae inveſtigatio cum omni attentione digna videatur, eam in ſequente problemate inſtituamus.

Problema 3.

242. In caſu praecedentis problematis ſi capiatur $k = \frac{5}{9}$ pro quacunque multiplicatione maiore m teleſcopium conſtruere, in quo non ſolum margo coloratus prorſus euaneſcat, ſed etiam confuſio ex apertura oriunda ad nihilum redigatur.

Solutio.

Cum hic ſit $k = \frac{5}{9}$ erit

$$\mathfrak{B} = \frac{4\cdot(9m+5)\cdot}{5\cdot 9\cdot(m+1)} = \frac{4(9m+5)}{45(m+1)}$$

$$B = \frac{4\cdot(9m+5)\cdot 9^2}{9\cdot 9(9m+25)} = \frac{4(9m+5)}{9m+25}.$$

Nunc igitur duos caſus euoluamus, in quorum priore ſit $m = 20$; in poſteriore vero $m = \infty$.

I. Ob $m = 20$ erit $\mathfrak{B} = \frac{148}{189}$ et $B = \frac{148}{41}$; vnde noſtra aequatio, quae eſt

$$\lambda = \frac{9\lambda'}{5\mathfrak{B}^3} + \frac{9\nu}{5\mathfrak{B}B} + \frac{\lambda''}{B^3 . m}.$$

ſi omnes lentes ex vitro communi pro quo $n = 1,55$ et $\nu = 0,2326$ lentem autem ocularem vtrinque aeque conuexam aſſumamus, vt ſit $\lambda'' = 1,6299$, ſequentem induet formam; in ſubſidium vocatis logarithmis

$$\text{Log.}\ \mathfrak{B} = 9.8937999$$

$$\text{Log.}\ B = 0.5574778$$

hincque

$$\text{Log.}\ \tfrac{1}{\mathfrak{B}} = 0.1062000$$

$$\text{Log.}\ \tfrac{1}{B} = 9.4425221$$

$$\text{et Log.}\ \tfrac{9}{5} = 0,2552725$$

$$\lambda = 3,7486\,\lambda' + 0,14812 + 0,00173.$$

$$\text{Log.}\ 3,7486\,\lambda' = 0,5738725 + \text{Log.}\ \lambda'.$$

Hic circa numerum λ' obſeruaſſe iuvabit, quod cum lens ſecunda maximam aperturam habere debeat, cuius ſcilicet ſemidiameter ſit $\frac{9}{5}x = \frac{9}{5}.my$ expediat hanc lentem vtrinque aeque conuexam reddere, quam ob cauſſam ſtatui oportet

$\surd(\lambda' - 1)$

$$\sqrt{(\lambda'-1)} = \frac{\sigma-\rho}{2\tau}\cdot\frac{\beta-b}{\beta+b} = \frac{\sigma-\rho}{2\tau}\cdot\frac{B-1}{B+1}.$$

hincque $\lambda' = 1 + \left(\frac{\sigma-\rho}{2\tau}\right)^2\cdot\left(\frac{B-1}{B+1}\right)^2.$

Cum autem conſtet eſſe $\left(\frac{\sigma-\rho}{2\tau}\right)^2 = 0,6299$

erit $\lambda' = 1 + 0,6299\cdot\left(\frac{107}{189}\right)^2$ ſeu

$\lambda' = 1,20189$; hincque

$\lambda = 4,50544 + 0,14812 + 0,00173$

$\lambda = 4,65529.$

Vnde fit $\lambda - 1 = 3,65529$ et $\tau\sqrt{\lambda-1} = 1,7304.$

Pro formatione igitur primae lentis habebimus

$$F = \frac{\alpha}{\sigma \pm 1,7304} = \frac{\alpha}{-0,1030} = -9,7087.\,\alpha$$

$$G = \frac{\alpha}{\rho + 1,7304} = \frac{\alpha}{+1,9211} = +0,52053.\,\alpha$$

cuius lentis aperturae ſemidiameter debet eſſe $x = my$
At interuallum ſecundae lentis ab hac eſt

$$\alpha + b = -0,8.\,\alpha.$$

Pro ſecunda autem lente, cum ſit eius diſtantia focalis $q = \mathfrak{B} b = -\frac{9}{5}\mathfrak{B}\alpha = -1,4095\,\alpha$ erit radius vtriusque faciei $= 1,10.\,q = -1,5504\,\alpha$ eius aperturae ſemidiameter $= \frac{9}{5}x = \frac{9}{5}my$. Ab hac autem lente ad tertiam, interuallum eſt

$$\beta + c = -B\alpha\left(\frac{m+k}{mk}\right) = -6,4974\,\alpha$$
$$-3,6097.\frac{\alpha}{m} = -6,6780.\,\alpha$$

 Pro

Pro lente tertia, cuius diſtantia focalis eſt

$$c = -\tfrac{148}{41}\cdot\tfrac{\alpha}{m} = 3,6097.\tfrac{\alpha}{m}$$

radius faciei vtriusque $= -1,1\,c = -3,9707.\tfrac{\alpha}{m}$;

hincque ad oculum vsque erit diſtantia

$$O = -\tfrac{(m+1).B\alpha}{(m+4).m} = -\tfrac{40,021}{50,01}.\tfrac{\alpha}{m} = -3,6878.\tfrac{\alpha}{m}.$$

II. Sit nunc $m = \infty$ erit $\mathfrak{B} = \tfrac{4}{5}$ $B = 4$ vnde noſtra aequatio induet hanc formam:

$$\lambda = \tfrac{225}{64}\lambda' + \tfrac{0}{16}\nu$$

Hic iterum lentem ſecundam aequaliter conuexam reddamus et ob $\beta = B\,b = 4\,b$ erit

$$\sqrt{\lambda' - 1} = \tfrac{\sigma-\varrho}{2\tau}.\tfrac{\beta-b}{\beta+b} = \tfrac{\sigma-\varrho}{2\tau}.\tfrac{3}{5}$$

Hincque $\lambda' = 1 + 0,6299.\tfrac{9}{25}$, $\lambda' = 1,2267$.
ex quo colligimus

$$\lambda = 4,3126 + 0,1308 = 4,4434$$

hincque $\lambda - 1 = 3,4434$

et $\tau\sqrt{(\lambda - 1)} = 1,6796$; quare ſequens habetur conſtructio:

I. Pro prima lente

$$F = \tfrac{\alpha}{\sigma - 1,6796} = \tfrac{\alpha}{-0,0522} = -19,1570.\,\alpha$$

$$G = \tfrac{\alpha}{\varrho + 1,6796} = \tfrac{\alpha}{+1,8703} = +0,5346.$$

Apertura eſt, vt ante, aeque ac diſtantia ad ſecundam lentem.

II. Pro

II. Pro ſecunda lente

Quum eius diſtantia focalis

$= -\frac{6}{5}\mathfrak{B}\alpha = -\frac{36}{25}.\alpha = -1.44\,\alpha$, fiet

radius vtriusque faciei $= -1,584.\alpha$

eiusque aperturae ſemidiameter $= \frac{9}{5}x$

at diſtantia ad lentem tertiam $= -7,2\,\alpha - 4.\frac{\alpha}{m}$.

III. Pro tertia lente

cuius diſtantia focalis $= -4\frac{\alpha}{m}$

radius vtriusque faciei $= -4,4.\frac{\alpha}{m}$

eiusque aperturae ſemidiameter $= -1,1\frac{\alpha}{m}$

Ab hac lente ad oculum vsque erit diſtantia

$O = -4.\frac{\alpha}{m}$.

His duobus caſibus euolutis ſolutionem quaeſtionis noſtrae generalis pro multiplicatione quacunque m maiore, quam 20, ita adſtruamus:

I. Pro prima lente

ſtatuamus

$$\text{radium faciei} \begin{cases} \text{anter} = -(19,1570 + \frac{f}{m}).\alpha \\ \text{poſter.} = +(0,5346 + \frac{g}{m})\alpha \end{cases}$$

et adplicatione ad caſum $m = 20$ facta reperietur

$19,1570 + \frac{f}{20} = 9,7087$

vnde fit $f = -188,966$

porro $0,5346 + \frac{g}{25} = 0,5205$

hinc $g = -0,2820$.

II. Pro ſecunda lente

ſtatuatur

radius vtriusque faciei $= -(1,584 + \frac{h}{m})\alpha$

cumque eſſe debeat $1,584 + \frac{h}{25} = 1,5504$

erit $h = -0,6720$.

Eius diſtantia focali exiſtente $= -(1,440 - \frac{0,6100}{m})\alpha$

et aperturae ſemidiameter $= \frac{9}{5}x$.

Pro diſtantia ad tertiam lentem inueniemus

$$-(7,2 - \frac{10,0520}{m})\alpha - (4,00 - \frac{7,8060}{m})\frac{\alpha}{m}$$

ſiue $-7,200\alpha + 10,0520 . \frac{\alpha}{m} + \frac{7,8060}{m^2} . \alpha$

ſiue $-(7,200 - \frac{10,0520}{m} - \frac{7,8060}{mm})\alpha$.

III. Pro tertia lente

cuius diſtantia focalis reperitur

$$= -(4,00 - \frac{7,8060}{m})\frac{\alpha}{m}$$

ſumi debet radius vtriusque faciei

$$= -(4,400 - \frac{8,1466}{m})\frac{\alpha}{m}$$

cuius parti quartae ſemidiameter aperturae aequalis ſtatui poteſt.

Diſtan-

Diſtantia denique oculi ab hac lente reperitur

$$O = -\left(4 - \frac{6,2170}{m}\right)\frac{\alpha}{m}$$

Campi vero apparentis ſemidiameter erit $\frac{850}{m+1}$ minut.

Coroll. 1.

243. Cum interuallum primae lentis et ſecundae ſit $= -0,8\,\alpha$, prodibit tota teleſcopii longitudo ab obiectiua vsque ad oculum

$$-\left(8 - \frac{6,0520}{m} - \frac{14,2180}{mm}\right)\alpha$$

ita, vt haec longitudo fere ſit octuplo maior, quam diſtantia focalis α qua circumſtantia haec teleſcopia non admodum commendari merentur.

Coroll. 2.

244. Cum primae lentis ſemidiameter aperturae debeat eſſe $x = my = \frac{m}{50}$ dig. qui autem maior eſſe nequit parte quarta radii minoris, quae eſt $0,1336.\alpha = \frac{1}{8}\alpha$ circiter; patet capi debere $-\alpha > \frac{16.m}{100}$ vel $\alpha > 0,16.m$.

Quia autem lentis ſecundae ſemidiameter aperturae eſſe debet $= \frac{9}{5}x = \frac{9m}{250}$ dig. hic quoque minor eſſe debet parte quarta radii, quae eſt $0,396.\alpha$; vnde eſſe debet $-\alpha > 0,0909.m$, qui limes cum minor ſit praecedente, illum obſeruari oportet.

Scho-

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Scholion.

245. Cum igitur $-\alpha$ maius eſſe debeat, quam $0,16.m$ ſtatuamus $-\alpha = \frac{2}{10}.m$ ſiue $\alpha = -0,2\,m$, atque ſequentem conſtructionem pro Teleſcopiis huius ſpeciei obtinebimus.

Conſtructio Teleſcopiorum pro quacunque multiplicatione m, lentibus ex vitro communi confectis.

I. Pro lente obiectiua

rad. fac. $\begin{cases} \text{anter.} = +3,8314\,m - 37,7932 \text{ dig.} \\ \text{poſter.} = -0,1069.m + 0,0564 \text{ dig.} \end{cases}$

Eius aperturae ſemidiameter $= \frac{m}{50}$ dig.

Interuallum ad lentem ſecundam $= 0,16.m$ dig.

II. Pro lente ſecunda, in digitis

diſtantia focalis $= +0,2880\,m. - 0,12.$

radius faciei vtriusque $= +0,3168\,m - 0,13.$

Eius aperturae ſemidiam. $= \frac{9}{250}.m = 0,036.m.$

Interuallum ad lentem tertiam

$= +1,4400\,m - 2,01 - \frac{1,56}{m}.$

III. Pro tertia lente in digitis

diſtantia focalis $= +0,800 - \frac{1,56}{m}.$

radius

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radius vtriusque faciei $= + 0,88 - \frac{1,72}{m}$
cuius pars quarta $= \frac{1}{5}$ dig. dat ſemidiametrum aperturae. Hinc ad oculum vsque diſtantia erit

$$O = 0,8 - \frac{1,24}{m} \text{ dig.}$$

Campi apparentis ſemidiameter $= \frac{850}{m+1}$ minut.
Tota autem Teleſcopii longitudo erit

$$= (-1,30 + 1,6 . m - \frac{2,8}{m}) \text{ dig.}$$

Ita v. gr. pro $m = 100$ erit longitudo $= 158\frac{2}{3}$ dig. ſiue 13 ped. $2\frac{2}{3}$ dig.

Cum igitur ſupra tubo vnum pedem vix ſuperante fere tantam multiplicationem produxerimus, haec teleſcopiorum ſpecies nunc quidem erit repudianda, etſi reſpectu vulgarium tuborum aſtronomicorum maxime foret aeſtimanda, cum quod nullum marginem coloratum praebeat, tum vero etiam quia confuſio ab apertura oriunda prorſus ſit ſublata. Quamobrem nobis inquiri conueniet, num duabus lentibus inter obiectiuam et imaginem collocandis hoc incommodum euitari poſſit.

Problema 4.

246. Inter lentem obiectiuam et imaginem eiusmodi duas lentes interponere, vt non ſolum margo coloratus, ſed etiam confuſio ab apertura oriunda penitus deſtruatur.

Solutio.

Cum hic quatuor lentes ſint conſiderandae, multiplicatio dabit hanc formulam $m = \frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$, quarum trium fractionum binae priores negatiuae, tertia vero affirmatiua eſſe debebit. Statuatur ergo

$$\frac{\alpha}{b} = -k;\ \frac{\beta}{c} = -k' \text{ eritque } \frac{\gamma}{d} = \frac{m}{kk'}.$$

Vnde erit $b = -\frac{\alpha}{k};\ c = -\frac{\beta}{k'};\ d = \frac{\gamma k k'}{m}$

praeterea vero eſt $\beta = \mathrm{B}\, b;\ \gamma = \mathrm{C}\, c$ vnde omnia haec elementa ex α ita definientur:

$$b = -\frac{\alpha}{k};\ \beta = -\frac{\mathrm{B}\alpha}{k};\ c = +\frac{\mathrm{B}\alpha}{k.k'}$$

$$\gamma = \frac{\mathrm{BC}\alpha}{kk'};\ d = \frac{\mathrm{BC}\alpha}{m}$$

hinc interualla lentium fient

$$1^\circ.\ \alpha + b = \frac{k-1}{k} \cdot \alpha.$$

$$2^\circ.\ \beta + c = \frac{\mathrm{B}\alpha}{k}\left(\frac{1-k'}{k'}\right)$$

$$3^\circ.\ \gamma + d = \mathrm{BC}\alpha\left(\frac{1}{kk'} + \frac{1}{m}\right)$$

vnde quia $k k'$ et m ſunt per ſe numeri poſitiui, hae ſequuntur conditiones:

$$1^\circ.\ \alpha(k-1) > 0;\ 2^\circ.\ \mathrm{B}\alpha(1-k') > 0;$$

$$3^\circ.\ \mathrm{BC}\alpha > 0;$$

quae

quae eliſo α reducuntur ad has duas

$$4^{\circ}.\ \frac{B(1-k')}{k-1} > 0;$$

$$5^{\circ}.\ \frac{C}{1-k'} > 0 \text{ ſeu } C(1-k') > 0.$$

Iam ex ſuperioribus vidimus, marginem coloratum deſtrui non poſſe, niſi ante fractiones π, π' et π'' definiantur quem in finem ſequentes aequationes conſiderari debent:

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi} = \frac{\alpha}{b} = -k$$

$$\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi} = \frac{B\alpha}{c} = kk'.$$

$$\text{et } \Phi = \frac{\pi-\pi'+\pi''}{m+1}$$

ex quibus aſſequimur:

$$\frac{\pi}{\Phi} = \frac{1-k}{\mathfrak{B}};$$

$$\frac{\pi'}{\Phi} = \frac{1-k}{\mathfrak{B}\mathfrak{C}} + \frac{kk'-1}{\mathfrak{C}}$$

$$\frac{\pi''}{\Phi} = \frac{\pi'}{\Phi} - \frac{\pi}{\Phi} + m + 1$$

$$= \frac{1-k}{\mathfrak{B}\mathfrak{C}} + \frac{kk'-1}{\mathfrak{C}} + m + 1$$

quibus valoribus ſubſtitutis ad marginem coloratum tollendum requiritur haec aequatio, diuiſione per $\frac{dn}{n-1}$ facta,

$$0 = -\frac{\pi}{\Phi}\cdot\frac{1}{k} + \frac{\pi'}{\Phi}\cdot\frac{1}{kk'} + \frac{\pi''}{\Phi}\cdot\frac{1}{m}$$

 ſiue

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ſiue

$$\mathrm{o} = \frac{k-1}{\mathfrak{B}k} + \frac{1-k}{\mathfrak{B}\mathfrak{C}kk'} + \frac{kk'-1}{\mathfrak{C}kk'}$$
$$+ \frac{1-k}{\mathfrak{B}Cm} + \frac{kk'-1}{\mathfrak{C}m} + \frac{m+1}{m}$$

ex qua aequatione vel 𝔅 vel ℭ definiri poteſt; tum vero vt ſemidiameter confuſionis ad nihilum redigatur, debet eſſe

$$\mathrm{o} = \lambda - \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right)$$
$$+ \frac{1}{B^3\mathfrak{C}kk'}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) + \frac{\lambda'''}{B^3C^3.m}$$

quae vt reſolui poſſit, littera 𝔅 debet eſſe poſitiua, vel ſi 𝔅 eſſet negatiuum ob B quoque negatiuum littera ℭ debet eſſe poſitiua.

Coroll. 1.

247. Aequatio pro margine colorato tollendo ad hanc formam reducitur:

$$\mathrm{o} = \frac{1-k}{\mathfrak{B}\mathfrak{C}}\left(\frac{1}{kk'} + \frac{1}{m}\right) - \frac{1-k}{\mathfrak{B}}\left(\frac{1}{k} + \frac{1}{m}\right)$$
$$+ \frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'} + \frac{1}{m}\right) + \frac{m+1}{m}$$

ſeu ad hanc:

$$\mathrm{o} = \frac{1-k}{\mathfrak{B}}\left(\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'} + \frac{1}{m}\right) - \frac{1}{k} - \frac{1}{m}\right)$$
$$+ \frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'} + \frac{1}{m}\right) + \frac{m+1}{m}$$

vnde

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vnde reperitur

$$\frac{k-1}{\mathfrak{B}} = \frac{\frac{kk'-1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)+\frac{m+1}{m}}{\frac{1}{\mathfrak{C}}\left(\frac{1}{kk'}+\frac{1}{m}\right)-\frac{1}{k}-\frac{1}{m}}$$

ſiue

$$\frac{k-1}{\mathfrak{B}} = \frac{(kk'-1)(m+kk')+(m+1)kk'\mathfrak{C}}{m+kk'-k'm\mathfrak{C}-kk'\mathfrak{C}}$$

$$\frac{k-1}{\mathfrak{B}} = \frac{(kk'-1)(m+kk')+\mathfrak{C}kk'(m+1)}{m+kk'-\mathfrak{C}k'(m+k)}$$

$$\frac{k-1}{\mathfrak{B}} = kk'-1+\frac{\mathfrak{C}k'(m(k-1)+kk'(k+m))}{m+kk'-\mathfrak{C}k'(m+k)}$$

Coroll. 2.

248. Si haec aequatio ſtatim a fractionibus liberetur, habebitur

$$0 = (1-k)(m+kk') - \mathfrak{C}(1-k)k'(m+k)$$
$$+ \mathfrak{B}(kk'-1)(m+kk')$$
$$+ \mathfrak{B}\mathfrak{C}kk'(m+1)$$

vnde reperitur

$$\mathfrak{C} = \frac{(m+kk')(k-1+\mathfrak{B}(1-kk'))}{\mathfrak{B}kk'(m+1)+(k-1)k'(m+k)}$$

Scholion.

249. Inprimis autem notatu dignus eſt caſus quo numerus B fit infinitus et numerus C = o, quem ſupra iam alia occaſione euoluimus; quae operatio cum ſupra difficilior ſit viſa, nunc ſequenti modo planiore

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expediatur. Conſiderabimus ſcilicet numerum B vt praegrandem ſitque $B = \frac{1}{\omega}$, denotante ω fractionem minimam, ita, vt ω loco B in calculum introducatur. Tum igitur erit $\mathfrak{B} = \frac{1}{1+\omega}$ iam ne ſecundum interuallum $\beta + c$ nimis excreſcat, ſtatuatur $\beta + c = \frac{\eta\alpha}{k}$

eritque $c = \frac{\eta\alpha}{k} - \beta$, $\frac{\beta}{c} = -k' = \frac{k\beta}{\eta\alpha - k\beta}$

et quia eſt

$$\beta k = -B\alpha = -\frac{\alpha}{\omega}; \text{ erit}$$

$$+k' = \frac{1}{\eta\omega + 1} \text{ et } 1 - k' = \frac{\eta\omega}{1 + \eta\omega}$$

ita vt nunc loco litterae k' in calculum introducatur littera η; denique ne tertium interuallum nimis excreſcat ob $B = \frac{1}{\omega}$, ſtatuamus $C = \vartheta\omega$, vt fiat $BC = \vartheta$; ita, vt hic loco litterae C, ϑ in calculum ingrediatur. Hinc erit $\mathfrak{C} = \frac{\theta\omega}{1+\theta\omega}$ atque hinc porro

$$\frac{\pi}{\Phi} = (1 - k) + \omega(1 - k)$$

$$\frac{\pi'}{\Phi} = \frac{1+\theta\omega}{(\eta\omega + 1)\theta}((1 - k - \eta k) + \eta\omega(1 - k))$$

$$\frac{\pi'}{\Phi} = \frac{1+\theta\omega}{(1 + \eta\omega)\theta}(1 - k - \eta k + \eta\omega(1 - k))$$

hincque poſito $\omega = 0$, erit

$$\frac{\pi}{\Phi} = 1 - k; \ \frac{\pi'}{\Phi} = \frac{1}{\theta}(1 - k - \eta k)$$

hincque

$$\frac{\pi''}{\Phi} = \frac{1 - k - \eta k}{\theta} + k + m.$$

Quare

Quare cum pro margine tollendo inuenta ſit hacc aequatio:

$$o = -\frac{\pi}{\Phi} \cdot \frac{1}{k} + \frac{\pi'}{\Phi} \cdot \frac{1}{kk'} + \frac{\pi''}{\Phi} \cdot \frac{1}{m}$$

ob $k' = 1$, ſi illi valores ſubſtituantur, prodibit

$$o = \frac{k-1}{k} + \frac{1-k-\eta k}{k\vartheta} + \frac{1-k-\eta k}{m\vartheta} + \frac{k+m}{m}$$

ſiue
$$\begin{aligned} o = m\vartheta(k-1) &+ m(1-k-\eta k) \\ &+ k(1-k-\eta k) \\ &+ k\vartheta(k+m) \end{aligned}$$

$$o = \vartheta(k^2 + (2k-1)m) + (k+m)(1-k-\eta k)$$

hincque inuenietur

$$\vartheta = \frac{+(k+m)(k+\eta k-1)}{(2k-1)m+k^2}$$

hinc autem noſtra elementa erunt

$$b = -\frac{\alpha}{k};\ \beta = \infty;\ c = \infty;$$

$$\gamma = \frac{\vartheta\alpha}{k};\ d = \frac{\vartheta\alpha}{m}$$

et interualla

$$\alpha + b = \frac{k-1}{k} \cdot \alpha$$

$$\beta + c = \frac{\eta\alpha}{k}$$

$$\gamma + d = \vartheta\alpha\left(\frac{1}{m} + \frac{1}{k}\right)$$

quae debent eſſe poſitiua; ideoque

$$\eta(k-1) > o;\ \vartheta(k-1) > o \text{ et } \vartheta\eta > o.$$

Pro

Pro loco oculi autem habebimus

$$O = \frac{\pi''}{m\Phi} d = \frac{1-k-\eta k}{mm} \cdot \alpha + \frac{(k+m)\theta z}{m^2}$$

et valore pro ϑ ſubſtituto

$$O = \frac{1-k-\eta k}{m^2} \cdot \alpha + \frac{(k+m)^2(k+\eta k-1)\alpha}{m^2((2k-1)m+k)}$$

$$O = \frac{m+1}{m} \cdot \frac{k+\eta k-1}{(2k-1)m+k^2} \cdot \alpha$$

his denique obſeruatis reſoluenda reſtat haec aequatio:

$$0 = \lambda - \frac{\lambda'}{k} + \frac{\lambda''}{\theta \cdot k} + \frac{\lambda'''}{\theta^3 . m}$$

quae cum ſecundum membrum per ſe ſit negatiuum facile reſolui poterit, id quod in ſequente problemate oſtendemus.

Problema 5.

250. In caſu praecedentis problematis ſi binae priores lentes ita fuerint comparatae, vt radii iterum paralleli euadant, conſtructionem huiusmodi teleſcopiorum exponere.

Solutio.

Cum hoc caſu fiat $B = \infty$ et $C = 0$ in ſcholio praecedente elementa iam ſunt definita; vnde ea hic repetere ſuperfluum foret; quo autem clarius ſolutionem euoluamus, duo ſunt caſus perpendendi; alter, quo diſtantia α eſt poſitiua; alter, quo ea eſt negatiua.

I. Sit

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I. Sit igitur $\alpha > 0$. debebitque eſſe $k > 1$; $\eta > 0$ et $\vartheta > 0$ quae vltima conditio ſponte impletur; ſitque etiam O poſitiuum, tum vero fiet $\frac{\pi}{\Phi} < 0$ et $\frac{\pi'}{\Phi} < 0$ nempe $\frac{\pi'}{\Phi} = \frac{1-k-\eta k}{\theta} = \frac{-(2k-1)m-k^2}{k+m}$.

Ex vltima igitur formula colligetur ſemidiameter campi viſi

$$\Phi = \frac{\pi''.\theta}{1-k-\eta k+(k+m.\theta} \text{ ſeu}$$

$$\Phi = \frac{(m+k)\pi''}{m(m+1)}$$

ſubſtituto ſcilicet valore ϑ, ſi modo praecedentes formulae non praebeant campum minorem. Ad quod diiudicandum comparentur valores π et π' cum π''

et ob $\frac{\pi''}{\Phi} = \frac{m(m+1)}{m+k}$

erit $\frac{\pi}{\pi''} = \frac{(1-k)(m+k)}{m(m+1)}$

et $\frac{\pi'}{\pi''} = \frac{-(2k-1)m-k^2}{m(m+1)}$ hincque $\frac{\pi-\pi'}{\pi''} = \frac{k}{m}$;

at ex illis formulis patet tam π, quam π' minores eſſe, quam π'' dummodo ſit k minus, quam $\frac{5}{12}m$ et cum ſit

$$\Phi = \frac{\pi-\pi'+\pi''}{m+1} \text{ ob } \frac{\pi-\pi'}{\pi''} > 0$$

campus apparens hinc aliquod augmentum accipiet eritque $\Phi = \frac{k+m}{m(m+1)}\pi''$ qui vtique maior eſt quam ſimplex, ſcilicet $\Phi = \frac{\pi''}{m+1}$ idque in ratione $m+k:m$.

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Iam porro aequatio reſoluenda eſt, vt ante:

II. Sin autem α ſit negatiuum fieri debet $k < 1$, $\eta < 0$; $\vartheta < 0$; ad quod neceſſarium eſt, vt ſit $k > \frac{1}{2}$.

Quia nunc pro caſu praecedente habuimus $\frac{\pi - \pi'}{\pi'} = \frac{k}{m}$, hinc campus apparens multo minus augmentum accipit in hoc caſu, quam in illo, quod adeo vix erit ſenſibile, et pro loco oculi diſtantia O etiam hoc caſu ſit poſitiua; quam ob cauſſam caſus prior huic poſteriori anteferendus videtur.

Etſi autem priori caſu campus apparens notabiliter augeri poſſe eſt inuentus, dum ſcilicet k vsque ad valorem $\frac{5}{13}\, m$ augetur, tamen reſolutio noſtrae aequationis hoc non permittit, quoniam numerus λ' nimis magnus accipi deberet, quocirca littera k vix vltra binarium vel ternarium ad ſummum creſcere poteſt; vti in ſubiunctis exemplis magis fiet manifeſtum, quae ex caſu priore deriuabimus, quoniam facile eſt praevidere, poſteriorem caſum eo etiam vitio eſſe laboraturum, quod longitudo teleſcopii nimis excreſcat.

Exemplum.

251. Statuamus $k = 2$ et multiplicationem $m = 50$, quandoquidem hic de tubis aſtronomicis agitur eritque

$$\vartheta = \frac{52.(1 + 2\eta)}{154} = \frac{26}{77}(1 + 2\eta)$$

qui

qui valor ne fiat nimis parvus, quia tum in noſtra aequatione terminus $\frac{\lambda''}{\theta^3 k}$ fieret nimis magnus, ita, vt λ' enormem adipiſceretur valorem, ſtatuamus inſuper $\eta = 1$, vt fiat $\vartheta = \frac{78}{77}$, hincque elementa noſtra ita ſe habebunt

$$b = -\frac{\alpha}{2};\ \beta = \infty;\ c = -\infty;$$

$$\beta + c = \frac{\alpha}{2};\ \gamma = \frac{39}{77}\cdot\alpha;\ d = \frac{35\cdot\alpha}{15\cdot 77};$$

tum vero aequatio reſoluenda ita eſt comparata;

$$0 = \lambda - \frac{\lambda'}{2} + \frac{77^3\cdot\lambda''}{78^3\cdot 2} + \frac{77^3\cdot\lambda'''}{78^3\cdot 50}$$

hincque

$$\lambda' = 2\lambda + \frac{77^3\cdot\lambda''}{78^3} + \frac{77^3\cdot\lambda'''}{78^3\cdot 25}$$

Iam vt tam prima lens, quam vltima maximam admittat aperturam, ponamus $\lambda = \lambda''' = 1,6299$, dum ſcilicet omnes lentes ex vitro communi $n = 1,55$ factae aſſumuntur; at λ'' ſit $= 1$. quibus poſitis colligemus

$$\lambda' = 3,2593 + 0,9620 + 0,0627$$

$$\lambda' = 4,2841;\ \text{hinc ergo}$$

$$\lambda' - 1 = 3,2841,\ \text{et}$$

$$\tau.\sqrt{(\lambda' - 1)} = 1,64023;$$

quare conſtructio ſingularum lentium ita ſe habebit:

I. Pro prima lente

quae cum ſit aeque vtrinque conuexa eiusque diſtantia

tia focalis $=\alpha$, erit radius vtriusque faciei $=1,10.\alpha$ tum vero eius ſemidiameter aperturae $x=my=1$ dig. ob $m=50$ et $y=\frac{1}{50}$ dig. et interuallum ab hac lente ad ſecundam $=\frac{1}{2}.\alpha$.

II. Pro ſecunda lente

ob $\beta=\infty$, erit

$$F=\frac{b}{\varrho\pm 1{,}64023}=\frac{b}{1{,}8309}$$

$$G=\frac{b}{\sigma+1{,}64023}=\frac{b}{-0{,}0128}$$

hinc $F=-0,2730.\alpha$

$G=+39,0625.\alpha$

tum vero ſemidiameter eius aperturae $=\frac{1}{2}$ dig. ex §. 23. et interuallum ad lentem ſequentem $=\frac{1}{2}.\alpha$.

III. Pro tertia lente

ob $c=\infty$ et $\lambda''=1$. erit

$$F=\frac{\gamma}{\sigma}=0,31123.\alpha$$

$$G=\frac{\gamma}{\varrho}=2,6559.\alpha$$

tum vero aperturae ſemidiameter $=\frac{1}{2}$ dig. et interuallum ad ſequentem lentem $=\frac{78,13}{77,23}\,\alpha$ ſeu $=\frac{1}{1}\,\alpha$ proxime.

IV. Pro quarta lente

radius vtriusque faciei $=1,10.d$, exiſtente $d=\frac{39}{85,77}.\alpha$, cuius pars quarta dat ſemidiametrum aperturae, hinc denique interuallum vsque ad oculum erit $=\frac{3,51}{30,154}.\alpha$

$=\frac{1}{50}\,\alpha$

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$= \frac{1}{35}\,\alpha$ proxime. Quod ad diſtantiam α attinet, ſi ad ſolam primam lentem reſpiceremus, quia ea aperturam admittit, cuius ſemidiameter $= \frac{1}{4}\,\alpha$, ſumi poſſet $\alpha = 4$ dig. ſed ad ſecundam lentem reſpiciendo, cuius minor radius eſt circiter $\frac{1}{4}\,\alpha$, huius pars quarta $\frac{1}{16}\,\alpha$ ſemidiametro aperturae $\frac{1}{2}$ dig. aequalis poſita, dabit $\alpha = 8$ dig., quam menſuram etiam retinere oportet; vnde longitudo teleſcopii excederet 12 dig. Huius rei cauſſa eſt, quod primam lentem vtrinque aeque conuexam aſſumſimus. Adiungamus igitur aliam inſuper ſolutionem ſumendo $\lambda = 1$; vnde fit

$$\lambda' = 2 + 0,9620 + 0,0627$$

$$\lambda' = 3,0247;\ \lambda' - 1 = 2,0247$$

$$\text{et}\ \tau\sqrt{(\lambda' - 1)} = 1,2878;$$

vnde haec ſequitur lentium conſtructio.

I. Pro prima lente

$$F = \frac{\alpha}{\sigma} = 0,6145.\,\alpha$$

$$G = \frac{\alpha}{\varrho} = 5,2439.\,\alpha$$

II. Pro ſecunda lente

$$F = \frac{b}{0.1507 \pm 1.2878} = \frac{[illegible]}{1.4785}$$

$$G = \frac{b}{1.6274 - 1.2878} = \frac{-0,5[illegible]}{0.3396}$$

$$F = -0,3382\,\alpha;\ G = -1,4723.\,\alpha$$

Reliqua manent, vt ante. Hic igitur ſtatim patet, ſecundam lentem debitam aperturam $\frac{1}{2}x$ recipere poſſe, ſi prima patiatur aperturam x. Primae autem radius minor, cum ſit circiter $\frac{6\alpha}{10}$, eius pars quarta $\frac{3}{20}\alpha$ ipſi $x = 1$ dig. aequata dat $\alpha = \frac{20}{3}$ dig. $= 6\frac{2}{3}$ dig. quin etiam tertia lens poſtulat, vt ſit $\frac{3\alpha}{40} = \frac{1}{2}$ dig. vnde α iterum $6\frac{2}{3}$ dig. ſicque tota teleſcopii longitudo vix ſuperabit 10 digit.

Quocirca notari merebitur ſequens

Conſtructio Teleſcopii quinquagies multiplicantis, lentibus ex vitro communi paratis.

I. Pro prima lente

radius faciei { anter. $= 4, 10$ dig.
poſter. $= 34, 96$ dig.

Aperturae ſemidiameter $= 1$ dig.

Interuallum ad ſecundam lentem $= 3\frac{1}{3}$ dig.

II. Pro ſecunda lente

radius faciei { anter. $= -2, 25$. dig.
poſter. $= -9, 82$. dig.

Semidiameter aperturae $= \frac{1}{2}$ dig.

Interuallum ad tertiam lentem $= 3\frac{1}{3}$ dig.

III. Pro tertia lente

radius faciei { anter. $= 2, 08$ dig.
poſter. $= 17, 71$. dig.

Aper-

Aperturae ſemidiameter $= \frac{1}{3}$ dig.

Interuallum ad lentem ocularem $= 3\frac{1}{3}$ dig.

IV. Pro quarta lente

radius vtriusque faciei $= 0,15$ dig.

Semidiameter aperturae $= \frac{3}{80}$ dig.

et diſtantia oculi $= \frac{2}{15}$ dig. proxime

vnde tota longitudo $= 10\frac{2}{15}$ dig.

Campi vero viſi ſemidiameter, vt hactenus,

$= \frac{859}{51}$ min. $=$ 16 min. 51 ſec.

Scholion.

252. Maiores multiplicationes calculo hic non ſubjicio, quia ab huiusmodi teleſcopiis etiam maior campus, quam vulgo, exſpectari ſolet. Quamobrem noſtram inueſtigationem ad campum apparentem augendum proſequamur, idque retentis commodis, quae ternae lentes priores nobis ſunt largitae. Hinc poſſemus valoribus hic aſſumtis vti, ſcilicet $k = 2$, $\eta = 1$ et $\vartheta = 1$ ſed quia hoc modo duo priora interualla ſatis fiunt magna, ſcilicet $\frac{1}{2}\alpha$; quo pacto tota longitudo non parum augetur, praeſtare videtur haec duo interualla multo minora efficere, ita, vt tantum non euaneſcant; neque lentes ſe immediate contingere debeant. Hunc in finem pro k numerus vnitatem vix ſuperans aſſumi debebit, vnde ſimul hoc lucrum nanciſci-

ciſcimur, vt pro λ' numerus binarium vix ſuperans reperiatur. Statuamus igitur $k = 1 + \omega$, denotante ω fractionem minimam, ita, vt ſit

$$b = -\frac{\alpha}{1+\omega} = -(1-\omega).\,\alpha;$$

$$\alpha + b = \omega\,\alpha;$$

ob eandemque rationem ſtatuatur etiam $\eta = \omega$, vt ſecundum interuallum etiam fiat $\omega\,\alpha$. Quod deinde ad litteram ϑ attinet, quae hic ex margine colorato eſt definita, factis his hypotheſibus multo minor vnitate eſſet proditura, ſcilicet $\vartheta = 2\,\omega$, qui valor maximis incommodis foret obnoxius; primo enim elementa γ et $\ddot{d}$ euaneſcerent, niſi α in immenſum augeretur; deinde etiam valor ipſius λ' fieret enormis. Sed probe hic notandum eſt, has hypotheſes non caſui hic tractato, vbi vnica lens ocularis admittitur, deſtinari, ſed propoſitum nobis eſſe iis vti in ſequentibus, vbi duae pluresue lentes oculares conſiderabuntur, quibus cum nouae litterae in calculum introducantur, non amplius opus erit, ex aequatione marginem coloratum tollente, hanc litteram ϑ definire, ſed eam poterimus vt arbitrariam contemplari; ita, vt iam nihil obſtet, quominus ponatur $\vartheta = 1$. Quod autem hunc valorem elegerim, duae ſunt cauſſae; altera eſt, quod cum diſtantia γ hic ſit $\vartheta\,\alpha$, ſi ϑ vltra vnitatem augeretur, longitudo teleſcopii maior eſſet proditura; altera autem ſuadet, ne ϑ minus vnitate capiatur, quia tum

λ'mox

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λ' mox enormem valorem eſſet obtenturum ſit igitur ratum ſtatuere

$$1^\circ.\ k = 1 + \omega;\ 2^\circ.\ \eta = \omega;\ 3^\circ.\ \vartheta = 1.$$

vnde quotcunque lentes adhibeantur, pro tribus primoribus ſemper erit

$$b = -(1-\omega)\alpha;\ \beta = \infty;\ c = -\infty$$

$$\beta + c = \omega . \alpha;\ \gamma = \alpha.$$

Deinde pro litteris π et π' erit quoque ſemper

$$\frac{\pi}{\Phi} = -\omega;\ \frac{\pi'}{\Phi} = -2\omega;$$

ceterum notetur, eſſe $B = \infty$, $\mathfrak{B} = 1$, $C = \mathfrak{C} = 0$, et $BC = 1$. atque hinc aequatio pro margine tollendo ſemper his duobus terminis exordietur $+\omega.-2\omega$; ita, vt hi duo termini ſemper coaleſcant in $-\omega$. Denique etiam aequatio pro confuſione tollenda ſemper incipiet ab his tribus terminis

$$0 = \lambda - \frac{\lambda'}{1+\omega} + \frac{\lambda''}{1+\omega} \dots\dots$$

vnde facile erit calculum pro quotuis lentibus ocularibus proſequi, vbi potiſſimum nobis erit propoſitum, campum apparentem multiplicare, idque quousque libuerit.

Problema 6.

253. Tribus lentibus prioribus ita ante imaginem realem diſpoſitis, vti §. praec. eſt indicatum, ſi poſt imaginem duae lentes conſtituantur, efficere, vt campus apparens euadat maximus.

 Solutio

Solutio.

Cum hic habeantur quinque lentes, formula pro multiplicatione erit $m = -\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}\cdot\frac{\delta}{e}$ quarum fractionum iſta $\frac{\gamma}{d}$ erit poſitiua, reliquae negatiuae. Cum igitur ſit $\frac{\alpha}{b} = -1 - \omega$, $\frac{\beta}{c} = -1$. ſtatuatur $\frac{\gamma}{d} = i$ et $\frac{\delta}{e} = -l$ habebimusque ſequentia elementa:

$$b = \frac{-\alpha}{1+\omega};\ \beta = \infty;\ c = -\infty;\ \gamma = \alpha.$$

$$d = \frac{\alpha}{i};\ \delta = +\frac{D\alpha}{i};\ e = -\frac{D\alpha}{il}$$

exiſtente $m = (1+\omega)\, i.\, l.$

Deinde diſtantiae focales

$$p = \alpha;\ q = b;\ r = \gamma;\ s = \mathfrak{D}\, d \text{ et } t = e.$$

Porro interualla lentium

$$\alpha + b = \omega\alpha;\ \beta + c = \omega\alpha;$$

$$\gamma + d = \frac{1+i}{i}\alpha;\ \delta + e = D\left(\frac{l-1}{il}\right).\ \alpha,$$

quorum trium priora cum per ſe ſint poſitiua, tantum ſupereſt, vt ſit $D(l-1)$ poſitiuum.

Pro fractionibus π, π' etc. iam habemus:

$$\frac{\pi}{\Phi} = -\omega;\ \frac{\pi'}{\Phi} = -2\omega; \text{ ideoque } \frac{\pi-\pi'}{\Phi} = \omega.$$

Pro binis reliquis vero habentur hae aequationes:

$$\frac{\mathfrak{D}\pi'' - \pi' + \pi - \mathfrak{P}}{\Phi} = \frac{\alpha}{d} = i$$

$$\frac{\pi''' - \pi'' + \pi' - \pi + \mathfrak{P}}{\Phi} = \frac{D\alpha}{e} = -il = -m$$

Ex

Ex quibus elicitur

$$\frac{\pi''}{\Phi} = \frac{1+i-\omega}{\mathfrak{D}} = \frac{1+i}{\mathfrak{D}}$$

$$\frac{\pi'''}{\Phi} = \frac{1+i}{\mathfrak{D}} + \omega - 1 - m.$$

Vnde pro loco oculi ſtatim habemus

$$O = -\frac{\pi'''}{m\Phi}\cdot t = \left(\frac{1+i}{\mathfrak{D}} + \omega - 1 - m\right)\frac{D\alpha}{m^2}$$

$$O = \left(\frac{1+i}{\mathfrak{D}} - 1 - m\right)\frac{D\alpha}{m^2}$$

quae vt fiat poſitiua, neceſſe eſt, vt D ſit negativum, adeoque ob $D(l-1) > 0$ erit quoque $l < 1$. Quare cum diſtantia O facta ſit poſitiua pro margine colorato tollendo habebitur haec aequatio:

$$0 = -\omega + \frac{\pi''}{\Phi}\cdot\frac{1}{i} - \frac{\pi'''}{\Phi}\cdot\frac{1}{m} \text{ ſeu}$$

$$0 = -\omega + \frac{1+i}{\mathfrak{D}i} - \frac{(1+i)}{\mathfrak{D}m} + \frac{1+m}{m}$$

ſeu reiectis ω

$$0 = \frac{1+i}{\mathfrak{D}}\cdot\frac{l-1}{il} + \frac{1+m}{m} \text{ ſeu}$$

$$0 = \frac{(1+i)(l-1)}{\mathfrak{D}} + 1 + m \text{ hinc}$$

$$\mathfrak{D} = \frac{-(1+i)(l-1)}{m+1} = \frac{(1+i)(1-l)}{m+1} \text{ et } D = \frac{(1+i)(1-l)}{2m-i+1},$$

qui valor debet eſſe negativus ob $l < 1$, hincque eſſe oporteret $l < \frac{1}{2l+1}$ ſiue $l < \frac{1}{2}$ ita, vt hinc eſſe debeat $i > 2m$, et $D = \frac{-(1+i)(1-l)}{i(1-2l)-l}$.

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His circa valores D et l definitis examinemus campum apparentem, cuius ſemidiameter Φ duplici modo exprimitur

$$1^\circ.\ \Phi = \frac{D\pi''}{1+i} = \frac{(1-l)\pi''}{m+1};$$

$$2^\circ.\ \Phi = \frac{D\pi'''}{(1+i)D-(m+1)},\ \Phi = \frac{(1-l)\pi'''}{(m+1)l}$$

quorum minor tantum locum habet, ſiquidem π'' et π''' maximum valorem, qui eſt circiter $\frac{1}{4}$, obtineant. Cum autem ſit $\pi'' : \pi''' = 1 : l$, tantum ſumi poterit $\pi'' = \frac{1}{4}$ fietque $\pi''' = \frac{l}{4}$ hincque campus prodiret $\Phi = \frac{1}{4} \cdot \frac{1-l}{m+1}$; ideoque minor, quam ſi lente oculari ſimplici vteremur, contra noſtrum inſtitutum; ita, vt hoc problema pro noſtro ſcopo reſolui nequeat.

Idem Problema praecedens.

254. Vbi ceteris manentibus omnibus, tantum quarta lens ante imaginem realem collocatur.

Solutio.

In ſolutione ergo etiam omnia manebunt, vt ante, niſi quod binarum quantitatum i et l ſigna ſint mutanda. Primo ergo erunt elementa

$$b = -\frac{\alpha}{1+\omega};\ \beta = \infty;\ c = -\infty;\ \gamma = \alpha$$

$$d = -\frac{\alpha}{i};\ \delta = -\frac{D\alpha}{i};\ e = \frac{-D\alpha}{il}.$$

Diſtantiae focales.

$$p = \alpha;\ q = -\alpha;\ r = \gamma = \alpha;$$

$$s = -\frac{D\alpha}{i};\ t = -\frac{D\alpha}{il}.$$

Len-

Lentium vero interualla

$$\alpha + b = \omega\,\alpha;\ \beta + c = \omega\,\alpha$$

$$\gamma + d = + \frac{(i-1)}{i}\,\alpha;$$

$$\delta + e = \frac{-(l+1)}{il}.\,D\,\alpha$$

vnde patet, eſſe debere D negatiuum, at $i > 1$; tum vero notetur eſſe $m = i\,l$.

Deinde inueniemus

$$\frac{\pi}{\Phi} = -\omega;\ \frac{\pi'}{\Phi} = -2\,\omega$$

$$\frac{\pi''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}};\ \frac{\pi'''}{\Phi} = -\frac{(i-1)}{\mathfrak{D}} - 1 - m$$

hincque pro loco oculi

$$O = \left(-\frac{(i-1)}{\mathfrak{D}} - 1 - m\right)\frac{D\,\alpha}{m\,m}$$

qui ergo valor eſt poſitiuus ob $D < 0$. Quare vt margo coloratus euaneſcat debet eſſe

$$\mathfrak{D} = \frac{-(i-1)(1+l)}{m+1}$$

$$D = \frac{-(i-1)(1+l)}{2m+i-l}$$

qui valor cum ſit negatiuus, conditionibus praecedentibus ſatisfit, ſi modo ſit $i > 1$. atque his valoribus ſubſtitutis erit

$$b = -\alpha;\ \beta = \infty;\ c = -\infty;\ \gamma = \alpha;\ d = -\frac{\alpha}{i}$$

$$\delta = \frac{(i-1)(1+l).\alpha}{(2m+i-l)i};\ e = \frac{(i-1)(1+l).\alpha}{(2m+i-l)il}$$

$$p = \alpha;\ q = -\alpha;\ r = \alpha;$$

$$s = \frac{(l-1)(1+l)\alpha}{(m+1)i};\ t = \frac{(l-1)(1+l)\alpha}{(2m+i-l)il}$$

$$a + b = \omega\alpha;\ \beta + c = \omega\alpha;\ \gamma + d = \frac{l-1}{i}\alpha$$

$$\delta + e = \frac{(l-1)(1+l)^2\alpha}{(2m+i-l)il}$$

$$\frac{\pi}{\Phi} = -\omega;\ \frac{\pi'}{\Phi} = -2\omega$$

$$\frac{\pi''}{\Phi} = +\frac{m+1}{l+1}$$

$$\frac{\pi'''}{\Phi} = +\frac{m+1}{l+1} - 1 - m = -\frac{l(m+1)}{1+l} \text{ hincque}$$

$$O = \frac{l(l-1)(m+1)}{2m+1-l}\cdot\frac{\alpha}{m^2}$$

Cum igitur ſit $\pi'' : \pi''' = 1 : -l$ pro campo duo caſus ſunt perpendendi.

I. Si $l > 1$. tum poterit capi $\pi''' = -\frac{1}{4}$ vt fiat $\pi'' = \frac{1}{4l} < \frac{1}{4}$ hincque ſemidiameter campi

$$\Phi = \frac{1}{4}\frac{(1+l)}{m+1}$$

II. Si $l < 1$, capi poterit $\pi'' = \frac{1}{4}$ vt fiat $\pi''' = -\frac{l}{4} < -\frac{1}{4}$ hincque $\Phi = \frac{1}{4}\cdot\frac{1+l}{m+1}$.

Vtroque ergo caſu campus maior erit, quam ſi vnica adeſſet lens ocularis, quo caſu inuenimus

$$\Phi = \frac{1}{4}\cdot\frac{1}{m+1}.$$

Maximus igitur campus obtinebitur ſi capiatur $l = 1$. quo caſu ob $il = m$ fit $i = m$ tum vero

$$\Phi = \frac{1}{2(m+1)} = \frac{1718}{m+1}\ \text{min.}$$

qui

qui eſt duplo maior. Conueniet igitur ſumi $l = 1$, ſi modo reſolutio poſtremae aequationis id permittat, quae eſt

$$0 = \lambda - \frac{\lambda'}{1+\omega} + \frac{\lambda''}{1+\omega} - \mathfrak{D}i\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu}{D}\right) - \frac{\lambda''''}{D i m}$$

vbi ſi capiatur $l = 1$, vt ſit $i = m$ ſit

$$\mathfrak{D} = -2.\frac{m-1}{m+1};\ D = -2.\frac{m-1}{3m-1}$$

quare ſi m ſit numerus praemagnus, erit $\mathfrak{D} = -2$; $D = -\frac{2}{3}$; ex quo manifeſtum eſt reſolutionem illius aequationis hoc modo non ſolum non impediri, ſed et adiuvari, ita, vt haec poſitio $l = 1$ noſtro ſcopo maxime conueniat. Hinc ergo conſequimur

$$\lambda' = (1+\omega)\lambda + \lambda'' - \frac{\lambda'''}{\mathfrak{D} i m} - \frac{\lambda''''}{D i m} - \frac{\nu}{\mathfrak{D} D m}$$

ad quam reſoluendam primo notetur, quia duae poſtremae lentes maximam requirunt aperturam, eas vtrinque aeque conuexas capi debere; vnde pro vltima lente ſumi debebit $\lambda'''' = 1,6299$; pro penultima vero habetur

$$\sqrt{(\lambda''' - 1)} = \frac{\sigma-\rho}{2\tau}.\frac{\delta-1}{\delta+1}$$

$$= \frac{\sigma-\rho}{2\tau}.\frac{-5m+3}{m+1}$$

Cum nunc ſit $\left(\frac{\sigma-\rho}{2\tau}\right)^2 = 0,6299$

erit $\lambda''' = 1 + 0.6299.\left(\frac{-5m+3}{m+1}\right)^2$

deinde vero ſumamus $\lambda = 1$ et $\lambda'' = 1$. pro ω autem commode ſumi poſſe videtur $\omega = \frac{2}{m}$, quoniam hoc

hoc modo interualla lentium priorum non fiunt nimis parva, quam vt in praxi locum habere queant.

Coroll. I.

255. Quodſi ergo ſtatuamus $l=1$, vt ſit $i=m$, tum vero $\omega=\frac{1}{m}$, noſtra elementa ita ſe habebunt:

$$b=-\frac{ma}{m+1};\ \beta=\infty;\ c=\infty$$

$$\gamma=a;\ d=-\frac{a}{m};\ \delta=\frac{2(m-1)a}{m(3m-1)};\ e=\frac{2(m-1)a}{m(3m-1)};$$

ita, vt ſit $\delta=e$ et imago realis inter binas lentes poſtremas media interiaceat.

Diſtantiae autem focales erunt

$$p=a;\ q=-\frac{ma}{m+1};\ r=a$$

$$s=\frac{2(m-1)a}{m(m+1)};\ t=\frac{2(m-1)a}{m(3m-1)}$$

Interualla vero lentium

$$a+b=\frac{a}{m};\ \beta+c=\frac{a}{m};\ \gamma+d=\frac{m-1}{m}a$$

$$\delta+e=\frac{4(m-1)a}{m(3m-1)}$$

$$\text{et } O=\frac{mm-1}{3m-1}\cdot\frac{a}{mm}.$$

Coroll. 2.

256. Adiecta igitur vnica lente hoc inſigne commodum feliciter ſumus adepti quod amplitudo campi duplo maior ſit facta, quam ſi vnica lente oculari vteremur, vbi probe notandum eſt, quod haec noua lens adiecta non poſt imaginem realem, ſed ante eam debeat collocari. Scho-

Scholion I.

257. Quo haec, quae inuenimus, commodiſſime ad praxin accommodemus, methodo iam ſupra tradita vtamur ac primo conſtructionem teleſcopii pro multiplicatione quapiam modica veluti $m = 25$ inueſtigemus; deinde vero pro $m = \infty$; ex quorum caſuum comparatione non difficulter pro qualibet multiplicatione media conſtructionem colligere licebit.

Exempl. I.

Pro $m = 25$.

258. Conſtructionem teleſcopii exhibere:

Cum hic ſit $m = 25$, erit

$$\mathfrak{D} = -\tfrac{24}{13} = -1,84615$$

$$D = -\tfrac{24}{37} = -0,64865$$

hinc erit $\frac{1-D}{1+D} = \frac{1,64865}{0,35134}$

hinc Log. $(\frac{1-D}{1+D})^2 = 1,3428018$

vnde colligitur $\lambda''' = 14,8699$.

Iam cum ſit

$$\text{Log.} -\mathfrak{D} = 0.2662669$$

$$\text{Log.} - D = 9.8120104$$

reperiemus

$$\lambda' = 1,04 + 1 + 0,094529 + 0,23888 - 0,00777$$

$\lambda' = 2,3656$

$\lambda' - 1 = 1,3656$ et

$\tau . \sqrt{(\lambda' - 1)} = 1,0577$

Conſtructio igitur lentium ita ſe habebit:

I. Pro prima lente.

radius faciei $\begin{cases} \text{anter.} = 0,6145 . \alpha \\ \text{poſter.} = 5,2439 . \alpha \end{cases}$

Semidiameter aperturae $= \frac{25}{50}$ dig. $= \frac{1}{2}$ dig.

Interuall. ad lentem ſequentem $= \frac{1}{25} . \alpha = 0,04 \alpha$.

II. Pro ſecunda lente

calculus ita ſe habebit

$F = \frac{b}{\varrho \pm 1,0577} = \frac{-0,96 \alpha}{1,2484}$

$G = \frac{b}{\sigma \mp 1,0577} = \frac{-0,96 . \alpha}{0,5697}$

ſeu $F = -0,7690 . \alpha$

$G = -1,6851 . \alpha$

Interuallum ad ſequentem, vt ante, $= 0,04 \alpha$

III. Pro tertia lente

cum eius diſtantia focalis ſit reuera

$\gamma = \frac{\alpha}{1 + \omega} = (1 - \omega) \alpha$ et $\lambda'' = 1$,

ex

ex prima lente haec ita definitur

radius faciei $\begin{cases} \text{anter.} = 0,5900.\,a \\ \text{poſter.} = 5,0342.\,a \end{cases}$

Interuallum ad quartam $= a - \frac{2a}{m} = 0,92.\,a$

IV. Pro quarta lente

cuius diſtantia focalis eſt $1,84615.\,\frac{a}{m}$

quia debet eſſe vtrinque aeque conuexa,

erit vtriusque faciei radius $= 2,03076.\,\frac{a}{m}$

Semidiameter aperturae $= 0,50769.\,\frac{a}{m}$

Interuallum ad ſequentem $= +1,29730\,\frac{a}{m}.$

V. Pro quinta lente

cuius diſtantia focalis $= 0,64865.\,\frac{a}{m}$

erit radius vtriusque faciei $= 0,71351.\,\frac{a}{m}$

Semidiameter aperturae $= 0,17838.\,\frac{a}{m}$

hinc interuallum ad oculum vsque erit $= 0,3372.\,\frac{a}{m}$, exiſtente $m = 25$ et campi apparentis ſemidiameter erit $\Phi = \frac{1718}{26}$. minut. $= 66$ min. et longitudo totius inſtrumenti

$$= a + 1,6345.\,\frac{a}{m} = 1,06538.\,a.$$

 Exem-

Exempl. II.

Si $m = \infty$.

259. Conſtructionem teleſcopii deſcribere.

Erit igitur $\mathfrak{D} = -2$; $D = -\frac{2}{3}$

et $\omega = 0$, $\frac{1-D}{1+D} = 5$; vnde fit

$$\lambda''' = 1 + 0,6299 \cdot 25 = 16,74.$$

Hincque colligitur

$$\lambda' = 1 + 1 = 2;\ \lambda' - 1 = 1.\ \text{ideoque}$$

$$\tau . \surd(\lambda' - 1) = \tau = 0.9051.$$

Conſtructio igitur lentium ita ſe habebit

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = 0,6145 \cdot \alpha \\ \text{poſter.} = 5,2439 \cdot \alpha \end{cases}$

Semidiameter aperturae $= \frac{m}{50}$ dig.
Interuallum ad lentem ſequentem $= \frac{\alpha}{m}$.

II. Pro ſecunda lente

cuius diſtantia focalis $b = -\alpha$ habebimus

$$F = \frac{b}{\varrho \pm 0,9051} = \frac{-\alpha}{1,0958}$$
$$G = \frac{b}{\sigma + 0,9051} = \frac{-\alpha}{0,7225}$$

Hinc $F = -0,91257 \cdot \alpha$
$G = -1,38446 \cdot \alpha$
Interuallum ad ſequentem $= \frac{\alpha}{m}$.

III. Pro

III. Pro tertia lente

radius faciei $\begin{cases} \text{anter.} = 0,6145 \cdot a \\ \text{poſter.} = 5,2439 \cdot a \end{cases}$

Interuallum ad ſequentem lentem $= a - \frac{2a}{m}$.

IV. Pro quarta lente

cuius diſtantia focalis eſt $\mathfrak{D}\, d = + \frac{2a}{m}$,

erit radius vtriusque faciei $= 2,2 \frac{a}{m}$

Interuallum $= \frac{4}{3} \cdot \frac{a}{m} = 1,333 \cdot \frac{a}{m}$.

V. Pro quinta lente

cuius diſtantia focalis $= 0,666 \cdot \frac{a}{m}$

erit radius vtriusque faciei $= 0,7333 \cdot \frac{a}{m}$

Interu...

Exempl. III.

260. Pro multiplicatione quacunque m conſtructionem huiusmodi teleſcopii deſcribere.

Hic aſſumſi omnes lentes ex ea vitri ſpecie parari, pro qua eſt $n = 1,55$. Ex praecedentibus autem ſequens conſtructio concinnabitur

I. Pro prima lente

erit, vt hactenus,

radius faciei $\begin{cases} \text{anter.} = 0,6145 \cdot a \\ \text{poſter.} = 5,2439 \cdot a \end{cases}$

 Semi-

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Semidiameter aperturae $x = \frac{m}{50}$ dig.

Interuallum ad ſequentem $= \frac{\alpha}{m}$.

II. Pro ſecunda lente

ponatur

$$F = -(0,91257 + \tfrac{f}{m})\,\alpha$$

$$G = -(1,38446 + \tfrac{g}{m})\,\alpha$$

erit autem

$$0,91257 + \tfrac{f}{25} = 0,7690$$

$$1,38446 + \tfrac{g}{25} = 1,6851$$

$g = +$[illegible]$,59$

Interuallum $= \frac{\alpha}{m}$.

III. Pro tertia lente

radius faciei $\begin{cases} \text{anter.} = 0,6145\,(1 - \tfrac{1}{m})\,\alpha \\ \text{poſter.} = 5,2439\,(1 - \tfrac{1}{m})\,\alpha \end{cases}$

Interuallum ad ſequentem $= \alpha - \frac{2\alpha}{m}$.

IV. Pro quarta lente

ſtatuatur radius vtriusque faciei

$$= (2,2 + \tfrac{b}{m})\,\tfrac{\alpha}{m}$$

eritque $2,2 + \frac{b}{25} = 2,03076$

hinc

hinc colligitur $b = -4,231$.

Interuallum ad ſequentem $= (1,333 + \frac{k}{m})\frac{\alpha}{m}$

hinc $k = -0,90$; adeoque interuallum erit

$$(1,333 - \frac{0,90}{m})\frac{\alpha}{m}.$$

V. Pro quinta lente

cuius diſtantia focalis $= (0,666 - \frac{0,465}{m})\frac{\alpha}{m}$

erit radius vtriusque faciei $= (0,7333 - \frac{0,465}{m})\frac{\alpha}{m}$

Diſtantia ad oculum $= (\frac{1}{3} + \frac{l}{m})\frac{\alpha}{m}$

vnde $l = 0,097$ adeoque haec diſtantia erit

$$(0,333 + \frac{0,097}{m})\frac{\alpha}{m}$$

et tota teleſcopii longitudo $=$

$$\alpha + (1,666 - \frac{0,803}{m})\frac{\alpha}{m}$$

vel $\alpha + 1,666.\frac{\alpha}{m} - 0,803.\frac{\alpha}{m^2}$

Perpendamus nunc quantum valorem ipſi α tribui conueniat et cum ternae priores lentes vt lens triplicata ſpectari queant, minimus radius eſt $0,6145.\alpha$, cuius pars quarta $\frac{3}{25}.\alpha$ ipſi $x = \frac{m}{50}$ aequalis poſita dat $\alpha = \frac{2m}{15}$. dig. $= \frac{4m}{35}$ dig. Ponamus igitur $\alpha = \frac{4}{30}m$ dig. et habebitur ſequens

Conſtructio huiusmodi Teleſcopiorum.

Circa diaphragma his teleſcopiis inſerendum videatur ſequens Scholion 3.

Pro

Pro multiplicatione quacunque m, lentibus ex vitro $n = 1,55$, factis

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = 0,08193.\,m.\ \text{dig.} \\ \text{poſter.} = 0,69918.\,m.\ \text{dig.} \end{cases}$

Semidiameter aperturae $= \frac{m}{50}$ dig.

Interuallum $= \frac{2}{15}$ dig.

II. Pro ſecunda lente

rad. fac. $\begin{cases} \text{anter.} = (-0,1217.m + 0,478)\ \text{dig.} \\ \text{poſter.} = (-0,18459.m - 1,003)\ \text{dig.} \end{cases}$

Interuallum $= \frac{2}{15}$ dig.

III. Pro tertia lente

rad. fac. $\begin{cases} \text{anter.} = (0,08193\,m - 0,0819)\ \text{dig.} \\ \text{poſter.} = (0,69918\,m - 0,699)\ \text{dig.} \end{cases}$

Interuallum $= (\frac{2}{15}\,m - \frac{4}{15})$ dig.

IV. Pro quarta lente

radius vtriusque faciei $= (0,2933 - \frac{0,5641}{m})$ dig.

Interuallum $= (0,1777 - \frac{0,12}{m})$ dig.

V. Pro quinta lente

radius vtriusque faciei $= (0,098 - \frac{0,066}{m})$ dig.

Hinc interuallum ad oculum $= (0,044 - \frac{0,012}{m})$ dig.

Longitudo tota $= (\frac{2}{15}\,m - 0,11)$ dig.

ita,

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ita, vt pro caſu $m = 100$ haec longitudo ſit $13\frac{1}{5}$ dig. campi denique apparentis ſemidiameter $= \frac{1718}{m+1}$ min. ſeu, quia etiam lentes priores aliquantillum ad campum augendum conferunt, $\Phi = \frac{1718}{m}$ min. ita, vt pro $m = 100$ fiat $\Phi = 17$ min. 10 ſec.

Scholion 2.

261. Teleſcopia haec in ſuo genere ita omnibus numeris abſoluta videntur, vt perfectiora vix deſiderari queant, niſi diuerſas vitri ſpecies adhibere velimus. Non ſolum enim confuſionis ab apertura oriundae ſunt expertia aeque ac marginis colorati, ſed etiam campum apparentem duplo maiorem pateſaciunt, quam ſimplicia ac praeterea tam ſunt brevia, vt breuiora ne ſperare quidem liceat. Deinde etiam in exſecutione inſigne commodum inde obtineri poteſt, quod inter tres priores lentes interualla aliquantillum variari poſſunt; ſi enim forte eueniat, vt ob tantillum errorem in praxi commiſſum hae lentes non exactiſſime ad interualla hic praeſcripta ſint accommodata, facile euenire poteſt, vt iis paulisper mutatis, egregium effectum ſint praeſtaturae. Interim tamen ſemper conſultum erit, ſecundam lentem concauam pluries elaborari, ſecundum easdem menſuras; cum enim ſemper aliquod diſcrimen deprehendatur, inter plures eiusmodi lentes optima facile eligi poterit. Nihilo minus vero conueniet noſtram inueſtigationem vlterius proſequi et in eiusmodi huius generis tele-

ſcopia inquirere, quorum campus adeo triplo vel quadruplo maior ſit prodituγus.

Scholion 3.

262. Quo haec teleſcopia meliorem effectum praeſtent, neceſſe eſt, vt in loco imaginis verae diaphragma ſiue ſeptum, quemadmodum ſupra iam eſt deſcriptum, cum foramine debitae magnitudinis conſtituatur. Cadit autem haec imago ob $\delta = e$ praeciſe in medium interualli quartae & quintae lentis ideoque ad diſtantiam $= (0,0888 - \frac{0,06}{m})$ dig. Deinde cum foramen magnitudini hujus imaginis debeat eſſe aequale et ſemidiameter imaginis ſit in genere $= \alpha \Phi . B C D = \alpha \Phi . D = -2 . \frac{m-1}{3m-1} . \alpha \Phi$; debet eſſe ſemidiameter foraminis $= -2 . \frac{m-1}{3m-1} \alpha \Phi$. Iam cum in noſtro exemplo euoluto ſit $\alpha = \frac{2}{15} m$ et $\Phi = \frac{1}{3m}$ colligitur iſte foraminis ſemidiameter $= \frac{1}{15} . \frac{2.(m-1)}{3m-1}$ dig. $= (\frac{2}{45} - \frac{1}{135 m} .)$ dig.

Ceterum etſi his teleſcopiis multo maiorem claritatis gradum conciliauimus quam vulgo fieri ſolet, dum ſumſimus $y = \frac{1}{50}$ dig., ex Hugenii regulis autem ſequitur $y = \frac{1}{75}$ dig. tamen ſi quis vereatur, ne hic ob multitudinem lentium claritas notabilem iacturam patiatur, huic incommodo facile medela afferetur, menſuras datas tantum quapiam ſui parte augendo ſeu, quod eodem redit, menſuram vnius digiti, quam hactenus indefinitam reliquimus, pro lubitu augendo.

Pro-

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Problema 7.

263. Si praeter tres lentes priores, vti in praecedente problemate ſunt conſtitutae, adhuc vna lens ante locum imaginis collocetur, poſt eam inſuper duas lentes ita diſponere, vt maximus campus obtineatur.

Solutio.

Cum hic occurrant ſex lentes, erit

$$m = \frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d} \cdot \frac{\delta}{e} \cdot \frac{\varepsilon}{f}.$$

quarum fractionum tres priores ſunt negatiuae, quarta poſitiua & quinta denuo negatiua. Pro prioribus iam ſumſimus eſſe $\frac{\alpha}{b} = -1 - \omega$; $\frac{\beta}{c} = -1$. Pro poſterioribus vero ſtatuamus $\frac{\gamma}{d} = -k$; $\frac{\delta}{e} = i$ et $\frac{\varepsilon}{f} = -l$ vt ſit $m = (1 + \omega) k l i$; vnde ob $B = \infty$, $C = 0$, et $BC = 1$ elementa noſtra erunt

$$b = -\frac{\alpha}{1+\omega};\ \beta = \infty;\ c = -\infty;$$

$$\gamma = \frac{\alpha}{1+\omega};\ d = -\frac{\alpha}{k};\ \delta = -\frac{D\alpha}{k};$$

$$e = -\frac{D\alpha}{ki};\ \varepsilon = -\frac{DE\alpha}{ki};\ f = \frac{DE\alpha}{kil}.$$

Atque hinc diſtantiae focales reperientur:

$$p = \alpha;\ q = -(1-\omega)\alpha;\ r = \frac{\alpha}{1+\omega};$$

$$s = -\frac{\mathfrak{D}\alpha}{k};\ t = -\frac{\mathfrak{E}D\alpha}{ki};\ u = \frac{DE\alpha}{kil}.$$

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Tum vero interualla lentium

$$\alpha + b = \omega\alpha;\ \beta + c = \omega\alpha;\ \gamma + d = \frac{k-1}{k}\cdot\alpha$$

$$\delta + e = -\mathrm{D}\,\alpha.\frac{i+1}{ki};\ \varepsilon + f = \mathrm{D\,E}\,\alpha\frac{1-l}{kil}$$

quae vt prodeant poſitiua, debet eſſe

$$1^\circ.\ k > 1;\ 2^\circ.\ \mathrm{D} < 0;\ 3^\circ.\ + \mathrm{E}(l-1) > 0$$

Litterae π, π' etc. ſequenti modo definientur:

$$\frac{\pi}{\Phi} = -\omega;\ \frac{\pi'}{\Phi} = -2\omega$$

et reliquae ex ſequentibus formulis determinari debent

$$\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{\mathrm{BC}\alpha}{d} = -k$$

$$\frac{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = \frac{\mathrm{BCD}\alpha}{e} = -ki$$

$$\frac{\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi}{\Phi} = m$$

hinc ergo colligimus

$$\frac{\pi''}{\Phi} = \frac{1-k}{\mathfrak{D}};\ \frac{\pi'''}{\Phi} = \frac{\pi''}{\mathfrak{E}\Phi} - \left(\frac{1+ki}{\mathfrak{E}}\right).$$

$$\text{et } \frac{\pi''''}{\Phi} = \frac{\pi'''}{\Phi} - \frac{\pi''}{\Phi} + m + 1$$

Nunc cum pro campo apparente ſit

$$\Phi = \frac{\pi - \pi' + \pi'' - \pi''' + \pi''''}{m+1}$$

is fiet maximus, ſi ſumatur $\pi'' = \frac{1}{4}$, $\pi''' = -\frac{1}{4}$, $\pi'''' = \frac{1}{4}$; inde enim fiet $\Phi = \frac{1}{4}\cdot\frac{1}{m+1}$; vnde illae aequationes dabunt

$$\frac{m+1}{1} = \frac{1-k}{\mathfrak{D}};\ \frac{-m-1}{1} = \frac{m+1}{1\,\mathfrak{E}} - \left(\frac{1+ki}{\mathfrak{E}}\right)$$

et

$$\text{et } \frac{m+1}{3} = \frac{-m-1}{3} - \frac{m-1}{3} + m + 1$$

quae eſt, vti debet, identica.

Vnde pro loco oculi ſequitur

$$O = \frac{m+1}{3} \cdot \frac{u}{m} = \frac{m+1}{3} \cdot \frac{DE\alpha}{kil.m}$$

quae diſtantia vt fiat poſitiua ob $D < 0$ debet etiam eſſe $E < 0$ adeoque $l < 1$, ſiquidem aſſumamus α poſitiuum. Patet autem, ſi caperemus $l = 1$, binas poſtremas lentes ſibi immediate iungi et prodire caſum lentis ocularis duplicatae iam ſupra conſideratum. Videamus autem ante quam aequationem pro margine colorato contemplemur, cuiusmodi valores litterae $\mathfrak{E}$ et $\mathfrak{D}$ ex binis aequationibus ſuperioribus obtineant et ex priori quidem inuenitur $\mathfrak{D} = -\frac{3(k-i)}{m+1}$ qui cum ſit negatiuus, etiam D fit negatiuum, vti oportet; et ex altera fit $\mathfrak{E} = \frac{3ki-m+2}{m+1}$ hincque $E = \frac{3ki-m+2}{2m-3ki-1}$, qui valor cum debeat eſſe negatiuus, duo caſus ſunt conſiderandi

I. Si numerator negatiuus et denominator poſitiuus, erit $3ki + 2 < m$ et $m > \frac{3ki+1}{2}$ ſeu ſimpliciter $m > 3ki + 2$.

II. Sin autem numerator ſit poſitiuus et denominator negatiuus, erit $m < 3ki + 2$ et $m < \frac{3ki+1}{2}$ ſeu ſimpliciter $m < \frac{3ki+1}{2}$.

Cum autem ſit $m = kil$, ob $l < 1$ erit $m < ki$, vnde patet priorem caſum locum habere non poſſe,

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ſed ſolum ſecundum, ita, vt ſit $m < ki$ quo pacto omnes conditiones ſunt adimpletae, ſicque nihil impedit, quominus reſolutionem aequationis pro margine tollendo ſuſcipiamus, quae praeter exſpectationem tam facilis euadet, vt nullae difficultates, quales ante occurrebant, negotium turbent. Haec autem aequatio omiſſis duobus primis terminis vtpote minimis ita ſe habebit:

$$0 = \frac{\pi''}{\Phi} \cdot \frac{d}{p} + \frac{\pi'''}{\Phi} \cdot \frac{e}{Dp} + \frac{\pi''''}{\Phi} \cdot \frac{f}{DEp}$$

quae ob $\pi'' = -\pi''' = +\pi''''$ abit in hanc:

$$0 = \frac{d}{p} - \frac{e}{Dp} + \frac{f}{DEp}$$

cum nunc ſit $p = \alpha$ et $\frac{d}{\alpha} = -k$, $\frac{e}{\alpha} = -\frac{D}{ki}$; $\frac{f}{\alpha} = \frac{DE}{k\alpha l}$

erit noſtra aequatio

$$0 = -\frac{1}{k} + \frac{1}{ki} + \frac{1}{kil}$$

ſeu per k multiplicando

$$0 = -1 + \frac{1}{i} + \frac{1}{il}$$

$$0 = -il + l + 1;$$

hincque $i = \frac{l+1}{l}$, vbi debet eſſe $l < 1$ hinc $il = 1 + l$ et $m = (1 + l)k$; ita vt ſit $k = \frac{m}{1+l}$.

Deinde erit

$$\mathfrak{D} = \frac{-3(k-1)}{m+1};\ \mathfrak{E} = \frac{3ki - m + 2}{m+1} \text{ ideoque}$$

$$D = \frac{-3(k-1)}{3k+m-2};\ E = \frac{3ki - m + 2}{2m - 3ki - 1}.$$

His

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His inuentis aequatio pro confuſione tollenda erit

$$0 = \lambda - \frac{\lambda'}{1+\omega} + \frac{\lambda''}{1+\omega} - \frac{1}{\mathfrak{D}k}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{v}{D}\right)$$
$$- \frac{1}{D^2.\mathfrak{E}ki}\left(\frac{\lambda''''}{\mathfrak{E}^2} + \frac{v}{E}\right) + \frac{\lambda'''''\iota}{D^3E^3.m}$$

quae, vt hactenus eſt factum, facile reſoluitur, quaerendo ſcilicet valorem ipſius λ'.

Scholion.

264. Solutio huius problematis ad ſequentes inueſtigationes expediendas maximum adiumentum nobis affert, dum ea nobis nouam methodum ſuppeditat aequationem pro margine colorato tollendo, quae ſupra inſignibus difficultatibus erat inuoluta, expeditiſſime reſoluendi. Huius methodi autem vis in eo conſiſtit, vt litteris π, π', π'' etc. ſtatim determinatos valores tribuamus qui quidem ita ſint comparati, vt maximum campum apparentem producant. Hoc enim facto iſtae litterae ex memorata aequatione ſtatim tolluntur et loco litterarum d, e, f valores ante inuentos ſubſtituendo etiam litterae maiuſculae ſponte ex calculo euaneſcunt; ita, vt tota aequatio nulla amplius alia elementa inuoluat praeter litteras k, i, l; quarum vna inde ſine vlla difficultate definitur; deinde vero ex illis valoribus pro litteris π, aſſumtis facile determinantur litterae $\mathfrak{D}$, $\mathfrak{E}$ etc. indeque etiam D, E etc. quarum valores in vltimam aequationem tranſlati totum negotium facile conficiunt; quin etiam haec metho-

thodus pro prioribus lentibus in vſum vocari poteſt, vbi autem notandum eſt, quia hae lentes quaſi ad obiectiuam conſtituendam concurrunt, ex earum litteris π et π' nihil vel perparum ad campum amplificandum redundare poſſe. Quocirca his litteris non vt ſequentibus valor $\frac{1}{4}$, ſed potius quam minimus, puta $\frac{1}{4} . \omega$ et $\frac{1}{4} . \omega'$ tribui debet, denotantibus ſcilicet ω et ω' fractiones quam minimas. Quare quo haec noua methodus clarius perſpiciatur, ea ad ſequens problema generale huc ſpectans ſoluendum vtemur.

Problema 8.

265. Teleſcopium huius generis ex ſex lentibus conſtruere, quarum tres priores inſeruiant omni confuſioni tollendae tres autem poſteriores campo triplicando, dum ſcilicet lenti oculari ſimplici campum ſimplicem aſſignamus.

Solutio.

Hic igitur quinque ſequentes fractiones conſiderandae veniunt:

$$\frac{\alpha}{b} . \frac{\beta}{c} . \frac{\gamma}{d} . \frac{\delta}{e} . \frac{\varepsilon}{f}$$

quarum omnes praeter vnicam debent eſſe negatiuae. Quare ſi ſtatuamus:

$$\frac{\alpha}{b} = -P; \ \frac{\beta}{c} = -Q; \ \frac{\gamma}{d} = -R;$$

$$\frac{\delta}{e} = -S \text{ et } \frac{\varepsilon}{f} = -T.$$

evidens

euidens eſt, harum quinque litterarum P, Q, R, S, T vnicam fore negatiuam, reliquis exiſtentibus poſitiuis. Quaenam autem ſit negatiua, hic nondum opus eſt definire. Hoc poſito noſtra elementa aeque ac diſtantiae focales cum interuallis lentium ſequenti modo conſpectui repraeſententur:

Diſtant. determinat.		Diſt. focales	Interualla lentium
		$p = a$	
$b = \frac{-\alpha}{P}$	$\beta = \frac{-B\alpha}{P}$	$q = \mathfrak{B} b$	$\alpha + b = \alpha(1 - \frac{1}{P}) > 0$
$c = \frac{B\alpha}{PQ}$	$\gamma = \frac{BC\alpha}{PQ}$	$r = \mathfrak{C} c$	$\beta + c = \frac{-B\alpha}{P}(1 - \frac{1}{Q}) > 0$
$d = \frac{-BC\alpha}{PQR}$	$\delta = \frac{-BCD\alpha}{PQR}$	$s = \mathfrak{D} d$	$\gamma + d = \frac{BC\alpha}{PQ}(1 - \frac{1}{R}) > 0$
$e = \frac{BCD\alpha}{PQRS}$	$\varepsilon = \frac{BCDE\alpha}{PQRS}$	$t = \mathfrak{E} e$	$\delta + e = \frac{-BCD\alpha}{PQR}(1 - \frac{1}{S}) > 0$
$f = \frac{-BCDE\alpha}{PQRST}$		$u = f$	$\varepsilon + f = \frac{BCDE\alpha}{PQRS}(1 - \frac{1}{T}) > 0$

vbi cum productum $PQRST$ ſit negatiuum, pro multiplicatione erit $m = -PQRST$.

Deinde cum pro campo apparente habeatur

$$\Phi = \frac{\pi - \pi' + \pi'' - \pi''' + \pi''''}{m + 1}$$

ſit ξ maximus valor, quem hae litterae π, π' etc. recipere poſſunt et ſtatuamus $\pi = \omega \xi$; $\pi' = -\omega' \xi$; $\pi'' = \xi$; $\pi''' = -\xi$; $\pi'''' = \xi$; vt ſit $\Phi = \frac{\omega + \omega' + 3}{m + 1} . \xi$

Cum igitur hinc ſit $\frac{\pi''''}{\Phi} = \frac{m + 1}{\omega + \omega' + 3}$ pro diſtantia oculi habebimus

$$O = \frac{\pi'''}{\Phi} \cdot \frac{u}{m} = \frac{m+1}{\omega+\omega'+1} \cdot \frac{-BCDE\alpha}{PQRS1 \cdot m}$$

$$\text{ſeu } O = \frac{m+1}{\omega+\omega'+1} \cdot \frac{BCDE \cdot \alpha}{mm}.$$

Vt igitur O fiat poſitiuum, quia $\frac{\pi'''}{\Phi}$ eſt poſitiuum, debet eſſe $u > 0$ ideoque vltima lens conuexa, quae conditio inſuper eſt probe obſeruanda.

Nunc igitur aequatio pro margine colorato tollendo ita ſe habebit:

$$0 = \omega \cdot \frac{b}{\alpha} - \omega' \cdot \frac{c}{B\alpha} + \frac{d}{BC\alpha} - \frac{e}{BCD\alpha} + \frac{f}{BCDE\alpha}$$

quae reducitur ad hanc formam:

$$0 = +\omega \cdot \frac{1}{P} + \omega' \cdot \frac{1}{PQ} + \frac{1}{PQR} + \frac{1}{PQRS} + \frac{1}{PQRST}$$

in qua duos priores terminos ob paruitatem negligere licet, ita, vt adhuc ſit

$$0 = 1 + \frac{1}{S} + \frac{1}{ST}$$

quae aequatio facile reſoluitur, dummodo litterarum S et T altera ſit negatiua; vnde patet, tres priores litteras P, Q, R neceſſario eſſe poſitiuas. Nunc ordo poſtulat, vt etiam litteras 𝔅, ℭ, 𝔇 etc. ex aequationibus fundamentalibus determinemus

$$\frac{\mathfrak{B}\pi - \Phi}{\Phi} = -P;$$

$$\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = PQ$$

$$\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = -PQR$$

$$\frac{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = PQRS.$$

ex

ex quibus, ſi breuitatis gratia ponamus, $\frac{\omega + \omega' + 1}{m + 1} = M$, colligimus:

$$\mathfrak{B} = \frac{(1 - P) \cdot M}{\omega} \qquad \Big| \qquad B = \frac{\mathfrak{B}}{1 - \mathfrak{B}}$$

$$\mathfrak{C} = \frac{(1 - PQ) M - \omega}{\omega'} \qquad \Big| \qquad C = \frac{\mathfrak{C}}{1 - \mathfrak{C}}$$

$$\mathfrak{D} = (1 - PQR) M - \omega' - \omega \qquad \Big| \qquad D = \frac{\mathfrak{D}}{1 - \mathfrak{D}}$$

$$\mathfrak{E} = (1 - PQRS) M - \omega' - \omega - 1 \qquad \Big| \qquad E = \frac{\mathfrak{E}}{1 - \mathfrak{E}}$$

adeoque

$$B = \frac{(1 - P) M}{\omega - (1 - P) M}$$

$$C = \frac{(1 - PQ) M - \omega}{\omega' + \omega - (1 - PQ) M}$$

$$D = \frac{(1 - PQR) M - \omega' - \omega}{1 + \omega + \omega' - (1 - PQR) M}$$

$$E = \frac{(1 - PQRS) M - \omega' - \omega - 1}{2 + \omega + \omega' - (1 - PQRS) M}$$

Nunc denique aequatio pro confuſione aperturae tollenda conſiderari debet, quae eſt

$$0 = \lambda - \frac{1}{\mathfrak{B} P} \left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B} \right)$$
$$+ \frac{1}{B^3 . \mathfrak{C} . PQ} \left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C} \right)$$
$$- \frac{1}{B^3 C^3 . \mathfrak{D} . PQR} \left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu}{D} \right)$$
$$+ \frac{1}{B^3 C^3 D^3 . \mathfrak{E} . PQRS} \left(\frac{\lambda''''}{\mathfrak{E}^2} + \frac{\nu}{E} \right)$$
$$- \frac{1}{B^3 C^3 D^3 E^3 . \mathfrak{F} . PQRST} . \lambda'''''$$

cui aequationi vt ſatisfieri queat, notandum eſt, terminos poſt tres priores ſequentes admodum fieri par-

 uos.

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uos. Cum enim ſit $PQRST = -m$, hoc eſt numero ingenti, primi autem factores P et Q vix ab vnitate diſcrepent, neceſſe eſt, vt productum RST numerum m fere totum producat; deinde quia inter S et T inuenimus aequationem: $0 = 1 + \frac{1}{S} + \frac{1}{ST}$, patet, numerum m neque in S neque in T contineri, ideoque factorem R maximam partem numerum m complecti. Quocirca huius aequationis membra quartum et ſequentia prae tribus prioribus quaſi euaneſcent ita, vt tria priora ſe mutuo propemodum deſtruere debeant, vnde proxime ſtatuendum erit

$$0 = \lambda - \frac{\lambda'}{\mathfrak{B}^3 P} + \frac{\lambda''}{B^3 \mathfrak{C}^3 PQ}$$

ideoque

$$\lambda' = \mathfrak{B}^3 P . \lambda + \frac{\mathfrak{G}^3 \lambda''}{B^3 \mathfrak{C}^3 Q}$$

vbi pro felici exſecutione optandum eſſet, vt prodiret $\lambda = 1$; $\lambda' = 1$ et $\lambda'' = 1$. quia tum leues errores in praxi commiſſi minimi ſunt momenti. Quare cum ſequentes termini parui ſint poſitiui, neceſſe erit, vt ſit $1 > \mathfrak{B}^3 P + \frac{\mathfrak{B}^3}{B^3 \mathfrak{C}^3 . Q}$ vnde ſi eſſet $P = 1$ et $Q = 1$, et, vt ſupra, $B\mathfrak{C} = 1$ deberet eſſe $1 > 2\mathfrak{B}^3$ ſeu $\mathfrak{B} < \sqrt[3]{\frac{1}{2}}$ ſiue $\mathfrak{B} < \frac{4}{5}$; quod praeceptum in adplicatione attendi meretur.

Coroll. I.

266. Cum igitur nunc certum ſit, quinque litteras PQRST omnes eſſe poſitiuas, praeter S vel T,

ſi

ſi ſumamus diſtantiam a ſemper eſſe poſitiuam, ex primo interuallo concludimus eſſe $P > 1$; et quia hoc interuallum ſtatuitur minimum, P parum tantum ſuperabit vnitatem & quia etiam ſecundum interuallum ſumitur minimum, littera Q parum quoque ab vnitate diſcrepabit. Deinde quia quoque F debet eſſe quantitas poſitiua, ideoque productum B C D E poſitiuum ob $\varepsilon = -Tf$ vltimum interuallum fit $(1-T)f$ ſtatimque hanc praebet conditionem $T < 1$ vnde ſi T ſit poſitiuum, conditio poſtulat, vt ſit $T < 1$; ſin autem T ſit negatiuum, nulla reſtrictione opus eſt.

Coroll. 2.

267. Quia in vltima aequatione omnia membra praeter ſecundum ſunt poſitiua, ita, vt ſolum ſecundum, omnia reliqua deſtruere debeat, neceſſe eſt, vt $\mathfrak{B}$ ſit poſitiuum et propemodum, vti notauimus, valorem habeat vnitate aliquantillum minorem. Quare cum inuentum ſit $\mathfrak{B} = \frac{(1-P)M}{\omega}$; littera autem M ſemper ſit poſitiua, at $1 - P$ negatiuum, ſequitur, fore particulam ω negatiuam.

Coroll. 3.

268. Si igitur interuallum primum ſtatuamus $= \eta a$, pariterque ſecundum etiam ηa exiſtente η fractione minima, quoniam tantum hic illum caſum euitare volumus, quo hae lentes quaſi in vnam coaleſcere deberent, η tam paruum aſſumi conuenit, quam

exſecutio permittit; ad quod ſufficere videtur, ſi ſit $\eta = 0,03$; hinc igitur erit $1 - \frac{1}{P} = \eta$ adeoque $P = \frac{1}{1-\eta}$ et nunc ω propius definire poterimus, ſcilicet $\omega = \frac{-\eta M}{(1-\eta)\mathfrak{B}}$ et quia $M = \frac{3}{m+1}$ erit $\omega = \frac{-3\eta}{(m+1)\mathfrak{B}}$ ſicque littera $\mathfrak{B}$ adhuc noſtro arbitrio relinquitur.

Coroll. 4.

269. Quia $\mathfrak{B} > 0$ et parumper minus vnitate erit B poſitiuum; vnde pro ſecundo interuallo habebimus $Q = \frac{B}{B+\eta P}$ ideoque $Q < 1$. ſiue $Q = \frac{(1-\eta)\mathfrak{B}}{(1-\eta)\mathfrak{B}+1}$ ſeu proxime $Q = 1 - \frac{\eta}{B}$; hincque definire licet ω' ſcilicet $\omega' = \frac{(1-PQ)M-\omega}{\mathfrak{C}}$ ſiue $\omega' = \frac{6\eta}{(m+1)B\mathfrak{C}}$, ita, vt etiam $\mathfrak{C}$ noſtro arbitrio relinquatur.

Scholion 1.

270. Hinc caſus ſupra tractatus, quo erat $B = \infty$ et $\mathfrak{C} = 0$, facile deducitur tum enim erit $Q = 1$; manente $P = 1 + \eta$, $\omega = \frac{-3\eta}{m+1}$; $\omega' = \frac{6\eta}{(m+1)B\mathfrak{C}}$ et quia tam $\beta = \infty$, quam $c = -\infty$ fiet γ diſtantia focalis tertiae lentis $r = \frac{B\mathfrak{C}\alpha}{PQ}$, vbi $B\mathfrak{C}$ ita definiri poterit, vt tertia lens primae perfecte euadat aequalis, quod ad praxin valde eſt conueniens; ſtatuatur nempe $B\mathfrak{C} = PQ = 1 + \eta$.

Quia autem tum fit $\mathfrak{B} = 1$, poſtremae conditioni ſatisfieri nequit; quae cum ſit maioris momenti, quam praecedens, ſtatuamus potius $\mathfrak{B} = \frac{4}{5}$ vt ſit $B = 4$ et

et ſumto $P = 1,03$ reperitur, ſumi debere $\mathfrak{C} > 0,2567$, quare ſumatur $\mathfrak{C} = 0,257$. eritque

$$C = \frac{0,257}{0,743} = 0,34589$$

quod notaſſe ſufficiat pro iis, qui tali reſolutione vti velint.

Coroll. 5.

271. Quia nunc tam BC, quam PQ ſunt numeri poſitiui, vt tertium quoque interuallum fiat poſitiuum, neceſſe eſt, vt ſit $R > 1$, quae conditio ſponte impletur, quoniam R erit numerus multo adhuc maior. Pro quarto interuallo $-\frac{BCD}{PQR}(1 - \frac{1}{S})\alpha$ neceſſe eſt, vt $-D(1 - \frac{1}{S})$ ſit poſitiuum; ex praecedentibus autem patet, $\mathfrak{D}$ ideoque et D eſſe negatiuum; vnde fieri debet $1 - \frac{1}{S} > 0$ quod fit, ſi fuerit vel S negatiuum vel $S > 1$, ſi ſit poſitiuum. De quinto interuallo iam ſupra vidimus.

Scholion 2.

272. Hic ergo duo caſus ſunt conſiderandi, alter, quo S eſt numerus negatiuus, alter vero, quo T eſt negatiuum.

I. Sit $S < 0$ et ponatur $S = -K$ habebiturque iſta aequatio $0 = 1 - \frac{1}{K} - \frac{1}{KT}$ eritque $K = 1 + \frac{1}{T}$ verum hic eſt $T < 1$, vti ſupra vidimus; vnde erit $K > 2$ et KT continetur intra limites 1 et 2; vnde cum ſit $RKT = m$, continebitur R intra limites m et $\frac{1}{2}m$ hoc porro caſu erit $\mathfrak{C} = (1 + RK)M - 1$; qui valor ob $RK > m$ et $M = \frac{1}{m+1}$ erit $\mathfrak{C} > \frac{3(m+1)}{m+1} - 1 > 2$;

ideo-

ideoque E ſemper negatiuum, vti reliqua conditio poſtulat, ſcilicet vt $BCDE$ ſit poſitiuum.

II. Sit iam $T < 0$ et ponatur $T = -K$ eritque $0 = 1 + \frac{1}{S} - \frac{1}{SK}$ ideoque $S = \frac{1}{K} - 1$, vbi quidem ratione vltimi interualli K pro lubitu accipi poſſet, nunc vero requiritur, vt ſit $K < 1$. Cum igitur ſit $RSK = m$, erit $RS = \frac{m}{K}$ adeoque $RS > m$ et littera $\mathfrak{E}$ manifeſto ſit negatiua ideoque etiam E; quibus notatis euolutio exemplorum nulla plane laborat difficultate; id tantum hic adhuc adiungere viſum eſt, vt tres poſteriores lentes maximam aperturam accipiant, eas vtrinque aeque conuexas confici debere; qua conditione numeri λ''', λ'''', λ''''' ſequenti modo determinantur, vti quidem iam ſupra eſt oſtenſum, ſcilicet ſi pro indole vitri ponatur $\frac{\sigma - \varrho}{2\tau} = N$, reperitur $\lambda''' = 1 + N^2 . (\frac{1 - D}{1 + D})^2$, quae forma ob $D = \frac{\mathfrak{D}}{1 - \mathfrak{D}}$ erit $\lambda''' = 1 + N^2 (1 - 2\mathfrak{D})^2$ ſimilique modo $\lambda'''' = 1 + N^2 (1 - 2\mathfrak{E})^2$ $\lambda''''' = 1 + N^2$.

Superfluum autem iudico hanc inueſtigationem exemplo illuſtrare, cum quia pro caſu quinque lentium plurima exempla iam ſunt allata, tum vero inprimis ſi quis campum maiorem deſideraverit, conſultum potius erit, duas vitri ſpecies adhibere, vt etiam vltima confuſionis ſpecies penitus remoueatur. Quod argumentum in ſequente adhuc capite fuſius nobis explicandum reſtat.

CAPVT

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CAPVT III.

DE VLTERIORI TELESCOPIORVM SECVNDI GENERIS PERFECTIONE, DIVERSAS VITRI SPECIES ADHIBENDO.

Problema I.

273.

Si teleſcopium ex tribus lentibus ſit componendum, invenire momenta, ad eius perfectionem facientia.

Solutio.

Incipiendum igitur eſt a duabus fractionibus, quae methodo ante expoſita ponantur $\frac{a}{b} = -P$; et $\frac{\beta}{c} = -Q$; ita vt litterarum P et Q altera ſit poſitiua, altera vero negatiua, ita, vt ſit $PQ = -m$. Tum igitur erunt diſtantiae determinatrices

$$b = -\frac{a}{P};\ \beta = -\frac{Ba}{P};\ c = \frac{Ba}{PQ}$$

diſtantiae focales

$$p = a;\ q = \frac{-\mathfrak{B}a}{P};\ r = \frac{Ba}{PQ} = \frac{-Ba}{m}$$

et bina interualla

$$a + b = a\left(1 - \frac{1}{P}\right);\ \beta + c = -\frac{Ba}{P}\left(1 - \frac{1}{Q}\right)$$

Deinde cum ſit pro campo $\Phi = \frac{\pi - \pi'}{m+1}$ et media lens parum confert, ſtatuamus $\pi = \omega\,\xi$ et $\pi' = -\xi$, vt fiat $\Phi = \frac{\omega+1}{m+1}.\,\xi$ vnde pro diſtantia oculi habetur

$$O = -\frac{\pi'}{\Phi}\cdot\frac{r}{m} = -\frac{(m+1)}{\omega+1}\cdot\frac{B\alpha}{mm}$$
$$= -\frac{B\alpha}{m}\cdot\frac{m+1}{m(1+\omega)}$$

ita, vt nunc $-B\alpha$ debeat eſſe poſitiuum; ſeu his tribus conditionibus erit ſatisfaciendum:

1°. $\alpha\left(1-\frac{1}{P}\right) > 0$

2°. $-\frac{B\alpha}{P}\left(1-\frac{1}{Q}\right) > 0$

3°. $-B\alpha > 0$

hincque $\frac{1}{P}\left(1-\frac{1}{Q}\right) > 0$

Porro autem fiet

$$\mathfrak{B}\,\omega = (1-P)\,M;\quad B = \frac{(1-P)M}{\omega-(1-P)M}$$

exiſtente $M = \frac{\omega+1}{m+1}$.

Iam pro margine colorato tollendo aequatio eſt, ſiquidem pro fractionibus $\frac{dn}{n-1}$; $\frac{dn'}{n'-1}$ et $\frac{dn''}{n''-1}$ litteras N, N′, N″ ſtatuamus.

$$0 = N'.\,\omega.\,\frac{1}{P} + N''.\,\frac{1}{PQ} \text{ ſeu}$$
$$0 = N'\omega + N''.\,\frac{1}{Q};\ \text{vnde fit } Q = -\frac{N''}{N'\omega}.$$

Vt autem haec confuſio penitus tollatur, requiritur, vt ſit

$$0 = N.\,\alpha - \frac{N'.\alpha}{\mathfrak{B}P} + \frac{N''\alpha}{B.PQ}$$

quae

quae loco Q valore ſubſtituto fit

$$0 = N - \frac{N'}{P}\left(\frac{1}{\mathfrak{B}} + \frac{\omega}{B}\right)$$

eſt vero $\omega = \frac{1-P}{(m+1)\mathfrak{B}+P-1}$

in qua ſi ponatur valor ipſius ω, erit

$$0 = N - \frac{N'}{P}\left(\frac{m+P}{(m+1)\mathfrak{B}+P-1}\right)$$

deinde aequatio pro Q inuenta ob $PQ = -m$ dabit quoque

$$m = \frac{N''.P((m+1)\mathfrak{B}+P-1)}{N'(1-P)}$$

ex qua ſi in praecedente aequatione pro $(m+1)\mathfrak{B}+P-1$ ſcribatur valor ipſius $\frac{N'.m(1-P)}{N''.P}$ orietur haec aequatio,

$$0 = NP((m+1)\mathfrak{B}+P-1) - N'(m+P)$$

$$0 = \frac{N.N'm(1-P)}{N''} - N'(m+P)$$

indeque porro

$P = \frac{m.(N-N'')}{Nm+N''}$ ideoque cum ſit

$\frac{mN'(1-P)}{N''P} = (m+1)\mathfrak{B}+P-1$, colligitur

$$\mathfrak{B} = \frac{N'}{N-N''} + \frac{N''}{Nm+N''} \text{ ſiue } \mathfrak{B} = \frac{NN'm+NN''+N'N''-N''N''}{(N-N'')(Nm+N'')}$$

hinc $1-\mathfrak{B} = \frac{NNm-NN'm-NN''m-N'N''}{(N-N'')(Nm+N'')}$

adeoque $B = \frac{NN'm+NN''+N'N''-N''N''}{NNm-NN'm-NN''m-N'N''}$

Iam videamus, quomodo hae determinationes cum ſuperioribus conditionibus ſubſiſtere queant et cum

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eſſe debeat $\frac{1}{P}(1-\frac{1}{Q})>0$; ſiue ob $Q=-\frac{m}{P}$, $\frac{1}{P}+\frac{1}{m}>0$ erit $\frac{N}{N-N''}>0$; et $N>N''$.

Primum autem interuallum $\alpha(1-\frac{1}{P})>0$ abit in hoc $\frac{-N''(m+1)}{m(N-N'')}.\alpha$; ideoque $\frac{-N''.\alpha}{N-N''}>0$. et quia denominator iam inuentus eſt poſitiuus, reſtat, vt ſit $-N''.\alpha>0$. Conditio vero $-B\alpha>0$ dabit nunc $B>0$ vnde etiam fiet $\mathfrak{B}>0$, quare quum ſit $N-N''>0$ erit quoque $\frac{NN'm+NN''+N'N''-N''N''}{Nm+N''}>0$, adeoque neceſſe eſt vt in hac fractione, tam numerator quam denominator ſimul ſit aut poſitiuus aut negatiuus, poni autem nequit $Nm+N''<0$, quia tum foret $m<-\frac{N''}{N}$, neque $NN'm+NN''+N'N''-N''N''>0$ nam inde ſequeretur eſſe $m<\frac{N''(N''-N-N')}{NN'}$, quod cum ſit impoſſibile, etiam impoſſibile eſt, vt ope trium lentium haec duo commoda, quibus altera confuſio penitus tollitur, obtineantur.

Scholion.

274. Hoc ergo problema reſolui nequit ſiquidem poſteriorem confuſionem penitus tollere velimus. Omiſſa autem vltima aequatione, ſolutio facilis fuiſſet, ſed tum plus non eſſemus conſecuti, quam in praecedente capite, vbi vnica vitri ſpecie ſumus vſi. Quoniam igitur non conuenit, duas vitri ſpecies adhibere, ad teleſcopia conficienda, quae ex vnica ſpecie aeque felici ſucceſſu obtineri poſſunt: huic inueſtigationi non immorabimur, ſed tantum eiusmodi in medium

dium producere conabimur, quae praeter ſuperiores qualitates etiam omni confuſione, quae ibi erat relicta, deſtituantur. Cauſſa autem, cur iſta inueſtigatio hic non ſucceſſit, in eo manifeſto conſiſtit, quod numerus litterarum indefinitarum erat nimis paruus, ſiquidem ad tres aequationes adimplendas tantum tres litterae praeſto erant. Quare ſi plures lentes conſtituamus, plures etiam habebimus eiusmodi litteras, quibus non ſolum his tribus aequationibus, ſed reliquis etiam conditionibus ſatisfieri poterit.

Problema 2.

275. Si Teleſcopium ex quatuor lentibus ſit componendum determinare momenta, ad eius perfectionem facientia.

Solutio.

Tres fractiones hic conſiderandae ponantur

$$\frac{\alpha}{b} = -P;\ \frac{\beta}{c} = -Q;\ \frac{\gamma}{d} = -R$$

ita, vt harum litterarum P, Q, R vna ſit negatiua et $m = -PQR$. vnde erunt diſtantiae determinatrices:

$$b = -\frac{\alpha}{P};\ \beta = -\frac{B\alpha}{P};\ c = \frac{B\alpha}{PQ}$$

$$\gamma = \frac{BC\alpha}{PQ};\ d = -\frac{BC\alpha}{PQR}$$

diſtantiae autem focales

$$p = \alpha;\ q = \frac{-\mathfrak{B}\alpha}{P};\ r = \frac{B\mathfrak{C}\alpha}{PQ};\ s = \frac{-BC\alpha}{PQR}$$

et interualla lentium

$$\alpha + b = \alpha\left(1 - \tfrac{1}{P}\right); \quad \beta + c = -\tfrac{B\alpha}{P}\left(1 - \tfrac{1}{Q}\right)$$

$$\gamma + d = \tfrac{BC\alpha}{PQ}\left(1 - \tfrac{1}{R}\right)$$

Pro campo autem apparente $\Phi = \frac{\pi - \pi' + \pi''}{m+1}$, ſtatuatur $\pi = \omega\xi$; $\pi' = -i\xi$ et $\pi'' = \xi$, exiſtente ξ valore maximo, quem hae litterae accipere poſſunt, ſcilicet $\frac{1}{4}$; ita vt ſit $\Phi = \frac{\omega + i + 1}{m+1} \cdot \xi$, ſiue $\Phi = M\xi$, poſito $M = \frac{\omega + i + 1}{m+1}$. Ex his igitur obtinemus

$$\mathfrak{B}\omega = (1 - P)M; \quad \mathfrak{C}i = (1 - PQ)M - \omega.$$

Pro loco autem oculi erit

$$O = \frac{\pi''}{\Phi} \cdot \frac{s}{m} = \frac{m+1}{m^2} \cdot \frac{BC.\alpha}{\omega + i + 1} = \frac{BC.\alpha}{m^2.M};$$

quod vt fiat poſitiuum debet eſſe $BC\alpha > 0$.

His poſitis tribus ſequentibus aequationibus ſatisfieri debet

$$\text{I.}\quad 0 = \frac{N'\omega}{P} + \frac{N''i}{PQ} + \frac{N'''}{PQR}$$

$$\text{II.}\quad 0 = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ} - \frac{N'''}{BCPQR}$$

$$\text{III.}\quad 0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right)$$

$$+ \frac{\mu''}{B^3\mathfrak{C}.PQ}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right) - \frac{\mu'''.\lambda'''}{B^3C^3.PQR}$$

quae reſolutio quo facilius inſtitui poſſit, conſideremus primo caſum, quo duae priores lentes ſibi immediate iunguntur, vt ſupra de lentibus duplicatis aſſumſimus.

Primus

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Primus caſus, quo $\alpha + b = 0$ ideoque $P = 1$, et $\omega = 0$ tum littera $\mathfrak{B}$ manet indeterminata hincque etiam B; quo facto reſolutio facile inſtitui poterit.

Prima enim aequatio dat. $0 = N'' i + \frac{N'''}{R}$ vnde ſequitur $R = -\frac{N'''}{N'' i}$, ita vt R ſit quantitas negatiua, ſiquidem i ſit poſitiuum, id quod ratio campi poſtulat. Hinc ergo cum ſit

$$P = 1, \text{ erit } PQR = \frac{-Q N'''}{N''. i} = -m$$

$$\text{ideoque } Q = \frac{N'' m. i}{N'''}.$$

Secunda autem aequatio, in qua iam duo poſtremi termini euadunt valde parui, ſiquidem multiplicatio m ſit magna, ſtatim dat $0 = N - \frac{N'}{\mathfrak{B}}$, adeoque $\mathfrak{B} = \frac{N'}{N}$; et hinc ſit $B = \frac{N'}{N - N'}$; vnde, ſi libuerit, valor ipſius $\mathfrak{B}$ adcuratius definiri poterit, habebitur nempe.

$$\frac{1}{\mathfrak{B}} = \frac{N}{N'} + \frac{N'''(N - N')^2}{N' N' \mathfrak{C}. m. i} + \frac{N'''(N - N')}{N' N'. C. m.}$$

plerumque autem ſufficit, his duabus aequationibus proxime ſatisfeciſſe.

Tertia autem adcurate reſolui debet, cuius ſecundus terminus cum ſit negatiuus, reliquis exiſtentibus poſitiuis, vt mox videbimus, ille reliquis aequalis eſſe debet: erit enim $\mathfrak{C} i = \frac{N''' - N'' m i}{N'''} . \frac{i + 1}{m + 1}$ ideoque $\mathfrak{C}$ eſt negatiuum, ſimulque etiam C vnde conditiones ſupra memoratae ſunt perpendendae.

Primum autem interuallum eſt per hypotheſin $= 0$. Secun-

Secundum ſit $\beta + c = \frac{-N'(N'' m i - N''')}{(N-N')N'' m i}.\alpha$ pro quo ſi α ſit poſitiuum, debet eſſe $-\frac{N'}{N-N'}$ poſitiuum, ſeu $N < N'$; contra autem ſi α ſit negatiuum, debet eſſe $N > N'$.

Tertium porro interuallum eſt $\frac{N' C \alpha}{(N-N')Q}\left(1 + \frac{N'' i}{N'''}\right)$; quia hic Q eſt poſitiuum; C negatiuum requiritur, vt ſit $-\frac{N'\alpha}{N-N'}$ poſitiuum vti pro ſecundo interuallo, ita, vt ſi ſecundum interuallum fuerit poſitiuum, tertium ſponte euadat poſitiuum.

Denique formula pro loco oculi $O = \frac{BC\alpha}{m \cdot M}$ etiam ſit poſitiua ſub conditionibus iisdem. Ex quibus ſequitur, ſi lens prima ſit ex vitro coronario, ſecunda vero ex chryſtallino, ſiue $N' > N$, tunc debere eſſe α poſitiuum; ſeu $p > 0$; $q < 0$; $r > 0$; $s > 0$.

Sin autem primam lentem ex vitro chryſtallino, ſecundam vero ex coronario faciamus, ita, vt ſit $N' < N$ debebit eſſe α negatiuum ideoque $p < 0$; $q > 0$; $r > 0$; $s > 0$. quare pro vtroque caſu facile erit tertiam aequationem reſoluere. His conditionibus perpenſis, quae etiam nunc locum habebunt, dummodo ω ſit fractio quam minima, ſtatuamus primum interuallum $\alpha\left(1 - \frac{1}{P}\right) = \eta\alpha$ exiſtente η fractione minima, ſiue poſitiua, ſi $\alpha > 0$, ſiue negatiua, ſi $\alpha < 0$ eritque $P = \frac{1}{1-\eta}$; deinde maneat $\mathfrak{B}$ adhuc indefinita et quaeratur ω; eritque

$$\mathfrak{B}\,\omega = \frac{-\eta}{(1-\eta)} \frac{(\omega + i + 1)}{(m+1)};$$

in

in quo poſtremo factore, ω tuto omittitur; ita, vt hinc ſit $\omega = \frac{-\eta}{(1-\eta)} \cdot \frac{i+1}{(m+1)\mathfrak{B}}$ qui valor ob duplicem cauſſam diminuitur, 1°. enim η eſt valde paruum, deinde ea diuiditur per $m+1$ numerum ſatis magnum; porro vero tam 𝔅 quam $i+1$ ab vnitate parum diſcrepant; quam ob cauſſam valor ω recte pro euaneſcente haberi poterit; vnde prima aequatio nobis dabit, vt ante,

$$o = N'' i + \frac{N'''}{R}; \text{ et } R = \frac{-N'''}{N'' i};$$

quem valorem ſiquis adhuc adcuratius deſideret, erit,

$$-\frac{1}{R} = \frac{N' \omega Q}{N'''} + \frac{N'' i}{N'''};$$

ita, vt nunc P et R ſint quantitates cognitae in primo termino vtpote minimo ſufficit Q proxime noſſe, quem adeo ex caſu praecedente deſumere licet quia ω iam eſt definitum et 𝔅 mox definietur. Hinc igitur $Q = \frac{-m}{PR}$. Secunda aequatio iterum, vt in caſu praecedente, proxime dabit, vt ante, $\mathfrak{B} = \frac{N'}{N}$; ſi quis eum vero exactius deſideret, erit ei hac aequatione vtendum:

$$\frac{N'}{\mathfrak{B}} = N P + \frac{N''}{B \mathfrak{C} Q} - \frac{N'''}{B C Q R}$$

vbi pro B ſufficit valorem prope verum noſſe, nempe $B = \frac{N'}{N - N'}$. Tum vero habebitur

$$\mathfrak{C} i = (1 + \frac{m}{R}) (\frac{\omega + i + 1}{m + 1}) - \omega$$

quem valorem manifeſtum eſt propter valorem ipſius R eſſe negatiuum, ideoque etiam C. Quocirca con-

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ditiones praeſcriptae iisdem caſibus implentur, vt in praecedente, vbi $\omega = 0$; ita, vt nunc tantum ſuperſit, aequationem tertiam reſoluere; ſi modo meminerimus, ob $\pi'' = \xi$ quartam lentem fieri debere aeque conuexam; quae forma etiam tertiae lenti tribui deberet, ſi eſſet $i = 1$. Verum ſi ſumeretur $i = 1$ vnde haec lens fieret vtrinque aeque conuexa, ob $\mathfrak{C} = -1$ propemodum, pro hac lente ſtatui deberet

$$\lambda'' = 1 + N^2 (1 - 2\,\mathfrak{C})^2 = 1 + N^2 . 9$$

vbi, vt ante ſumſimus, eſt $N = \frac{\sigma - \rho}{2\tau}$ ſicque numerus λ'' ſatis magnum obtineret valorem; quod incommodum euitabimus, ſumendo $i < 1$. et plerumque ſufficiet ſtatuere $i = \frac{1}{2}$.

Corollarium.

276. Hic ergo differentia refractionis vitri tantum in duabus prioribus lentibus in conſiderationem venit ideoque ſufficiet, vnicam tantum lentem ex vitro chryſtallino conficere, et reliquas omnes ex vitro coronario ſicque duos tantum caſus habebimus euoluendos; alterum, quo prima lens ex vitro chryſtallino conficitur; alterum, quo ſecunda.

CASUS I.

277. Sit igitur prima lens chryſtallina, reliquae omnes ex vitro coronario factae erit $n = 1,58$; et $n' = n'' = 1,53$ tum vero ſecundum Dollondi experimenta

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rimenta $N = 10$, $N' = N'' = N''' = 7$. His poſitis et ſumto $i = \frac{1}{3}$ erit α negatiuum ideoque etiam η. Sumatur autem $\eta = -0,03$ hinc ergo habebimus

$P = \frac{1}{1,03} = \frac{100}{103}$; et quia erit proxime

$\mathfrak{B} = \frac{7}{15}$ et $B = \frac{7}{3}$, inuenimus $\omega = \frac{45}{721(m+1)}$.

Deinde cum ſit proxime $R = -2$; ideoque $Q = \frac{103.m}{200}$ adcuratius habebimus

$-\frac{1}{R} = \frac{1}{3} + \frac{577.m}{36050(m+1)}$ ſeu

$R = \frac{-36050(m+1)}{18952m+18025}$

qui eſt valor correctus ipſius R, ex quo etiam adcuratius Q definiri poterit ſcilicet $Q = \frac{-m}{PR}$.

Vt nunc etiam $\mathfrak{B}$ adcuratius definiatur, erit

$\frac{1}{\mathfrak{B}} = \frac{10.P}{7} + \frac{3.200}{7.10.m.\mathfrak{C}} + \frac{3.200}{7.3.03.m.C}$.

Eſt vero $\mathfrak{C} = \left(1 + \frac{m}{R}\right)\left(\frac{3+2\omega}{m+1}\right) - 2\omega$ et $C = \frac{\mathfrak{C}}{1-\mathfrak{C}}$; vnde etiam B definiri poteſt.

His igitur valoribus definitis, tertia aequatio principalis reſolui debet, ſtatuendo $\lambda''' = 1 + \left(\frac{\sigma-\varrho}{2\tau}\right)^2$, vt quarta lens aeque conuexa vtrinque reddatur. Pro tertia autem lente videtur ſtatui poſſe $\lambda'' = 1$.

Denique inuentis ſingulis litteris λ etiam ſingularum lentium conſtructio habebitur. Vtemur autem methodo iam aliquoties vſitata, ſcilicet pro caſu quodam determinato, puta $m = 25$, deinde pro caſu $m = \infty$ euolutionem inſtituamus, indeque conſtructionem pro quacunque multiplicatione deriuemus.

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Exempl. I.

278. Si prima lens ex vitro chryſtallino, tres ſequentes autem ex coronario ſint parandae, pro multiplicatione $m = 25$ conſtructionem teleſcopii inueſtigare.

Sumto igitur $\eta = -0,03$, vt interuallum duarum priorum lentium fiat $-\frac{3\alpha}{100}$ ob α negatiuum, habemus ſtatim $P = \frac{100}{103} = 0.97087$

$$\text{Log. } P = 9.9871628$$

hinc $\omega = 0,00240$

deinde $R = \frac{-36050 \cdot 26}{18 \cdot 52 \cdot 25 + 18025}$

ſeu $R = \frac{-36050 \cdot 26}{491825} = -1,90576$

$$\text{Log. } R = 0,2800680. (-)$$

hincque $Q = 13,51160$

$$\text{Log. } Q = 1,1307092$$

nunc pro $\mathfrak{C}$ inueniendo notetur eſſe $PQ = 13,11810$

$$\text{Log. } PQ = 1,1178720$$

hincque erit

$$\mathfrak{C} = -12,1181 \frac{(1,00480)^2}{26} - 0,00480$$

ſeu $\mathfrak{C} = -1,40528$

$$\text{Log. } \mathfrak{C} = 0,1477628 (-)$$

hincque $C = -0,58425$.

$$\text{Log. } C = 9,7665972 (-)$$

Nunc deique pro B inueniendo nulla approximatione vtamur, quia ob $\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ adcurate habemus

1 —

$$1 - \frac{1}{\mathfrak{C}\mathfrak{Q}} + \frac{1}{\mathfrak{C}\mathfrak{Q}\mathfrak{R}} = (\tfrac{10}{7} P - 1) B \text{ ſiue}$$

$$1 + 0,05266 + 0,06647 = (1,38696 - 1) B$$

adeoque $1,11913 = 0,38696. B.$

ideoque $B = 2,89210.$

Log. $B = 0,4612145.$

conſequenter

$$\mathfrak{B} = \tfrac{2,89210}{3,89210} = 0,74307$$

$$\text{Log. } \mathfrak{B} = 9.8710305$$

His valoribus definitis primo quaeramus noſtra elementa, ac reperiemus

$b = - 1,03000. \alpha.$

Log. $\frac{b}{\alpha} = 0.0128371 \, (-)$

Log. $B = 0,4612145$

Log. $\frac{\beta}{\alpha} = 0,4740517 (-)$; $\beta = -2,97887 \alpha$

Log. $Q = 1,1307092$

Log. $\frac{c}{\alpha} = 9,3433425$; $c = + 0,22046 \alpha$

Log. $C = 9,7665972 \, (-)$

Log. $\frac{\gamma}{\alpha} = 9,1099397$; $\gamma = - 0,12880 \alpha$

Log. $R = 0,2800680$

Log. $\frac{d}{\alpha} = 8.8298717$; $d = - 0,06759 \alpha.$

 Hinc

Hinc porro etiam diſtantias focales

$$p = \alpha;\; q = -0,76536.\alpha$$

$$\text{Log. } q = 9.8838677\,(-)$$

$$r = -0,30982\,\alpha$$

$$\text{Log. } r = 9,4911053\,(-)$$

$$s = -0,06759.\alpha$$

$$\text{Log. } s = 8,8298717.$$

Porro interualla lentium erunt.

$$\alpha + b = -0,03000.\alpha.$$

$$\beta + c = -2,75841.\alpha.$$

$$\gamma + d = -0,19639.\alpha.$$

Diſtantia denique oculi ab vltima lente

$$O = -1,12480.\tfrac{m+1}{mm}.\alpha$$

ideoque interuallum inter primam et vltimam lentem $= -2,98480.\alpha$.

Tertiam iam conſideremus aequationem, quae reſoluta et per μ diuiſa pro hoc caſu dabit

$$0 = \lambda - \frac{\mu'}{\mu}.\frac{\lambda'}{\mathfrak{B}^3 P} + \frac{\mu'}{\mu}.\frac{\lambda''}{B^3\mathfrak{C}^3 PQ}$$
$$-\frac{\mu'}{\mu}.\frac{\nu'}{B\mathfrak{B}P} + \frac{\mu'}{\mu}.\frac{\nu'}{B^3 C\mathfrak{C} PQ} - \frac{\mu'}{\mu}.\frac{\lambda'''}{B^3.C^3.P.Q.R}$$

vbi $\mu = 0,8724$, et $\mu' = 0,9875$ et $\nu' = 0,2196$, vnde habebitur haec aequatio ad numeros reducta

$0 = \lambda$

$$0 = \lambda - 2,84162\,\lambda' - 0,00128\,\lambda''$$
$$(\text{Log. } 7,1090175)$$
$$- 0,00938\,\lambda''' - 0,11913$$
$$(\text{Log. } 7,9724463) + 0,00095.$$

Vt nunc hanc aequationem reſoluamus, primo notandum eſt, quartam lentem eſſe vtrinque aeque conuexam ideoque $\lambda''' = 1,60006$ vnde cum eius diſtantia focalis ſit $s = -0,06759.\,\alpha$, erit radius vtriusque faciei $= 1,06.\ s = -0,07164.\,\alpha$. Pro tertia autem lente, cuius diſtantia focalis eſt $r = \mathfrak{C}c$, quia eius ſemidiameter aperturae eſſe debet $= \frac{1}{4}\mathfrak{C}c$ cui ergo quarta pars minoris radii aequalis eſſe debet; inde minor radius eſſe debet $\frac{1}{4}\mathfrak{C}c$, ex quo λ'' definiri oportet. Hunc in finem hanc lentem nunc definiamus. Eius autem radius anterioris faciei eſt

$$= \frac{c\gamma}{\varrho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda'' - 1)}}$$
$$= \frac{Cc}{\varrho C + \sigma \mp \tau(1+C)\sqrt{\lambda'' - 1}}$$

et radius faciei poſterioris

$$= \frac{Cc}{\sigma C + \varrho \pm \tau(1+C)\sqrt{\lambda'' - 1}}$$

ita, vt habeamus

$$\text{rad. fac.} \begin{cases} \text{anter.} = \dfrac{Cc}{1,3277 \mp 0,41575\,\tau\sqrt{\lambda'' - 1}} \\ \text{poſter.} = \dfrac{Cc}{-0,7432 \pm 0,41575\,\tau\sqrt{\lambda'' - 1}} \end{cases}$$

vbi

vbi ſigna ſuperiora valere debent; tum vero prior radius eſt minor; ideoque ponatur $= \frac{1}{2}\mathfrak{C}c$; vnde colligitur

$$\frac{c}{1,5277 - 0,41575\,\tau\sqrt{\lambda'' - 1}} = \frac{1}{2}\mathfrak{C}$$

ſicque erit $0,41575\,\tau\sqrt{\lambda'' - 1} = 0,6961$ adeoque

$$\text{rad. fac.} \begin{cases} \text{anter.} = \frac{\gamma}{0,6316} = -0,15489.\,\alpha \\ \text{poſter.} = \frac{\gamma}{-0,0471} = +2,73475.\,\alpha \end{cases}$$

cuius ergo lentis ſemidiameter aperturae erit $= 0,03872\,\alpha$ at ex valore ipſius x colligemus $\sqrt{(\lambda'' - 1)} = 1,80969$; hincque $\lambda'' = 4,27497$.

Nunc revertamur ad noſtram aequationem, quae erit

$$\lambda = 2,84162\,\lambda' - 0,00549 - 0,11818 \\ - 0,01501$$

quia nunc λ' vnitate minus eſſe nequit, ſtatuamus $\lambda' = 1$, eritque

$$\lambda = 2,98030;\quad \lambda - 1 = 1,98030$$

$$\text{et}\quad \tau\sqrt{\lambda - 1} = 1,23484;$$

ex quo prima lens ita erit conſtruenda

$$F = \frac{\alpha}{\sigma \pm 1,23484} = \frac{\alpha}{0,3479} = 2,87439.\,\alpha$$

$$G = \frac{\alpha}{\rho + 1,23484} = \frac{\alpha}{1,3762} = 0,72664.\,\alpha$$

Tandem

Tandem pro ſecunda lente habemus

$$F = \frac{b\beta}{\varrho\beta + \sigma b} = \frac{Bb}{2,3157} = -1,28638.\alpha$$

$$G = \frac{b\beta}{\sigma\beta + \varrho b} = \frac{Bb}{5,0229} = -0,59247.\alpha$$

cuius minoris radii pars quarta ſeu $0,14812.\alpha$ dat ſemidiametrum aperturae tam pro lente prima, quam pro lente ſecunda. Quare hinc deducitur ſequens

Conſtructio Teleſcopii
pro multiplicatione $m = 25$.

I. Pro lente prima. Flint Glaſſ.

radius faciei $\begin{cases} \text{anter.} = 2,87439.\alpha \\ \text{poſter.} = 0,72664.\alpha \end{cases}$

Semidiameter aperturae $= 0,14812.\alpha$

Interuallum ad ſecundam $= -0,03.\alpha$

II. Pro ſecunda lente. Crown Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -1,28638.\alpha \\ \text{poſter.} = -0,59247.\alpha \end{cases}$

Semidiameter aperturae vt ante.

Interuallum $= -2,75841.\alpha$

III. Pro tertia lente. Crown Glaſſ.

radius faciei $\begin{cases} \text{anter.} = -0,15489.\alpha \\ \text{poſter.} = +2,73475.\alpha \end{cases}$

Semidiameter aperturae $= 0,03872\,\alpha$.

Interuallum $= -0,19639.\alpha$.

IV. Pro quarta lente. Crown Glaſſ.

radius vtriusque faciei $= -0,07164.\,a$.

Semidiameter aperturae $= 0,01791.\,\alpha$.

Diſtantia ad oculum $= -1,1248.\frac{m+1}{m^{2}}\,\alpha$

ſeu $O = -1,1248\,(1+\frac{1}{m}).\frac{\alpha}{m}$

$O = -0,04679.\,\alpha$

vbi notandum eſt, eſſe α negatiuum ac ſi ſemidiametrum aperturae ſumamus $= \frac{m}{10}$ dig. $= \frac{1}{2}$ dig. fiet $\alpha = -\frac{7}{8}$ dig. circiter vel maius, et longitudo teleſcopii $= 10\frac{1}{2}$ dig. Denique ſemidiameter campi erit $\Phi = 49\frac{1}{2}$ min. circiter.

Exempl. II.

279. Si prima lens ex vitro chryſtallino, reliquae ex coronario ſint parandae, pro multiplicatione maxima conſtructionem teleſcopii inueſtigare.

Sit iterum $\eta = -0,03$; habebimus

vt ante, $P = 0,97087$

Log. $P = 9.9871628$

tum vero $\omega = 0$. et $R = -\frac{16050}{18532}$

ſeu $R = -1,90217$

Log. $R = 0,2792503\,(-)$

$Q = 0,54148.\,m$

Log. $\frac{Q}{m} = 9.7335868$

Porro

Porro $\mathfrak{C} = \frac{1}{R} = -1,57714$

Log. $\mathfrak{C} = 0.1978710\,(-)$

et $C = -0,61197$.

Log. $C = 9,7867330\,(-)$

Pro B autem inueniendo habetur haec aequatio

$$1 - \frac{1}{\mathfrak{C}Q} + \frac{1}{CQR} = (\tfrac{10}{7}P - 1)B$$

quae quia termini per Q diuiſi euaneſcunt, abit in hanc $1 = 0,3869.B$

hinc $B = 2,58464$.

Log. $B = 0,4124012$

et $\mathfrak{B} = 0,72103$

Log. $\mathfrak{B} = 9.8579557$.

Hinc habebimus diſtantias determinatrices

$b = -1,03000.\alpha$; Log. $b = 0,0128371\,(-)$

$\beta = -2,66218\alpha$; Log $\beta = 0,4252383\,(-)$

$c = +4,91645\frac{\alpha}{m}$; Log. $c = 0.6916515$

$\gamma = -3,00874\frac{\alpha}{m}$. Log. $\gamma = 0,4783845\,(-)$

$d = -1,58173.\frac{\alpha}{m}$; Log. $d = 0,1991342\,(-)$

atque interualla lentium

$\alpha + b = -0,03000.\alpha$

$\beta + c = -2,66218.\alpha - 4,91645.\frac{\alpha}{m}$

$\gamma + d = -4,59047.\frac{\alpha}{m}$

et diſtantia oculi poſt vltimam lentem

$$O = -0,63269 \cdot \frac{\alpha}{m}$$

tum vero diſtantias focales

$$p = \alpha;\ q = -0,74267 \cdot \alpha;$$

$$r = -7,75394 \cdot \frac{\alpha}{m};\ s = -1,58173 \cdot \frac{\alpha}{m}.$$

Nunc igitur conſideremus aequationem tertiam

$$0 = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda'}{\mathfrak{B}^3 P} - \frac{\mu' \nu'}{\mu \cdot \mathfrak{B} B P}$$

$$0 = \lambda - 3,11022\, \lambda' - 0,13738$$

quae ſumto, vt ante, $\lambda' = 1$, dat $\lambda = 3,24760$; hinc $\lambda - 1 = 2,24760$ et $\tau \sqrt{\lambda - 1} = 1,31555$; vnde lentium conſtructio ſequenti modo expedietur.

I. Pro prima lente ex vitro chryſtallino.

$$F = \frac{\alpha}{\sigma - 1,31555} = \frac{\alpha}{0,2672} = 3,74251 \cdot \alpha$$

$$G = \frac{\alpha}{\varrho + 1,31555} = \frac{\alpha}{1,4569} = 0,68639 \cdot \alpha$$

II. Pro ſecunda lente ex vitro Coronario

$$F = \frac{b\beta}{\varrho\beta + \sigma b} = \frac{\beta}{2,2461} = -1,18525 \cdot \alpha$$

$$G = \frac{b\beta}{\sigma\beta + \varrho b} = \frac{\beta}{4,2175} = -0,58931 \cdot \alpha$$

III. Pro tertia lente ex Crown Glaſſ.

$$F = \frac{Cc}{C\varrho + \sigma + x};\ G = \frac{Cc}{C\sigma + \varrho \pm x}$$

$$F = \frac{\gamma}{1,5214 + x};\ G = \frac{\gamma}{-0,7892 \pm x}$$

Cum

Cum nunc, vt ante, radius faciei anterioris fiat minor, is ſemiſſi diſtantiae focalis $= \frac{1}{2} \mathfrak{C} c$ aequalis ponatur, eritque

$$1,5214 - x = \frac{2C}{\mathfrak{C}} = 2(C+1) \text{ ſeu } x = 0,7454$$

Vnde ſtatim habetur

$$F = -3,87697 \cdot \frac{\alpha}{m}$$

$$G = \frac{\gamma}{-0,0438} = +68,69266 \cdot \frac{\alpha}{m}.$$

IV. Denique pro quarta lente,

cuius diſtantia focalis $s = -1,58173 \frac{\alpha}{m}$

radius vtriusque faciei erit

$$= 1,06 \cdot s = -1,67663 \cdot \frac{\alpha}{m}$$

ſicque omnia momenta pro hoc caſu ſunt definita.

Exempl. III.

280. Si prima lens ex vitro chryſtallino, reliquae ex coronario ſint parandae, pro multiplicatione quacunque conſtructionem teleſcopii exponere.

Solutio.

Ex comparatione duorum exemplorum praecedentium ſingula momenta methodo ſupra indicata facile definiemus. Primo pro diſtantiis determinatricibus erit

$b = -1,0300 \cdot \alpha$; pro reliquis autem ſtatuatur

 $\beta =$

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$\beta = -2,6622\,\alpha - \beta'.\frac{\alpha}{m}$; erit $\beta' = 7,9150$

$c = +4,9164.\frac{\alpha}{m} + \frac{c'}{m}.\frac{\alpha}{m}$; $c' = 14,8812$

$\gamma = -3,0087.\frac{\alpha}{m} - \frac{\gamma'}{m}.\frac{\alpha}{m}$; $\gamma' = 5,281$

$d = -1,5817\frac{\alpha}{m} - \frac{d'}{m}.\frac{\alpha}{m}$; $d' = -3,531$

ſimili modo pro diſtantiis focalibus eſt $p = \alpha$; pro reliquis ſtatuatur

$q = -0,7427.\alpha - \frac{q'}{m}.\alpha$;

$r = -7,7539.\frac{\alpha}{m} - \frac{r'}{m}.\frac{\alpha}{m}$;

$s = -1,5817.\frac{\alpha}{m} + \frac{3,531}{m}.\frac{\alpha}{m}$;

eritque $q' = 0,5665$; $r' = -0,2062$
vnde lentium interualla ſunt

$\alpha + b = -0,0300.\alpha$

$\beta + c = -2,6622.\alpha - \frac{2,008}{m}.\alpha + \frac{14,88}{m}.\frac{\alpha}{m}$

$\gamma + d = -4,5904.\frac{\alpha}{m} - \frac{1,750}{m}.\frac{\alpha}{m}$

Pro loco oculi ſtatuatur

$O = -0,63269.\frac{\alpha}{m} - \frac{O'}{m}.\frac{\alpha}{m}$

erit $O' = 13,431$.

I. Pro lente prima ſtatuatur radius faciei

anter. $= 3,7425.\alpha + F'.\frac{\alpha}{m}$;

poſter. $= 0,6864.\alpha + G'.\frac{\alpha}{m}$

erit $F' = -21,70$; $G' = 1,005$.

II. Pro

II. Pro lente ſecunda ſtatuatur

radius faciei

anter. $= - 1,1853 . \alpha - F' . \frac{\alpha}{m}$;

poſter. $= - 0,5893 . \alpha - G' \frac{\alpha}{m}$

erit $F' = 2,52$; $G' = 0,08$.

III. Pro lente tertia

radius faciei

anter. $= - 3,8769 . \frac{\alpha}{m} - \frac{F'}{m} . \frac{\alpha}{m}$

poſter. $= + 68,6926 . \frac{\alpha}{m} + \frac{G'}{m} . \frac{\alpha}{m}$

erit $F' = - 0,116$; $G' = - 8,096$.

IV. Denique pro quarta lente

radius vtriusque faciei $= - 1,6766 . \frac{\alpha}{m} - \frac{H}{m} . \frac{\alpha}{m}$

erit $H = + 2,862$. ex quibus conficitur ſequens.

Conſtructio Teleſcopii pro multiplicatione quacunque m.

I. Pro prima lente Chryſtall. Glaſſ.

radius faciei

anter. $= 3,7425 . \alpha - 21,70 . \frac{\alpha}{m}$

poſter. $= 0,6864 . \alpha + 1,005 . \frac{\alpha}{m}$

Eius diſtantia focalis $= \alpha$

Semidiameter aperturae $= \frac{m}{50}$ dig.

Interuallum ad lentem ſecundam $= - 0,03 . \alpha$.

II. Pro

II. Pro ſecunda lente. Crown Glaſſ.

radius faciei

anter $= -1,1853\,\alpha - 2,52\,\frac{\alpha}{m}$

poſter. $= -0,5893\,\alpha - 0,08.\,\frac{\alpha}{m}$

Diſtantia focalis $= -0,7427.\,\alpha - 0,\frac{5665\,\alpha}{m}$

Semidiameter aperturae quoque $= \frac{m}{53}$ dig.

Interuallum ad tertiam

$= -2,6622\,\alpha - 2,998.\,\frac{\alpha}{m} + \frac{14,88}{m}.\,\frac{\alpha}{m}.$

III. Pro tertia lente Crown Glaſſ.

radius faciei

anter. $= -3,8769\,\frac{\alpha}{m} + 0,116.\,\frac{\alpha}{mm}$

poſter. $= +68,6926.\,\frac{\alpha}{m} - 8,096.\,\frac{\alpha}{mm}$

Diſtantia focalis $= -7,7539.\,\frac{\alpha}{m} + 0,206.\,\frac{\alpha}{mm}$

cuius pars octaua dat ſemidiametrum aperturae

Interuallum ad quartam

$-4,5904.\,\frac{\alpha}{m} - 1,750.\,\frac{\alpha}{mm}$

At diſtantia imaginis realis poſt hanc lentem

$= \gamma = -3,0087.\,\frac{\alpha}{m} - 5,281.\,\frac{\alpha}{mm}.$

IV. Pro quarta lente Crown Glaſſ.

radius vtriusque faciei $= -1,6766\,\frac{\alpha}{m} - \frac{2,862\,\alpha}{mm}.$

Eius diſtantia focalis $= -1,5817.\,\frac{\alpha}{m} - \frac{2,57.\,\alpha}{mm}.$

cuius pars quarta dat ſemidiametrum aperturae.

Et interuallum ad oculum

$O = -0,6326.\,\frac{\alpha}{m} - 13,43.\,\frac{\alpha}{mm}.$

Hinc

Hinc totius Teleſcopii longitudo erit

$$= -0,03\alpha - 2,998\tfrac{\alpha}{m} + 14,88\tfrac{\alpha}{mm}$$
$$-2,6622.\alpha - 4,590\tfrac{\alpha}{m} - 1,75\tfrac{\alpha}{mm}$$
$$-0,632\tfrac{\alpha}{m} - 13,43\tfrac{\alpha}{mm}$$

$$= -2,6922.\alpha - 8,220\tfrac{\alpha}{m} - 0,30.\tfrac{\alpha}{mm}$$

Semidiameter vero campi apparentis erit $= \frac{1280}{m+1}$ min.

Hic vero notandum eſt, α eſſe negatiuum, cuius valor definietur ex radio poſteriore lentis ſecundae, cuius pars quarta 0, 1473. α ſeu circiter $= \frac{1}{7}.\alpha$ ipſi $\frac{m}{10}$ aequalis poſita dabit $\alpha = -\frac{7.m}{10}$ ſtatui igitur poterit $\alpha = -\frac{m}{7}$.

Corollarium.

281. Si igitur ſtatuamus $\alpha = -\frac{m}{7}$, habebitur ſequens

Conſtructio determinata teleſcopii in ratione $m : 1$ multiplicantis:

I. Pro prima lente. Flint Glaſſ.

radius faciei in digitis

anter. $= -0,5346.m + 3,10.$

poſter. $= -0,0981.m - 0,14.$

Diſtantia focalis $= -\frac{m}{7}$

Semidiameter aperturae $= \frac{m}{10}$

Interuallum $= 0,0043.m.$

II. Pro ſecunda lente. Crown Glaſſ.

radius faciei

anter. $= + 0,1693.m + 0,36$.

poſter. $= + 0,0842.m + 0,01$.

Diſtantia focalis $= + 0,1061.m + 0,080$.

Semidiameter aperturae $= \frac{m}{50}$ dig.

Interuallum $= + 0,3803.m + 0,43 - \frac{2,12}{m}$.

III. Pro tertia lente. Crown Glaſſ.

radius faciei

anter. $= + 0,55 - \frac{0,01}{m}$.

poſter. $= - 9,81 + \frac{1,1}{m}$.

Diſtantia focalis $= + 1,11 - \frac{0,05}{m}$.

Interuallum $= 0,65 + \frac{0,1}{m}$.

Diſtantia imaginis realis poſt hanc lentem

$= 0,43 + \frac{0,7}{m}$.

IV. Pro quarta lente. Crown Glaſſ.

radius vtriusque faciei $= + 0,24 - \frac{0,4}{m}$.

Diſtantia focalis $= 0,22 + \frac{0,5}{m}$.

Semidiameter aperturae $= 0,05 + \frac{0,1}{m}$.

Interuallum vsque ad oculum $= 0,09 - \frac{1,8}{m}$.

et

et tota teleſcopii longitudo

$$= 0,3846.m + 1,18 + \frac{0,26}{m}.$$

et ſemidiameter campi $= \frac{1289}{m+1}$ min.

Scholion.

282. Haec itaque teleſcopia multo fiunt longiora, quam ea, quae praecedente capite ſunt inuenta, pro eadem vtrinque multiplicatione. Hic enim pro multiplicatione $m = 100$ prodit longitudo $= 40$ dig., dum in praecedente capite ſufficiebat longitudo $= 13\frac{1}{3}$ dig. hoc eſt triplo minor. Hic igitur merito quaeritur, vtrum qualitas, qua etiam ſpatium diffuſionis a diuerſa radiorum refrangibilitate oriundae ad nihilum redigitur, tanti ſit habenda, vt longitudo teleſcopii triplicetur, quae quaeſtio non niſi per praxin dijudicari poterit quae autem eo erit difficilior, quo minus accuratiſſimam exſecutionem horum praeceptorum exſpectare licet. Quocirca niſi haec conditio praeſcripta feliciſſime ſuccedat, ſemper praeſtabit, teleſcopia praecedentis capitis praeferre; ita, vt duplici vitri ſpecie carere poſſimus. Ceterum in euolutione caſus ſecundi huiusmodi euolutionibus numericis non immorabimur, cum quia methodus tales calculos tractandi iam ſatis eſt explicata, tum vero quia propoſitum eſt, huic operi ſingularem librum ſubiungere, in quo ſola praecepta pro praxi dirigenda in gratiam artificum colligentur.

 CASVS II.

CASVS II.

283. Sit iam ſecunda lens ex vitro chryſtallino, reliquae vero ex vitro coronario, ita, vt ſit $n' = 1,58$ et $n = n'' = n''' = 1,53$, indeque etiam $N = N'' = N'''$ et ponatur $\frac{N'}{N} = \zeta$, vt ſit ſecundum Dollondum $\zeta = \frac{10}{7}$. His poſitis et ſumto $i = \frac{1}{2}$ erit α poſitiuum ideoque etiam η. Sumatur igitur $\eta = 0,03$, vnde habebimus $P = \frac{1}{1-\eta}$. Quia nunc, vt vidimus, eſt $\mathfrak{B} = \zeta$ proxime, erit

$$\omega = \frac{-\eta}{1-\eta} \cdot \frac{i+1}{(m+1)\zeta} = \frac{-9}{194 \cdot \zeta (m+1)}.$$

Deinde cum ſit proxime $R = -2$, ideoque $Q = \frac{m}{2P}$, vbi $P = +\frac{100}{97}$; adcuratius habebimus ex prima aequatione, $0 = \zeta Q \omega + i + \frac{1}{R}$, quae abit in hanc

$$-\frac{1}{R} = \frac{1}{2} - \frac{9m}{388 \cdot P(m+1)} \text{ ſeu } -\frac{1}{R} = \frac{1}{2} - \frac{9m}{400(m+1)}$$

ex quo etiam Q adcuratius definiri poteſt, cum ſit $Q = \frac{-m}{PR}$.

Vt nunc etiam $\mathfrak{B}$ adcuratius definiatur, colligimus ex aequatione ſecunda,

$$0 = 1 - \frac{\zeta}{\mathfrak{B}P} + \frac{1}{B\mathfrak{C}PQ} - \frac{1}{BCPQR}$$

quae ob $\frac{1}{\mathfrak{B}} = \frac{1}{B} + 1$ dabit

$$-B(\zeta - P) = \zeta - \frac{1}{\mathfrak{C}Q} + \frac{1}{CQR}.$$

Erat autem proxime $B = \frac{-\zeta}{\zeta - 1}$ ideoque negatiuum. Ex valore porro adcurato ipſius B erit $\mathfrak{B} = \frac{B}{B+1}$.

His

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His igitur definitis tertia aequatio erit

$$0 = \lambda - \frac{\mu'.\lambda'}{\mu \mathfrak{B}^3 P} + \frac{\lambda''}{B^3 \mathfrak{C}^3 PQ} - \frac{\lambda'''}{B^3 C^3 PQR}$$
$$- \frac{\mu' \nu'}{\mu B \mathfrak{B} P} + \frac{\nu}{B^3 C \mathfrak{C} PQ}.$$

Hic duo primi termini vtpote valde magni dabunt proxime $\lambda' = \frac{\mu}{\mu'} . \mathfrak{B}^3 P . \lambda$ vnde intelligere licet pro lente concaua eius valorem λ maiorem prodire, quam caſu primo ita, vt caſus primus ad praxin ſit aptior iudicandus.

Vt nunc etiam de longitudine teleſcopii iudicare poſſimus, quia praecedente caſu ea nimis magna prodierat, conſiderari debet eius pars praecipua, quae eſt littera β, cuius valor ante fuerat $= - 2,6622 . \alpha$ circiter, qui autem nunc ob $\beta = \frac{-B\alpha}{P}$ et $B = \frac{-\zeta}{\zeta - P}$ proxime ſeu $B = -\frac{10}{3}$ et $P = 1$ reperitur $\beta = \frac{10}{3} . \alpha = 3,33 . \alpha$, ita, vt teleſcopia hinc nata adhuc fiant longiora, quae propterea hic fuſius euoluere operae non erit pretium.

Scholion.

284. Quia longitudo teleſcopii, quae hinc tanta reſultat, perfectioni non parum obſtat, disquirendum eſt, vtrum huic incommodo non aliquod remedium adferri poſſit, quod autem ex hypotheſi hic facta, qua lens obiectiua quaſi ex duabus lentibus conſtare eſt aſſumta, neutiquam ſperare licet. Quamobrem lentem obiectiuam quaſi ex tribus lentibus conſtantem aſſumi conueniet, quarum vel vna vel duae

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ſint concauae et ex vitro chryſtallino formatae. Ne autem nouam inueſtigationem circa maiorem campum ſuſcipere cogamur, hic ſtatim quoque tres lentes quaſi oculares introducamus, vt hoc modo ſi negotium ſucceſſerit, non ſolum breuiora teleſcopia obtineantur, ſed etiam ſimul campus apparens notabile incrementum accipiat.

Problema 2.

285. Si tres lentes priores ad obiectiuam conſtituendam referantur, tum vero tres quaſi lentes oculares adiungantur, definire momenta, ad teleſcopii conſtructionem neceſſaria.

Solutio.

Cum igitur hic occurrant ſex lentes, ſtatuamus $\frac{\alpha}{b} = -P$; $\frac{\beta}{c} = -Q$; $\frac{\gamma}{d} = -R$; $\frac{\delta}{e} = -S$; $\frac{\varepsilon}{f} = -T$. vt ſint noſtra elementa

$$b = -\frac{\alpha}{P};\ c = \frac{B\alpha}{PQ};\ d = \frac{-BC\alpha}{PQR};\ e = \frac{BCD\alpha}{PQRS}$$

$$\beta = \frac{-B\alpha}{P};\ \gamma = \frac{BC\alpha}{PQ};\ \delta = \frac{-BCD\alpha}{PQR};\ \varepsilon = \frac{BCDE\alpha}{PQRS}$$

et $f = \frac{-BCDE\alpha}{PQRST}$

adeoque diſtantiae focales

$$p = \alpha;\ q = \frac{-\mathfrak{B}\alpha}{P};\ r = \frac{B\mathfrak{C}\alpha}{PQ};\ s = \frac{-BC\mathfrak{D}\alpha}{PQR};$$

$$t = \frac{BCD\mathfrak{E}\alpha}{PQRS};\ u = \frac{-BCDE\alpha}{PQRST};$$

exiſtente $m = -PQRST$.

Horum

Horum iam numerorum P, Q, R, S, T vnicus debet eſſe negatiuus; interualla autem lentium ita exprimentur

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right); \quad \beta + c = \frac{-B\alpha}{P}\left(1 - \frac{1}{Q}\right)$$

$$\gamma + d = \frac{BC\alpha}{PQ}\left(1 - \frac{1}{R}\right); \quad \delta + e = \frac{-BCD\alpha}{PQR}\left(1 - \frac{1}{S}\right)$$

$$\text{et } \varepsilon + f = \frac{BCDE\alpha}{PQRS}\left(1 - \frac{1}{T}\right)$$

quod vltimum interuallum ob $\varepsilon = -Tf$, etiam ita exhibetur $\varepsilon + f = f(1 - T)$; vnde quia f debet eſſe quantitas poſitiua, ſtatim patet, eſſe debere $T < 1$.

Quia nunc tres priores lentes exiguis interuallis a ſe inuicem diſtare debent, ſtatuamus vtrumque interuallum $= \eta\alpha$ hincque habebimus

$$P = \frac{1}{1-\eta} \text{ et } Q = \frac{B}{B + \eta P}.$$

Nunc vero cum ſit campi ſemidiameter

$$\Phi = \frac{\pi - \pi' + \pi'' - \pi''' + \pi''''}{m+1}$$

ſtatuamus $\pi = \omega\xi$; $\pi' = -\omega'\xi$; $\pi'' = i\xi$; $\pi''' = -\xi$; et $\pi'''' = \xi$, ita, vt ſit

$$\Phi = \frac{\omega + \omega' + i + 2}{m+1}\xi = M\xi, \text{ exiſtente}$$

$$M = \frac{\omega + \omega' + i + 2}{m+1} \text{ ſeu proxime } M = \frac{i+2}{m+1}$$

ob ω et ω' fractiones minimas, vt mox videbimus. Hincque ob $\frac{\pi''''}{\Phi} = \frac{1}{M}$, ſtatim oritur diſtantia oculi

$$O = \frac{1}{M} \cdot \frac{r}{m} = \frac{m+1}{m(i+2)} \cdot r.$$

Porro

Porro conſiderari oportet ſequentes aequationes

$$\mathfrak{B}\,\omega = (1-P)\,M;\ \mathfrak{C}\,\omega' = (1-PQ)\,M-\omega$$
$$\mathfrak{D}\,i = (1-PQR)\,M-\omega'-\omega$$
$$\mathfrak{E} = (1-PQRS)\,M-\omega'-\omega-i.$$

Ex harum aequationum duabus primis quaerantur particulae ω et ω', ſcilicet

$$\omega = \frac{(1-P)M}{\mathfrak{B}} = \frac{-\eta M}{(1-\eta)\mathfrak{B}}$$
$$\omega' = \frac{(1-PQ)M-\omega}{\mathfrak{C}} \text{ ſeu}$$
$$\omega' = \frac{\eta(1-B).M}{(B+(1-B)\eta)\mathfrak{C}} + \frac{\eta M}{(1-\eta)\mathfrak{B}\mathfrak{C}} \text{ ſeu}$$
$$\omega' = \frac{\eta M}{\mathfrak{C}}\left(\frac{2B+\eta(1-B)}{(B+\eta(1-B))(1-\eta)B}\right.$$

quare cum η ſit fractio valde parua erit proxime

$$\omega = \frac{-\eta M}{\mathfrak{B}};\ \omega' = \frac{2\eta M}{B\mathfrak{C}}$$

atque litterae $\mathfrak{B}$ et $\mathfrak{C}$ etiamnum manent indeterminatae, dum ſequentes $\mathfrak{D}$ et $\mathfrak{E}$ per formulas hic allatas determinantur. Nunc igitur conſiderentur tres aequationes, quas adimpleri oportet

$$\text{I. } 0 = +\frac{N'\omega}{P} + \frac{N''\omega'}{PQ} + \frac{N'''i}{PQR} + \frac{N''''}{PQRS} + \frac{N'''''}{PQRST}$$
$$\text{II. } 0 = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ} - \frac{N'''}{BC\mathfrak{D}PQR}$$
$$+ \frac{N''''}{BCD\mathfrak{E}.PQRS} - \frac{N'''''}{BCDE\mathfrak{F}PQRST}$$

III. o =

$$\text{III.}\quad 0 = \mu\lambda - \frac{\mu'}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu'}{B}\right)$$
$$+ \frac{\mu''}{B^3\mathfrak{C}PQ}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu''}{C}\right)$$
$$- \frac{\mu'''}{B^3C^3\mathfrak{D}PQR}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu'''}{D}\right)$$
$$+ \frac{\mu''''}{B^3C^3D^3.\mathfrak{E}.PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^2} + \frac{\nu''''}{E}\right)$$
$$- \frac{\mu'''''.\lambda'''''}{B^3C^3D^3E^3.PQRST}.$$

In prima autem aequatione duo membra priora prae ſequentibus manifeſto ſunt valde parua, dum etiam per m multiplicata adhuc multo minora manent ſequentibus. Iis omiſſis habebitur

$$0 = N'''' i + \frac{N''''}{S} + \frac{N'''''}{ST}$$

vbi quia tres poſteriores lentes ex eadem vitri ſpecie fieri conuenit erit $0 = iST + T + 1$; vnde patet, vel S vel T eſſe debere quantitatem negatiuam. Hanc ob rem ſtatuatur, $S = -k$ vt vnica harum lentium ante imaginem realem cadat, vnde fiet $k = \frac{T+1}{iT}$; ante autem iam vidimus, eſſe $T < 1$ ideoque $K > 2$ ob $i < 1$ et ſi $i = \frac{1}{2}$ fiet adeo $K > 4$ ex quo erit

$$KT = \frac{1+T}{i} = -ST.$$

Iam ob P et Q vnitati proxime aequales erit quoque proxime $RST = -m$ ideoque $R = \frac{im}{1+T}$ adeoque numerus magnus. Ex his autem valoribus proximis facile erit valores adcuratos ex eadem aequatione prima deducere. Iam in aequatione ſecunda ſatis euidens eſt, tres terminos priores multo maiores eſſe ſequentibus.

His ergo omiſſis habebitur

$$o = N - \frac{N'}{\mathfrak{B}P} + \frac{N''}{B\mathfrak{C}PQ}$$

vnde ob P et Q proxime $= 1$, colligitur fore etiam proxime

$$o = N - \frac{N'}{\mathfrak{B}} + \frac{N''}{B\mathfrak{C}}$$

vnde colligitur

$$\mathfrak{C} = \frac{N''.\mathfrak{B}}{B(N' - N\mathfrak{B})} = \frac{N''(1-\mathfrak{B})}{N' - N\mathfrak{B}} \text{ hincque}$$

$$C = \frac{N''(1-\mathfrak{B})}{N' - N\mathfrak{B} - N'' + N''\mathfrak{B}}$$

$$C = \frac{N''(1-\mathfrak{B})}{N' - N'' + \mathfrak{B}(N'' - N)}$$

vbi littera $\mathfrak{B}$ adhuc arbitrio noſtro permittitur.

Pro $\mathfrak{C}$ autem ſequentes caſus ſunt notandi;

1°. Si $\mathfrak{B} > 1$ et $\mathfrak{B} < \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negatiuum, adeoque etiam C negatiuum.

2°. Si $\mathfrak{B} > 1$ et $\mathfrak{B}$ ſimul $> \frac{N'}{N}$; tunc erit $\mathfrak{C}$ poſitiuum.

3°. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} < \frac{N'}{N}$; tunc erit $\mathfrak{C}$ poſitiuum.

4°. Si fuerit $\mathfrak{B} < 1$ et $\mathfrak{B} > \frac{N'}{N}$; tunc erit $\mathfrak{C}$ negatiuum; adeoque et C negatiuum.

Pro ſecundo autem caſu erit C negatiuum ſi $N' > N$. Sin autem fuerit $N' < N$, erit C poſitiuum. Pro caſu tertio erit C poſitiuum, ſi ſit $N' > N$; ſin autem ſit $N' < N$, erit C negatiuum.

Circa

Circa duas autem reliquas litteras $\mathfrak{D}$ et $\mathfrak{E}$ notandum eſt, fore $\mathfrak{D}$ negatiuum, ideoque et D, tum vero $\mathfrak{E}$ eſſe poſitiuum; fit enim proxime

$$\mathfrak{E} = \frac{i+1}{i} - i. \text{ ideoque}$$

$$E = \frac{\frac{i+1}{i} - i}{1 + i - \frac{i+1}{i}}; \text{ ideoque E negatiuum.}$$

Examinemus iam interualla lentium ac primo quidem cum ſit $d < \gamma$ neceſſe eſt, vt ſit γ poſitiuum ideoque debet eſſe $BC\alpha > 0$.

Eſt vero $BC = \frac{N''\mathfrak{B}}{N' - N'' + \mathfrak{B}(N'' - N)}$.

Cum igitur diſtantia γ maximam partem totius longitudinis contineat, $\mathfrak{B}$ ita accipi debet, vt quantitas B C vnitatem vix ſuperet, quia ante hic coefficiens modo binarium tum vero et ternarium ſuperaverat.

Statim autem ac γ reddita eſt quantitas poſitiua, erit d negatiuum et manifeſto $\gamma + d > 0$.

Porro vero ſit δ poſitiuum, nempe $\delta = Dd$ atque etiam e poſitiuum; ita, vt ſit $\delta + e$ poſitiuum.

Denique $\varepsilon = Ee$ adeoque negatiuum et f poſitiuum; eritque etiam $\varepsilon + f > 0$ ob $T < 1$, vt iam ante notauimus; ex quo etiam diſtantia oculi ſit poſitiua, ſicque omnes conditiones ſunt adimpletae.

Denique in tertia aequatione etiam manifeſtum eſt, tres tantum terminos priores potiſſimum in com-

putum venire, vtpote prae reliquis multo maiores; vnde fit

$$o = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}} + \frac{\mu''\lambda''}{B\,\mathfrak{C}}$$

vbi efficiendum quoque eſt, vt nulla litterarum λ, λ', λ'' vnitatem notabiliter ſuperet. Quemadmodum igitur haec omnia commodiſſime praeſtentur, diuerſos caſus euolui oportet, prouti inter tres lentes priores vel vna vel duae chryſtallinae occurrant et quo loco.

CASVS I.

286. Quo prima lens chryſtallina, ſecunda et tertia vero ex vitro coronario eſt parata. Erit ergo $N = 10$, $N' = 7$, et $N'' = 7$. adeoque primo

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ et } C = \frac{-7(1-\mathfrak{B})}{3\mathfrak{B}} \text{ ideoque}$$

$$B\,\mathfrak{C} = \frac{7\mathfrak{B}}{7-10\mathfrak{B}} \text{ et } B\,C = -\tfrac{7}{3}$$

hinc ergo fit $\gamma = -\frac{7}{3}\alpha$; vnde euidens eſt, ſumi debere α negatiuum, qui valor cum non minor ſit, quam in problemate praecedente; hunc caſum vlterius non proſequimur.

CASVS II.

287. Quo lens ſecunda eſt chryſtallina; prima vero et tertia ex vitro coronario; erit $N = 7$, $N' = 10$; $N'' = 7$; atque hinc

$$C = \frac{7(1-\mathfrak{B})}{3} \text{ adeoque } B\,C = \tfrac{7}{3}\mathfrak{B}$$

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ et } B\,\mathfrak{C} = \frac{7\mathfrak{B}}{10-7\mathfrak{B}}$$

vnde

vnde fit $\gamma = \frac{7}{3} \mathfrak{B} . \alpha$. Quo igitur teleſcopium contrahatur, ſumi debet $\mathfrak{B} < 1$; tum vero erit B poſitiuum; at ex tertia aequatione habebimus

$$0 = \mu \lambda - \frac{\mu' \lambda'}{\mathfrak{B}^3} + \frac{\mu'' \lambda''}{B\, \mathfrak{C}^3} \text{ ſiue}$$

$$0 = \mu \lambda - \frac{\mu' \lambda'}{\mathfrak{B}^3} + \frac{\mu \lambda'' (10 - 7 \mathfrak{B})^3}{7^3 . \mathfrak{B}^3} \text{ vnde fit}$$

$$\frac{\mu' \lambda'}{\mu} = \mathfrak{B}^3 \lambda + \frac{(10 - 7 \mathfrak{B})^3 . \lambda''}{7^3}$$

hic ergo videamus, an litterae λ et λ' et λ'' poſſint ad vnitatem redigi; ad quod requiritur, vt ſit

$$\frac{\mu'}{\mu} = \mathfrak{B}^3 + \left(\frac{10}{7} - \mathfrak{B}\right)^3$$

cuius euolutio ob $\frac{\mu'}{\mu}$ propemodum $= 1$ dat

$$\left(\frac{10}{7}\right)^3 - 3 \left(\frac{10}{7}\right)^2 \mathfrak{B} + 3 \left(\frac{10}{7}\right) \mathfrak{B}^2 = 1$$

cuius radices ſunt prior $\mathfrak{B} = 0,97$; qua autem hic nihil in longitudine lucramur; altera vero eſt $\mathfrak{B} = 0,46$ ſiue $\mathfrak{B} = \frac{5}{11}$ hocque modo fiet $\gamma = \frac{7}{9} \alpha$; quod inſigne lucrum eſt qui ergo caſus inprimis meretur, vt adcuratius euoluatur.

CASVS III.

288. Sit nunc tertia lens chryſtallina, prima et ſecunda ex vitro coronario, vt ſit $N = 7$, $N' = 7$, at $N'' = 10$. Hinc igitur ſequitur

$$\mathfrak{C} = \frac{10(1 - \mathfrak{B})}{7 - 7 \mathfrak{B}} = \frac{10}{7}; \; C = -\frac{10}{3} \text{ indeque}$$

$$B \mathfrak{C} = \frac{10}{7} B; \text{ et } B C = -\frac{10}{3} B$$

vnde fit $\gamma = -\frac{10}{3} B \alpha$. Deberet ergo eſſe $- B < 1$

 vel

vel $B > -1$. hinc $\mathfrak{B} < 1$. Hinc ergo tertia aequatio vnitate loco cuiuslibet λ ſcripta erit

$$0 = 1 - \frac{1}{\mathfrak{B}^3} + \frac{343(1-\mathfrak{B})^3}{1000\,\mathfrak{B}^3} \text{ ſeu}$$

$$0 = \mathfrak{B}^3 - 1 + \frac{343}{1000}(1-\mathfrak{B})^3 \text{ ſeu}$$

$$0 = 657 - 1029\,\mathfrak{B} + 1029\,\mathfrak{B}^2 + 657\,\mathfrak{B}^3$$

qui ergo caſus etiam euolutione dignus videtur.

CASVS IV.

289. Si prima et ſecunda lens ſint chryſtallinae et tertia coronaria; erit

$$N' = N = 10; \text{ et } N'' = 7; \text{ hincque}$$

$$\mathfrak{C} = \frac{7(1-\mathfrak{B})}{10(1-\mathfrak{B})} = \frac{7}{10} \text{ et } C = \frac{7}{3} \text{ hincque } BC = \frac{7B}{3}.$$

Nunc quia eſt α negatiuum, γ autem poſitiuum, debet eſſe $\gamma = + \frac{7}{3} B \alpha$. Ponamus nunc $\frac{7}{3} B = -1$ erit $B = -\frac{3}{7}$ et $\mathfrak{B} = -\frac{3}{4}$ et $B\mathfrak{C} = -\frac{3}{10}$ vnde tertia aequatio fiet

$$0 = \lambda + \frac{\lambda'.4^3}{3^3} - \frac{\mu''}{\mu}.\frac{\lambda''.10^3}{3^3}$$

ita, vt eſſe debeat

$$\lambda + \frac{64\lambda'}{27} = \frac{\mu''}{\mu}.\frac{1000\lambda''}{27} + \text{etc.}$$

quod cum fieri nequeat, niſi pro λ et λ' numeri maximi accipiantur, hic caſus nullo modo in praxi admitti poteſt.

CASVS V.

290. Sint prima et tertia lens chryſtallinae, ſecunda ex vitro coronario, erit $N = N'' = 10$

et

et $N' = 7$, adeoque

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{7-10\mathfrak{B}} \text{ et } C = -\frac{10(1-\mathfrak{B})}{3} \text{ hinc}$$

$$BC = -\tfrac{10}{3}\mathfrak{B} \text{ et } \gamma = -\tfrac{10}{3}\mathfrak{B}\alpha.$$

Ponatur ergo $\frac{10}{3}\mathfrak{B} = 1$ eritque $\mathfrak{B} = \frac{3}{10}$ et $B = \frac{3}{7}$ et $B\mathfrak{C} = \frac{3}{4}$; vnde aequatio tertia dabit

$$0 = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda' \cdot 1000}{27} + \frac{61\lambda''}{27} \text{ ſeu}$$

$$\lambda + \frac{61\lambda''}{27} = \frac{\mu'}{\mu} \cdot \frac{1000\lambda'}{27} + \text{etc.}$$

quae aeque parum ad praxin eſt idonea.

CASVS VI.

291. Sint ſecunda et tertia lens chryſtallinae, prima ex vitro coronario confecta; erit $N = 7$, et $N' = N'' = 10$, vnde colligitur

$$\mathfrak{C} = \frac{10(1-\mathfrak{B})}{10-7\mathfrak{B}} \text{ et } C = \frac{10(1-\mathfrak{B})}{3\mathfrak{B}}; \text{ hincque}$$

$$BC = \tfrac{10}{3} \text{ et } \gamma = \tfrac{10}{3}\alpha,$$

qui ergo caſus iam ſponte cadit.

Evolutio vlterior caſus ſecundi.

292. Quod ad valorem litterae η attinet, pro quauis multiplicatione m, quae lentis obiectiuae aperturam poſtulat, cuius ſemidiameter ſit circiter $\frac{m}{50}$ dig. ſumamus accipi $\alpha = \frac{m}{5}$ dig. quia lens plerumque fere eſt plano-conuexa; eritque huius lentis craſſities circiter $\frac{1}{24}\alpha$, quare ſi interuallum binarum lentium priorum ſtatuamus $\frac{1}{50}\alpha$; metuendum non eſt, ne duae lentes ſe mutuo tangant, ſed ſatis relinquetur ſpatii, vt etiam

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etiam quodammodo moueri poſſint. Ponamus ergo $\eta = \pm \frac{1}{50} = \pm 0,02$.

Quia nunc prima lens eſt ex vitro coronario, ideoque conuexa erit α poſitiuum et $\eta = + \frac{1}{50} = + 0,02$. Hinc reperimus ſtatim $P = \frac{50}{49}$ et $Q = \frac{45 \cdot B}{45 B + 1}$, vbi de B infra diſpiciemus. Hic notaſſe ſufficiat, eſſe proxime $Q = 1$ et $P = 1$.

His praemiſſis ſumta fractione $i = \frac{1}{2}$ et $T = \frac{1}{2}$, quandoquidem eſſe debet $T < 1$. prima aequatio noſtra dabit $k = 6 = - S$ et ob $PQ = 1$ proxime $R = \frac{1}{3} m$; neque opus eſt, vt hic valor accuratius eruatur.

Secunda autem aequatio, cui pariter proxime tantum ſatisfeciſſe ſufficit, quia hoc caſu eſt $N = 7$, $N' = 10$, $N'' = N''' = N'''' = N''''' = 7$. nobis praebet

$$0 = 7 - \frac{10}{\mathfrak{B} P} + \frac{7}{B \mathfrak{C} P Q}.$$

quae ſumto $P = 1$ et $PQ = 1$ dat

$$\mathfrak{C} = \frac{-7(1 - \mathfrak{B})}{(7 \mathfrak{B} - 10)} = \frac{7(1 - \mathfrak{B})}{10 - 7 \mathfrak{B}} \text{ indeque}$$
$$C = \frac{7(1 - \mathfrak{B})}{3} \text{ et } BC = \frac{7 \cdot \mathfrak{B}}{3}.$$

Cum dein ex primis elementis ſit $\gamma = \frac{BC\alpha}{PQ}$, quae diſtantia praecipuam partem totius longitudinis continet, faciamus $\gamma = \alpha$ ſiue proxime ſaltem $= \alpha$ adeoque $BC = 1$; vnde ſequitur $\frac{7}{3} \mathfrak{B} = 1$ et $\mathfrak{B} = \frac{3}{7}$; vnde porro $B = \frac{3}{4}$; $\mathfrak{C} = \frac{4}{7}$ et $C = \frac{4}{3}$ ideoque $B \mathfrak{C} = \frac{3}{7}$.

Nunc

Nunc ex valore B inuento habebimus praeter $P = \frac{60}{59}$ etiam $Q = \frac{147}{151}$ et $PQ = \frac{150}{151}$ ſicque adcurate iam erit $R = \frac{151.m}{3.150}$.

Nunc igitur ad aequationem tertiam progrediamur:

$$
\begin{aligned}
0 = \lambda - \frac{\mu'}{\mu} \cdot \frac{\lambda'}{\mathfrak{B}^3 P} &+ \frac{\lambda''}{B^3 \mathfrak{C}^3 PQ} \\
- \frac{\mu' \nu'}{\mu} \cdot \frac{1}{B \mathfrak{B} P} &+ \frac{\nu}{B^3 C \mathfrak{C} PQ} \\
- \frac{\lambda'''}{B^3 C^3 \mathfrak{D}^3 PQR} &+ \frac{\lambda''''}{B^3 C^3 D^3 \mathfrak{E}^3 PQRS} \\
- \frac{\nu}{B^3 C^3 D \mathfrak{D} PQR} &+ \frac{\nu}{B^3 C^3 D^3 E \mathfrak{E} PQRS} \\
+ \frac{\lambda'''''}{B^3 C^3 D^3 E^3 . m}
\end{aligned}
$$

quae vt in numeros euolui poſſit, ante neceſſe eſt, valores litterarum D et E inueſtigare, qui ex formulis ſupra datis inueniuntur:

$$\tfrac{1}{3} \mathfrak{D} = (1 - PQR) M; \text{ ſeu}$$

$$\mathfrak{D} = (1 - \tfrac{1}{3} m) \frac{-5}{m+1} = - \tfrac{5}{3}$$

ob m vt valde magnum aſſumtum praecipue cum $\mathfrak{D}$ tantum occurrat in numeris per ſe minimis; adeoque $D = - \frac{5}{8}$. Simili modo erit

$$\mathfrak{E} = \frac{5(1+2m)}{2(m+1)} - \tfrac{1}{2} = \tfrac{9}{2} \text{ et } E = \tfrac{-9}{7}.$$

His

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His igitur valoribus ſubſtitutis habebimus

$$0 = \lambda - \underset{(1,0413348)}{10,9985\,\lambda'} + \underset{(1,1068156)}{12,7884.\,\lambda''}$$

$$- \underset{(9,8332690)}{0,68119} + \underset{(9,8374334)}{0,68775}$$

$$+ \underset{(9,8115752)}{\frac{0,64800.\,\lambda'''}{m}} + \underset{(8,3516924)}{\frac{0,02247.\,\lambda''''}{m}}$$

$$- \underset{(9,8010248)}{\frac{0,363245}{m}} + \underset{(8,8906053)}{\frac{0,07773}{m}}$$

$$+ \underset{(0,2849264)}{\frac{1,92720\,\lambda'''''}{m}}$$

ex qua aequatione ſi ſumatur $\lambda = 1$ et $\lambda'' = 1$, colligitur

$$\lambda' = 0,09092 + 0,0589\,\lambda''' : m - \frac{0,05044}{m}$$

$$+ 1,16275 + 0,0020\,\lambda'''' : m$$

$$\underline{+ 0,00060} + 0,1752\,\lambda''''' : m$$

$$1,25427$$

Circa litteras λ''', λ'''', λ''''' obſeruandum eſt, quia binae poſtremae plenam aperturam admittere debent, eſſe debere

$$\lambda''''' = 1,60006 \text{ et } \lambda'''' = 1 + 0,60006.$$

$$\text{et } \lambda'''' = 1 + 0,60006\,(1 - 2\,\mathfrak{E})^2$$

$$= 1 + 0,60006.\,64$$

vnde hae duae lentes ſtatim computari poſſunt.

Pro

Pro quarta autem lente in ipſo radiorum calculo valor numeri λ''' definiatur. Tum vero lens prima et tertia quoque per calculum determinantur. Quo facto quaeratur valor ipſius λ', qui cum etiam m inuoluat, primo pro valore determinato ipſius m. v. g. $m = 25$, deinde pro $m = \infty$ radii facierum huius lentis inueſtigentur ex iisque pro multiplicatione quacunque eorum valores concludantur, vt iam ſupra aliquoties eſt factum.

Interualla autem lentium cum diſtantiis focalibus ſequenti modo ſe habebunt:

$$b = -0,98.\,\alpha$$

$$\beta = -0,73500.\,\alpha \quad \log.\,\beta = 9,8662874;$$

$$c = 0,75500.\,\alpha;$$

$$\gamma = 1,00667.\,\alpha. \quad \log.\,\gamma = 0,0028856$$

$$d = -\tfrac{3\alpha}{m}$$

$$\delta = +\tfrac{1,875}{m}.\,\alpha$$

$$e = +\tfrac{0,3125.\,\alpha}{m}$$

$$\varepsilon = -\tfrac{0,40178.\,\alpha}{m}$$

$$f = +\tfrac{0,80357.\,\alpha}{m}$$

et diſtantiae focales

$$p = \alpha; \; q = -0,42000.\,\alpha; \; r = 0,43144$$

$$s = \tfrac{5\alpha}{m}; \; t = +\tfrac{1,40625.\,\alpha}{m}; \; u = \tfrac{0,80357}{m}.\,\alpha$$

Hinc-

Hincque interualla lentium

$$\alpha + b = 0,02.\alpha;\ \beta + c = 0,0200.\alpha$$

$$\gamma + d = 1,00667.\alpha - \frac{s}{m}.\alpha$$

$$\delta + e = 2,1875.\frac{\alpha}{m};\ \varepsilon + f = \frac{0,40170.\alpha}{m}$$

et pro loco oculi $O = 0,5626.\frac{\alpha}{m}$.

Conſtructio lentium.

Inueſtigemus primo conſtructionem pro ſingulis lentibus ex vitro coronario parandis poſitisque pro quauis lente

radiis faciei $\begin{cases} \text{anter.} = F. \\ \text{poſter.} = G. \end{cases}$

haec determinatio ſequenti modo ſe habebit

I. Pro prima lente

ob $\lambda = 1$ reperietur

$$F = \frac{\alpha}{\sigma} = \frac{\alpha}{1,66011} = 0,60237.\alpha$$

$$G = \frac{\alpha}{\varrho} = \frac{\alpha}{0,22670} = 4,41111.\alpha$$

III. Pro tertia lente.

ob $\lambda'' = 1$ erit

$$F = \frac{c\gamma}{\gamma\varrho + c\sigma} = \frac{Cc}{C\varrho + \sigma} = \frac{\gamma}{C\varrho + \sigma}$$

$$G = \frac{c\gamma}{\gamma\sigma + c\varrho} = \frac{Cc}{C\sigma + \varrho} = \frac{\gamma}{C\sigma + \varrho}$$

$$F = \frac{\gamma}{1,95624} = 0,51298.\alpha$$

$$G = \frac{\gamma}{2,44601} = 0,41255.\alpha$$

IV.

IV. Pro quarta lente

ob λ''' etiamnunc incognitum ponatur breuitatis gratia

$$\tau(1+D)\sqrt{(\lambda'''-1)} = x \text{ eritque}$$

$$F = \frac{\delta}{D\varrho+\sigma \pm x};\; G = \frac{\delta}{\varrho+D\sigma \mp x}$$

adeoque

$$F = \frac{\delta}{1,5184 \pm x};\; G = \frac{\delta}{-0,8109 \mp x}$$

Vt nunc haec lens aperturam $\frac{1}{2}\xi$ admittat, hoc eueniet, ſi poſterior facies fuerit plana, ſeu denominator $= 0$; valeant igitur ſigna inferiora et ponatur

$$x = 0,8109 \text{ vnde fiet}$$

$$G = \infty \text{ et } F = \frac{\delta}{0,7075} \text{ ſeu } F = \frac{2,650.\alpha}{m}$$

vti debet eſſe, quia $F = (n-1)5$. Cum igitur ſit

$$\tau(1+D)\sqrt{(\lambda'''-1)} = 0,8109 \text{ erit}$$

$$\sqrt{\lambda''-1} = \frac{0,8109}{0,3469} \text{ hincque } \lambda''' = 6,4642.$$

V. Pro quinta lente

eſt $\lambda'''' = 39,40384$ et quia haec lens eſt vtrinque aeque conuexa erit

radius faciei vtriusque $= 1,06.\, t = 1,4906.\frac{\alpha}{m}$.

VI. Pro ſexta lente

eſt, vti vidimus, $\lambda''''' = 1,60006$ ideoque

radius vtriusque faciei $= 1,06.\, u = 0,8518.\frac{\alpha}{m}$.

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II. Pro ſecunda lente

reperietur nunc primo

$$\lambda' = 1,25427 + \frac{0,6\,94}{m}.$$

Statuamus nunc eſſe $m = 25$ eritque

$$\lambda' = 1,28184.$$

Quare cum pro ſecunda lente ſit

$$F = \frac{\beta}{B\varrho' + \sigma' \pm \tau'(1+B)\surd(\lambda'-1)}$$

$$G = \frac{\beta}{B\sigma' + \varrho' + \tau'(1+B)\surd(\lambda'-1)}$$

erit $\tau'(1+B)\surd\overline{\lambda'-1} = 0,81524$

vnde colligitur

$$F = \frac{\beta}{1,6888 \pm 0,81524} = \frac{\beta}{0,8736}$$

$$G = \frac{\beta}{1,3284 + 0,81524} = \frac{\beta}{2,1436}$$

hinc $F = -0,84134 . \alpha$

$G = -0,34286 . \alpha$

ſit nunc $m = \infty$ erit

$\lambda' = 1,25427$ et $\tau(1+B)\surd\overline{\lambda'-1}$

$= 0,77434$, vnde radii facierum

$$F = \frac{\beta}{1,6888 \pm 0,7743} = \frac{\beta}{0,9145}$$

$$G = \frac{\beta}{1,3284 + 0,7743} = \frac{\beta}{2,1027}$$

hinc

hinc $F = -0,80373 . \alpha$

$G = -0,34955 . \alpha$

Ex his igitur duobus caſibus pro multiplicatione quacunque concludimus

$$F = -0,80373 . \alpha - \frac{f}{m}$$

$$F = -0,80373 . \alpha - 0,940 \frac{\alpha}{m}$$

et $G = -0,34955 \, \alpha - \frac{g}{m}$

$$G = -0,34955 \, \alpha + 0,167 . \frac{\alpha}{m}$$

Denique ſemidiameter campi viſi erit

$$\Phi = \frac{2147}{m+1} \text{ minut.}$$

Scholion.

293. Quia ternae lentes priores communem poſtulant aperturam, cuius ſemidiameter ſit $\frac{m}{50}$ dig. hic ad radium minimum iſtarum lentium, qui eſt $0,343 . \alpha$; reſpici debet, cuius pars quarta $0,086 . \alpha$ eſt circiter $\frac{1}{12} \alpha$ ipſi $\frac{m}{50}$ dig. aequalis poſita dabit $\alpha = \frac{6}{25} m$ ſiue $\alpha = \frac{1}{4} m$ cum ante licuiſſet ſtatuere $\alpha = \frac{1}{7} m$ neque ergo voti noſtri compotes ſumus facti, dum longitudinem teleſcopii contrahere ſumus conati, etſi enim hic longitudo teleſcopii minorem tenet rationem ad α, tamen ipſa quantitas α fere tanto maior hic prodiit. Ex quo intelligitur, ſi omnes plane perfectiones deſideremus, neceſſe prorſus eſſe, maiorem longitudinem admittere. Interim tamen longitudo hinc

hinc reſultans aliquanto minor eſt, quam ſupra inuenta, ſed probe hic eſt perpendendum, hoc caſu elaborationem lentium multo maioribus difficultatibus eſſe obnoxiam, quam ante. Ita vt artifex non niſi poſt plurima tentamina ſcopum attingere poſſit. Quocirca his inueſtigationibus non vlterius immoror, cum ex calculis allatis facile ſit huiusmodi teleſcopiorum conſtructionem in vſum artificum depromere.

LIBRI

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LIBRI SECVNDI,

DE

CONSTRVCTIONE TELESCOPIORVM

SECTIO TERTIA.

DE

TELESCOPIIS TERTII GENERIS,

QVIBVS

OBIECTA ITERVM SITV ERECTO REPRAESENTANTVR.

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CAPVT I.

DE
TELESCOPIIS SIMPLICIORIBUS
TERTII GENERIS EX VNICA VITRI
SPECIE PARATIS.

Problema 1.

294.

Teleſcopium ſimpliciſſimum huius generis, quod tribus tantum conſtat lentibus conſtruere, quod obiecta ſecundum datam rationem aucta et ſitu erecto repraeſentet.

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Solutio.

Pro duobus interuallis, quae hic occurrunt, ponamus vt ſemper fractiones $\frac{\alpha}{b} = -P$ et $\frac{\beta}{c} = -Q$ et quia hic duae imagines reales habentur, quarum altera in prius interuallum cadens eſt inuerſa, altera vero in poſterius interuallum cadens erecta, ita, vt ſit ſemidiameter illius $= \alpha \Phi$, huius vero $= B \alpha \Phi$; ambae litterae P et Q debent eſſe negatiuae, vnde ſtatuamus $-P = k$ et $-Q = k'$, vt ſit multiplicatio $m = k k'$. Hinc elementa noſtra ita ſe habebunt:

$$b = \frac{\alpha}{k};\ \beta = \frac{B\alpha}{k} \text{ et } c = \frac{B\alpha}{kk'}$$

et diſtantiae focales

$$p = \alpha;\ q = \frac{\mathfrak{B}\alpha}{k} \text{ et } r = \frac{B\alpha}{kk'} = \frac{B\alpha}{m}$$

tum vero bina interualla

$$\alpha + b = \alpha\left(1 + \frac{1}{k}\right);\ \beta + c = \frac{B\alpha}{k}\left(1 + \frac{1}{k'}\right)$$

quae per ſe ſunt poſitiua, ſiquidem eſſe debet $B > 0$ ideoque et $\mathfrak{B}$. Pro campo porro apparente cum ſit eius ſemidiameter $\Phi = \frac{-\pi + \pi'}{m-1}$; ponamus $\pi = -i\xi$ et $\pi' = \xi$, denotante ξ maximum valorem, quem litterae π et π' recipere poſſunt et i fractionem vnitate minorem eritque $\Phi = \frac{i+1}{m-1} . \xi$ atque hinc pro loco oculi fiet

$$O = \frac{\pi'}{\Phi} \cdot \frac{r}{m} = \frac{m-1}{i+1} \cdot \frac{B\alpha}{mm}$$

quae

quae diſtantia etiam per ſe eſt poſitiua. His poſitis aequationes pro litteris π, π' ſupra datae dabunt

$$\frac{\mathfrak{B}\pi - \Phi}{\Phi} = \frac{a}{b} = k; \text{ ſeu}$$

$$\mathfrak{B}. \frac{-i(m-1)}{i+1} = k + 1 \quad \text{vnde}$$

$$i = \frac{-k-1}{k+1+(m-1)\mathfrak{B}}$$

qui valor debet eſſe vnitate minor. Cum igitur hinc valor ipſius i neceſſario ſit negatiuus et vnitate minor, erit campi ſemidiameter

$$\Phi = \frac{\mathfrak{B}\xi}{k+1+(m-1)\mathfrak{B}},$$

qui certe eo minor eſt, quam $\frac{\xi}{m-1}$, quo k eſt maius et quo minus eſt $\mathfrak{B}$. Quo igitur campum maiorem obtineamus, in id eſt incumbendum, vt litterae k quam minimus, litterae $\mathfrak{B}$ vero quam maximus valor concilietur; at cum ſit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ et debeat eſſe $B > 0$ hinc euidens eſt, $\mathfrak{B}$ non vltra vnitatem augeri poſſe. Caſu autem quo ſit $\mathfrak{B} = 1$ ſit $\Phi = \frac{\xi}{k+m}$. Tum vero ob $B = \infty$ longitudo tubi fieret infinita. Diminutio vero numeri k quum parum conferat ad campum augendum; videamus nunc etiam an margo coloratus deſtrui poſſit, quem in finem eſſe deberet.

$$0 = \frac{\pi}{\Phi} . \frac{b}{p} + \frac{\pi'}{\Phi} . \frac{c}{Bp}$$

$$0 = -i . \frac{1}{k} + . \frac{1}{kk'}$$

quae aequatio ob $i < 0$ nullo modo ſubſiſtere poteſt; vnde

vnde haec teleſcopiorum ſpecies vitio marginis colorati quam maxime laborabit. Ceterum pro ſemidiametro confuſionis habebimus hanc aequationem

$$\frac{\mu m x^3}{\alpha^3}\left(\lambda + \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{\lambda''}{B^3 . m}\right) = \frac{1}{k^3}$$

vnde colligitur

$$\alpha = k x \sqrt[3]{\mu m \left\{ \lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 . m} + \frac{\nu}{B \mathfrak{B} k} \right\}}$$

qui ſumto $x = \frac{m}{50}$ dig. et $k = 50$ abit in hunc valorem

$$\alpha = m \sqrt[3]{\mu m \left\{ \lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 . m} + \frac{\nu}{B \mathfrak{B} k} \right\}}$$

in qua expreſſione cum omnia membra ſint poſitiua, nullum eſt dubium, quin diſtantia focalis α multo fiat maior, quam caſu duarum lentium.

Coroll. 1.

295. Cum iam ſit animaduerſum, ſi $\mathfrak{B}$ caperetur $= 1$ longitudinem inſtrumenti in infinitum excreſcere ideoque $\mathfrak{B}$ capi debere minus vnitate; ſecundum membrum in aequatione valde increſcet pariter ac vltimum, ex quo diſtantia α augebitur.

Coroll. 2.

296. Sin autem huic incommodo mederi vellemus augendo numerum k, tunc campus apparens reſtringeretur.

Scho-

Scholion 1.

297. Nullum igitur eſt dubium, quin haec prima iſtiusmodi teleſcopiorum ſpecies penitus ſit repudianda, non ſolum quod nimis exiguum campum oſtendat, tubusque fiat valde longus, ſed eam ob cauſſam praecipue, quod repraeſentatio margine colorato ſit inquinata neque etiam reperimus huiusmodi teleſcopia vnquam vſu fuiſſe recepta. Interim tamen caſum quendam in ſequente exemplo proponamus.

Exemplum.

298. Si ſumatur $\mathfrak{B} = \frac{4}{5}$ et $k = 2$ teleſcopium huius generis deſcribere pro multiplicatione quacunque m.

Cum igitur hinc ſit $B = 4$ erunt elementa

$$b = \tfrac{2}{5}\alpha;\ \beta = 2\alpha;\ c = \tfrac{4\alpha}{m}$$

et diſtantiae focales

$$p = \alpha;\ q = \tfrac{2}{5}\alpha;\ \text{et}\ r = \tfrac{4\alpha}{m}\ \text{et}$$

$$\alpha + b = \tfrac{3}{5}\alpha;\ \beta + c = 2\alpha + \tfrac{4\alpha}{m}$$

quorum ſumma $\frac{7}{5}\alpha + \frac{4\alpha}{m}$ dat tubi longitudinem.

Tum vero reperitur $i = \frac{-15}{11 + 4m}$

et ſemidiameter campi $\Phi = \frac{4\xi}{11 + 4m}$

ſeu in menſura anguli $\Phi = \frac{3436}{11 + 4m}$. minut.

qui non multo eſt minor, quam campus ordinarius.

Pro

Pro loco oculi vero erit $O = \frac{4m+11}{mm} \cdot a$

Denique vero pro diſtantia focali α habebimus

$$\alpha = m.\sqrt[3]{\mu\, m\left(\lambda + \frac{\lambda' \cdot 125}{128} + \frac{\lambda''}{64 \cdot m} + \frac{5\nu}{32}\right)}$$

vbi circiter eſt $\mu = 1$ et $\nu = \frac{1}{5}$, quare ſi litteris λ, λ', λ'' valor minimus ſcilicet 1 tribuatur; erit

$$\alpha = m\sqrt[3]{m\left(2 + \frac{1}{128} + \frac{1}{64 \cdot m}\right)}$$

$$\alpha = m\sqrt[3]{\left(\left(2 + \frac{1}{128}\right)m + \frac{1}{64}\right)} \text{ digit.}$$

hinc ſi eſſet $m = 25$, erit

$$\alpha = 25.\sqrt[3]{50.\tfrac{27}{128}} = 92,23 \text{ dig.}$$

hincque tota longitudo erit $= 340$ dig. $= 28$ ped. 4 dig. quae longitudo ratione multiplicationis vtique tam eſt magna, vt in praxi nullo modo admitti poſſit, etiamſi vitium marginis colorati non adeſſet.

Scholion 2.

299. Cum igitur hinc nihil in vſum practicum trahi poſſit, haecque ſpecies ſimpliciſſima penitus rejici debeat, ad ſpecies ſimpliciores progrediamur, quae ſcilicet oriuntur, ſi tribus lentibus inſuper vna lens quarta adiungatur, ex quo variae ſpecies naſcentur, prouti haec noua lens vel inter obiectiuam et priorem imaginem, vel inter priorem et poſteriorem, vel inter hanc poſteriorem et lentem ocularem conſtituatur; quos ergo caſus ſeorſim hic euolui conueniet.

Problema 2.

300. Si inter lentem obiectiuam et primam imaginem noua lens ponatur, indolem horum teleſcopiorum indagare eorumque conſtructionem deſcribere.

Solutio.

Cum hic quatuor lentes ſint, ſtatuantur ternae fractiones, vt ſemper, $\frac{\alpha}{b} = -P$; $\frac{\beta}{c} = -Q$ et $\frac{\gamma}{d} = -R$ et quia in primum interuallum nulla imago cadit retinebit P valorem poſitiuum, reliquae vero Q et R fient negatiuae.

Quare ponatur $Q = -k$ et $R = -k'$ vt fiat multiplicatio $m = Pkk'$ elementaque noſtra ſint

$$b = -\frac{\alpha}{P};\ c = -\frac{B\alpha}{Pk};\ d = -\frac{BC\alpha}{Pkk'} = -\frac{BC\alpha}{m}$$

$$\beta = -\frac{B\alpha}{P};\ \gamma = -\frac{BC\alpha}{Pk};$$

$$p = \alpha;\ q = -\frac{\mathfrak{B}\alpha}{P};\ r = -\frac{B\mathfrak{C}\alpha}{Pk};\ s = -\frac{BC\alpha}{m}$$

vnde prodeunt interualla

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right)$$

$$\beta + c = -\frac{B\alpha}{P}\left(1 + \frac{1}{k}\right)$$

$$\gamma + d = -\frac{BC\alpha}{Pk}\left(1 + \frac{1}{k'}\right)$$

ſicque patet, $B\alpha$ eſſe debere negatiuum, vt et $BC\alpha$ ideoque C debet eſſe poſitiuum; vnde ſi $\alpha > 0$, debet eſſe $P > 1$; $B < 0$ et $C > 0$, ſin autem $\alpha < 0$, debet eſſe $P < 1$, $B > 0$ et $C > 0$.

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Nunc cum pro campo apparente ſit

$$\Phi = \frac{-\pi + \pi' - \pi''}{m - 1} \text{ ſtatuatur}$$

$$\pi = -\omega\,\xi;\ \pi' = i\,\xi;\ \pi'' = -\xi \text{ vt ſit}$$

$$\Phi = \frac{\omega + i + 1}{m - 1}.\ \xi = \mathrm{M}\,\xi \text{ exiſtente } \mathrm{M} = \frac{\omega + i + 1}{m - 1}.$$

Atque ſtatim pro loco oculi ſequitur

$$\mathrm{O} = -\frac{\pi''}{\Phi}\cdot\frac{s}{m} = -\frac{1}{\mathrm{M}}\cdot\frac{\mathrm{BC}\alpha}{mm}$$

quae diſtantia per conditiones ſuperiores iam eſt poſitiua. Aequationes autem pro litteris π ſupra datae praebent:

$$\frac{\mathfrak{B}\pi - 1}{\Phi} = -\frac{\mathfrak{B}\omega - 1}{\mathrm{M}} = -\mathrm{P};$$

$$\frac{\mathfrak{C}i}{\mathrm{M}} + \frac{\omega}{\mathrm{M}} + 1 = -\mathrm{P}\,k$$

vnde colligitur

$$\omega = \frac{(\mathrm{P} - 1)(i + 1)}{\mathfrak{B}(m - 1) - \mathrm{P} + 1}$$

vt maneat $\mathfrak{B}$ indefinitum; et

$$\mathfrak{C} = \frac{-(1 + \mathrm{P}k)\mathrm{M} - \omega}{i}$$

quae quantitas cum debeat eſſe poſitiua, debet eſſe vel i negatiuum vel ſi eſſet i poſitiuum, deberet eſſe

$$-(1 + \mathrm{P}\,k)\,\mathrm{M} - \omega > 0 \text{ ſiue}$$

$$-(1 + \mathrm{P}\,k)\left(\frac{\omega + i + 1}{m - 1}\right) - \omega > 0 \text{ ſeu}$$

$$-\omega(m + \mathrm{P}\,k) - (1 + \mathrm{P}\,k)(i + 1) > 0$$

vnde patet, fractionem ω negatiuam eſſe debere; ita, vt hinc campus apparens diminuatur.

Videa-

Videamus iam, an marginem coloratum tollere vel huic aequationi ſatisfacere poſſimus:

$$0 = +\omega . \frac{1}{P} - \frac{i}{PR} + \frac{1}{Pkk'}$$

vnde colligimus

$$0 = \omega - \frac{i}{k} + \frac{1}{kk'} \text{ adeoque}$$

$$k' = \frac{-1}{k\omega - i} = \frac{1}{i - k\omega}$$

qui valor debet eſſe poſitiuus adeoque $k\omega - i < 0$, de quo deinceps videbimus. Nunc adhuc aequationem pro confuſione aperturae tollenda contemplemur, quae ſequenti modo exhibebitur:

$$\alpha = kx\sqrt[3]{\mu m}\left(\lambda - \frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{1}{B^3 \mathfrak{C} Pk}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) - \frac{\lambda'''}{B^3 C^3 . m}\right)$$

pro qua expreſſione hactenus ſumſimus $x = \frac{m}{25}$ dig. et $k = 50$.

Coroll. I.

301. Pro dijudicandis litteris ω et i, vtrum valores habere queant poſitiuos an negatiuos, conſiderandae ſunt hae duae formulae:

$$\text{I. } \mathfrak{C} = \frac{-(1+Pk)\mathfrak{A} - \omega}{i}$$

$$\text{II. } k' = \frac{1}{i - k\omega}$$

ex quarum prima patet, ambas litteras i et ω ſimul poſitiuas eſſe non poſſe, quia alioquin $\mathfrak{C}$ foret negatiuum; quae littera tamen valorem poſitiuum habere debet. Ex ſecunda vero euidens eſt, fieri non poſſe,

vt ſit $\omega > 0$ et $i < 0$, quia alioquin k' prodiret negatiuum.

Coroll. 2.

302. Ex his duobus caſibus ſequitur litteram ω nunquam poſitiuam eſſe poſſe, quae conditio ita enunciari poteſt, vt ſecunda lens ſemper campum apparentem imminuere debeat.

Coroll. 3.

303. Cum igitur ω ſemper debeat eſſe negatiuum, ponatur $\omega = -\zeta$, vt ſit $\mathfrak{B}\zeta = (1 - P)M$ et $M = \frac{1+i-\zeta}{m-1}$. Noſtrae vero formulae, neceſſario poſitiuae, erunt

I. $\mathfrak{C} = \frac{-(1+Pk)M+\zeta}{i}$

II. $k' = \frac{1}{i+k\zeta}$

vnde ſi ſit i fractio poſitiua, debet eſſe $\zeta > (1+Pk)M$.

Sin autem i ſit fractio negatiua, puta $i = -y$, per primam debet eſſe $\zeta < (1 + Pk)M$ et ſimul $\zeta > \frac{y}{k}$.

Coroll. 4.

304. Praeterea etiam manifeſtum eſt, fractionem $\zeta = -\omega$ nunquam euaneſcere poſſe, ſi enim ſit $i > 0$ debet eſſe $\zeta > (1+Pk)M$. Sin autem ſit $i < 0$ ſeu $i = -y$ debet eſſe $\zeta > \frac{y}{k}$.

Coroll. 5.

305. Quia caſu $i = -y$, duplicem inuenimus conditionem, priorem $\zeta < (1 + Pk)M$ et poſteriorem $\zeta > \frac{y}{k}$; ex earum comparatione neceſſe eſt, vt ſit $(1 + Pk)M > \frac{y}{k}$ ſeu $y < (1 + Pk)kM$.

Scholion.

306. Tot autem caſus diuerſi ideo potiſſimum habent locum, quod in ſolutione problematis non definitur, vtrum lens obiectiua habeat ſuam diſtantiam focalem α poſitiuam an negatiuam. Vtrumque autem vſu venire poteſt, ſiquidem circa litteram P nihil aliud praecipitur, niſi quod ſit poſitiua ideoque eius valor a ciphra vsque in infinitum augeri queat.

Quamdiu autem littera P intra limites o et 1 continetur, α valorem habere debet negatiuum ſeu lens obiectiua erit concaua, et littera B poſitiua ideoque et $\mathfrak{B}$; vnde fit $\zeta = \frac{(1-P)M}{\mathfrak{B}}$ adeoque poſitiuum. Sin autem ſtatuatur $P = 1$, quo caſu binae lentes priores ſibi immediate iunguntur, fit $\zeta = o$, qui caſus, vti vidimus, penitus excluditur, ita, vt lens obiectiua duplicata eſſe nequeat. At ſi ſit P maior vnitate, neceſſario fit α poſitiuum ſeu lens obiectiua conuexa; vnde B fit negatiuum, neque vero hinc definitur $\mathfrak{B}$. At quia nouimus eſſe ω negatiuum ſeu ζ poſitiuum ob $\zeta = -\frac{(P-1)M}{\mathfrak{B}}$ patet, litteram $\mathfrak{B}$ negatiuam eſſe debere, hincque porro concluditur, $-B$ eſſe vnitate minus.

 nus.

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nus. Si denique P ſit numerus infinitus, ſecunda lens in ipſo loco prioris imaginis conſtituetur et ex eius diſtantia focali q concluditur

$$\mathfrak{B} = -P.\frac{q}{\alpha} = -\infty \text{ hincque } B = -1;$$

atque ſic contemplati ſumus obiter omnes caſus pro littera P; qui autem nunc diligentius perpendi merentur. Ante omnia autem notari conuenit, ſumi non poſſe $P = 0$, quia iam primum interuallum fieret infinitum, niſi diſtantia α eſſet infinite parua, quod autem foret aeque abſurdum, quia prima lens aperturam definitam admittere debet.

I. Evolutio caſus, quo $P < 1$.

307. Pro hoc caſu iam animaduertimus, fore $\alpha < 0$, quae negatio ne turbet ponamus $\alpha = -a$ eritque

$$b = \frac{a}{P};\ c = \frac{Ba}{Pk};\ d = \frac{BCa}{m}$$

$$\beta = \frac{Ba}{P};\ \gamma = \frac{BCa}{Pk}$$

vnde patet, ambas litteras B et C debere eſſe poſitiuas; vnde litterae germanicae $\mathfrak{B}$ et $\mathfrak{C}$ non ſolum erunt quoque poſitiuae, ſed etiam vnitate minores; quare cum ſit $\mathfrak{B}\zeta = (1-P)M$, manifeſto ſequitur fore $\zeta > (1-P)M$. Deinde ob

$$\mathfrak{C} = \frac{-(1+Pk)M+\zeta}{i} \text{ et } k' = \frac{1}{1+k\zeta},$$

non ſolum eſſe debet

$$\frac{-(1+Pk)M+\zeta}{i} > 0, \text{ ſed etiam } \frac{-(1+Pk)M+\zeta}{i} < 1.$$

quod

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quod quo clarius explicetur, duos caſus examinari conueniet

I. Si i ſit poſitiuum

ex valore $\mathfrak{C}$ nanciſcimur has conditiones

$$\zeta > (1 + Pk) M \quad \text{et} \quad \zeta < (1 + Pk) M + i$$

conditio autem litterae k' ſic ſponte impletur. Quia autem iam inuenimus $\zeta > (1 - P) M$; nunc inde patet, eſſe debere $(1 + Pk) M + i > (1 - P) M$ ideoque $i > - P(k + 1) M$; id quod ſemper eſt verum, dummodo i ſit poſitiuum, vti ſupponimus.

II. Si i ſit negatiuum.

ponatur $i = -y$ eritque

$$\mathfrak{C} = \frac{(1 + Pk)M - \zeta}{y}, \; k' = \frac{1}{k\zeta - y}.$$

Inde igitur ſequuntur hae conditiones,

$$\zeta < (1 + Pk) M$$
$$\zeta > (1 + Pk) M - y; \text{ hinc vero } \zeta > \frac{y}{k};$$

at ſupra iam inuenimus, $\zeta > (1 - P) M$; vnde ſequitur fore $(1 + Pk) M > \frac{y}{k}$ ſiue $y < (1 + Pk) k M$.

Iſto igitur caſu, quo $P < 1$, fractio i tam poſitiue capi poterit, quam negatiue, ac ſi poſitiue accipiatur, eius valorem nulla limitatione reſtringi. Quare cum i vnitatem ſuperare nequeat, poterit ſine haeſitatione ſtatim poni $i = 1$ ita, vt pro campo apparente fiat $\Phi = \frac{2 - \zeta}{m - 1} . \xi$, dummodo ζ non ſuperet vnitatem. Nulla autem ratio ſuadet, capere i negatiuum, quia tum campus nimium diminueretur.

II. Euo-

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II. Euolutio caſus, quo $P > 1$.

308. Quia hic eſt α quantitas poſitiua ideoque b negatiua, debet eſſe B negatiuum, at C, vt ante, poſitiuum. Deinde etiam vidimus eſſe $\mathfrak{B}$ negatiuum ideoque $-B < 1$; vnde fit $\zeta = \frac{(1-P)M}{\mathfrak{B}}$ adeoque poſitiuum, vbi tantum notetur $\mathfrak{B}$ tam paruum accipi non debere, vt ζ ſuperet vnitatem. Deinde habetur

$$\mathfrak{C} = \frac{-(1+Pk)M+\zeta}{i} \text{ et } k' = \frac{1}{i+k\zeta};$$

ex quibus formulis plane eadem ſequuntur, quae in caſu praecedente ſunt allata; vnde videtur etiam ſtatui poſſe $i = 1$; dummodo ex valore pro ζ ante dato ſit $\frac{1-P}{\mathfrak{B}} > 1 + Pk$ ſiue $-\mathfrak{B} < \frac{P-1}{Pk+1}$ et

$$-\mathfrak{B} > \frac{(P-1)M}{(1+Pk)M+1}.$$

III. Euolutio caſus, quo $P = \infty$.

309. Hoc ergo caſu, vt iam ſupra notauimus, erit $B = -1$ et $\mathfrak{B} = -\frac{Pq}{\alpha}$.

Nunc autem euidens eſt, ſtatui debere $k = 0$, ita tamen vt ſit $Pk = \vartheta$ ex quo elementa erunt

$$b = 0;\ \beta = 0;\ c = \frac{\alpha}{\theta};\ \gamma = \frac{C\alpha}{\theta};\ d = \frac{C\alpha}{m}.$$

Deinde cum ſit $\mathfrak{B}\zeta = (1-P)M$ habebitur nunc $\zeta = \frac{M\alpha}{q}$; vnde $q = \frac{M\alpha}{\zeta}$. Deinde binae noſtrae formulae erunt $\mathfrak{C} = \frac{-(1+\theta)M+\zeta}{i}$ et $k' = \frac{1}{i}$; vbi cum nihil impediat, quominus ponatur $i = 1$, erit hoc caſu $k' = 1$ et $Pk = \vartheta = m$ ita, vt ſit $\mathfrak{C} = -(1+m)M+\zeta$;

ex

ex quo valore hi limites colliguntur: $\zeta > (1+m)M$; $\zeta < (1+m)M + 1$, at vero eſt $M = \frac{2-\zeta}{m-1}$; ideoque $\zeta > \frac{(1+m)(2-\zeta)}{m-1}$ ideoque $\zeta > \frac{m+1}{m}$ qui valor etſi vnitatem ſuperat, tamen in praxi locum habere poteſt, dummodo littera ξ in eadem ratione diminuatur; ita vt $\zeta\xi$ non ſuperet valorem $\frac{1}{4}$, ſiquidem $\frac{1}{4}$ pro apertura maxima accipiatur. Sin autem ſumſiſſemus $i = \frac{1}{2}$ prodiiſſet $k' = 2$ hincque $m = 2\vartheta$ ſeu $\vartheta = \frac{m}{2}$ ſicque haberemus $\zeta > (1 + \frac{1}{2}m)M$ et $\zeta < (1 + \frac{1}{2}m)M + \frac{1}{2}$; quia autem eſt $M = \frac{3-2\zeta}{2(m-1)}$; prior conditio dat

$$\zeta > (1 + \tfrac{1}{2}m)\left(\frac{3-2\zeta}{2(m-1)}\right) \text{ ſiue } \zeta > \frac{2+m}{2m};$$

ideoque multo magis $\zeta > \frac{1}{2}$. Ex quo patet, campum apparentem ob valorem ζ magis imminui, quam ob valorem i augeri, ſicque eum ſemper aliquanto minorem fieri, quam in tubis aſtronomicis communibus. Supra iam obſeruauimus, talem lentis locum in praxi vitari oportere.

IV. Euolutio caſus prorſus ſingularis quo $i = 0$.

310. Cum ſit $i = 0$ et $\mathfrak{C}$ vnitatem ſuperare nequeat, ob

$$\mathfrak{C}\, i = -(1 + Pk)M + \zeta \text{ erit}$$

$$\zeta = (1 + Pk)M \text{ hincque } Pk = \frac{\zeta}{M} - 1;$$

at eſt $k' = \frac{1}{k\zeta}$, ob $Pkk' = m$ erit $Pk = mk\zeta$; ideoque $P = m\zeta$. Quare ille valor pro Pk inuentus huic aequalis poſitus dabit $mk\zeta = \frac{\zeta}{M} - 1$; hincque

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$k = \frac{1}{Mm} - \frac{1}{m\zeta}$ ex quo porro habetur $k' = \frac{Mm}{\zeta - M}$ quia vero eſt $M = \frac{1-\zeta}{m-1}$, naſcetur

$$k = \frac{m-1}{(1-\zeta)m} - \frac{1}{m\zeta} = \frac{m\zeta - 1}{m(1-\zeta)\zeta}$$

$$k' = \frac{m(1-\zeta)}{m\zeta - 1} \text{ et } P = m\zeta \text{ atque } Pk = \frac{m\zeta - 1}{1-\zeta},$$

qui valores cum neutiquam a $\mathfrak{C}$ pendeant, hoc inſigne lucrum iam ſumus adepti, vt littera C penitus arbitrio noſtro relinquatur, ſicque efficere poterimus, vt poſteriores diſtantiae determinatrices ipſaeque lentes poſteriores, quae hactenus plerumque nimis paruae ſunt repertae, nunc datae magnitudinis fieri queant, in quo certe maximum commodum conſiſtit; quod denique ad litteras $\mathfrak{B}$ et B attinet, duos caſus conſiderari oportet, prouti $P = m\zeta$ fuerit vel vnitate minor vel vnitate maior.

I. Sit igitur $m\zeta < 1$, ſeu $\zeta < \frac{1}{m}$ et habebitur

$$\mathfrak{B} = \frac{(1-m\zeta)M}{\zeta} = \frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta};$$

ibi autem vidimus, $\mathfrak{B}$ eſſe debere poſitiuum et vnitate minus; quocirca hoc caſu, quo $\zeta < \frac{1}{m}$ ob

$$\mathfrak{B} = \frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta} \text{ debet eſſe}$$

$$(1 - m\zeta)(1 - \zeta) < (m - 1)\zeta \text{ ſeu}$$

$$m\zeta^2 - 2m\zeta + 1 < 0;$$

vnde colligitur, capi debere intra limites $\frac{1}{m}$ et $\frac{1}{2m}$.

Cum autem litterae k et k' neceſſario ſint poſitiuae, ad hoc neceſſario requiritur, vt ſit $m\zeta > 1$ ſeu

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ſeu $\zeta > \frac{1}{m}$; ob quam conditionem caſus primus ſtatim excludi debuiſſet.

II. Sit igitur $P (= m\zeta) > 1$ ſeu $\zeta > \frac{1}{m}$, prouti valores k et k' poſtulant, atque ad caſum ſecundum recurrere debemus, pro quo cum iterum ſit

$$\mathfrak{B} = \frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta}$$

ſimulque notetur $\mathfrak{B}$ eſſe debere negatiuum ſine vlla alia conditione, niſi quod eſſe debeat $\zeta < 1$, vti quidem ratio campi abſolute poſtulat; ita, vt iam contineatur intra limites 1 et $\frac{1}{m}$; manifeſtum autem eſt, expedire, vt ζ quam minime limitem $\frac{1}{m}$ ſuperet. Ex quo operae pretium videtur, duo exempla adiungere, in quorum altero ζ limiti priori $\frac{1}{m}$, in altero vero limiti poſteriori 1 propius accipiatur.

Exempl. I.

311. Pro caſu poſtremo, quo $i = 0$ ſi ſtatuatur $\zeta = \frac{2}{m}$, teleſcopium inde oriundum deſcribere.

Hoc igitur caſu habebimus

$$\mathfrak{B} = \frac{-(m-2)}{2(m-1)} \text{ et } B = \frac{-(m-2)}{3m-4}.$$

Porro $P = 2$; $k = \frac{m}{2(m-2)}$; $k' = m-2$; $M = \frac{m-2}{m(m-1)}$ vnde diſtantiae noſtrae determinatrices ob α poſitiuum erunt

$$b = \frac{-\alpha}{2};\ c = \frac{(m-2)^2}{m(3m-4)}.\alpha$$

$$\beta = \frac{m-2}{2(3m-4)}.\alpha;\ \gamma = \frac{(m-2)^2}{m(3m-4)}C\alpha.$$

 $d =$

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$$d = \frac{m-2}{m(3m-4)} \mathfrak{C} \alpha$$

et diſtantiae focales

$$p = \alpha; \; q = \frac{m-2}{4(m-1)} . \alpha$$

$$r = \frac{(m-2)^2}{m(3m-4)} \mathfrak{C} \alpha; \; s = \frac{m-2}{m(3m-4)} \mathfrak{C} \alpha.$$

Tum vero interualla lentium

$$\alpha + b = \tfrac{1}{2} \alpha; \; \beta + c = \frac{(m-2)(3m-4)}{2m(3m-4)} \alpha = \frac{m-2}{2m} . \alpha$$

$$\gamma + d = \frac{(m-1)(m-2)}{m(3m-4)} . \mathfrak{C} \alpha$$

et diſtantia oculi $O = \frac{m-1}{m(3m-4)} \mathfrak{C} \alpha$ et campi ſemidiameter $\Phi = \frac{m-2}{m(m-1)} . \xi$. qui ſi in menſura angulorum deſideretur ſumi poteſt $\xi = 859$. min. ob $\xi = \frac{1}{4}$.

Diſtantia denique focalis lentis obiectiuae α definiri debet ex formula in problemate data, vbi notandum eſt, ipſius λ' coëfficientem circiter fore 4, et ipſius λ'' coëfficiens ſemper maior erit, quam 27, qui termini cum omnes ſint poſitiui, euidens eſt, pro α ſemper ingentem valorem reperiri, ita, vt haec teleſcopia valde longa euadant.

Exempl. II.

312. Pro caſu poſtremo, quo $i = 0$, ſi ſumatur $\zeta = \frac{1}{4}$, teleſcopium inde oriundum deſcribere.

Hoc igitur caſu erit $\mathfrak{B} = \frac{-(m-2)}{2(m-1)}$ et $B = \frac{-(m-2)}{3m-4}$; $P = \frac{m}{2}$; $k = \frac{2(m-2)}{m}$; $k' = \frac{m}{m-2}$; $M = \frac{1}{2(m-1)}$; vnde

diſtan-

diſtantiae determinatrices

$$b = -\frac{2\alpha}{m};\ c = \frac{\alpha}{3m-4}$$

$$\beta = \frac{2(m-2)\alpha}{m(3m-4)};\ \gamma = \frac{C\alpha}{3m-4}$$

$$d = \frac{m-2}{(3m-4)m} C\alpha.$$

et diſtantiae focales

$$p = \alpha;\ q = \frac{m-2}{m(m-1)}\alpha$$

$$r = \frac{\mathfrak{C}\alpha}{3m-4};\ s = \frac{m-2}{(3m-4)m} C\alpha$$

et interualla

$$\alpha + b = \frac{m-2}{m}\alpha;\ \beta + c = \frac{\alpha}{m}$$

$$\gamma + d = \frac{2(m-1)C\alpha}{m(3m-4)} \text{ et}$$

$$O = \frac{2(m-1)(n-2)C\alpha}{mm(3m-4)}$$

nunc vero campi ſemidiameter erit tantum

$$\Phi = \frac{470}{m-1} \text{ minut.}$$

In formula autem pro diſtantia α definienda notandum eſt, coefficientem λ' fore $\frac{16}{m}$, ipſius vero $\lambda'' > \frac{27}{m\mathfrak{C}r}$ ſiquidem multiplicatio ſit praemagna; vnde patet, pro α valorem multo minorem prodire, ita vt hinc teleſcopia ſatis idonea obtinerentur, ſi modo campus non eſſet tam exiguus.

Coroll. I.

313. Quia pro lente tertia ſumſimus r hincque et $\pi' = 0$, eius apertura ex formulis generalibus definiri

 debet,

debet, cuius ſemidiameter erit $= \frac{rx}{B\mathfrak{C}\alpha}$, qui ergo pro priori exemplo fit $\frac{m-2}{m}x$ pro ſecundo autem $\frac{x}{m-2}$ vnde ſi ſumatur $x = \frac{m}{50}$ dig. hic ſemidiameter erit circiter $\frac{1}{50}$ dig. quae ergo lens commodiſſime locum diaphragmatis tenebit.

Coroll. 2.

314. Si quaſi medium ſumendo inter duo exempla allata ſtatuatur $\zeta = \frac{1}{\sqrt{n}}$, erit $P = \sqrt{m}$ et $k = 1$ et $k' = \sqrt{m}$; porro $\mathfrak{B} = -\frac{(\sqrt{m}-1)}{\sqrt{m}+1}$; $B = \frac{-(\sqrt{m}-1)}{2\sqrt{m}}$; $M = \frac{1}{m+\sqrt{n}}$ atque hinc

$$b = -\frac{\alpha}{\sqrt{n}};\ \beta = \frac{+(\sqrt{m}-1)\alpha}{2m};\ c = \frac{\sqrt{m}-1}{2m}.\alpha$$

$$\gamma = \frac{\sqrt{m}-1}{2m}C\alpha;\ d = \frac{\sqrt{m}-1}{2m\sqrt{n}}C\alpha \text{ ergo}$$

$$\alpha + b = (1 - \tfrac{1}{\sqrt{n}})\alpha;\ \beta + c = \frac{\sqrt{m}-1}{m}\alpha$$

$$\gamma + d = \frac{\sqrt{m}-1}{2m}C\alpha(1 + \tfrac{1}{\sqrt{m}}) = \frac{m-1}{2m\sqrt{m}}C\alpha$$

et diſtantia oculi $O = +\frac{(m-1)}{2mm}.\alpha$

quare longitudo teleſcopii erit $\frac{m-1}{m}(1 + \frac{1+\sqrt{m}}{2m}C)\alpha$

ac denique ſemidiameter campi $\Phi = \frac{\xi}{m+\sqrt{m}} = \frac{859}{m+\sqrt{m}}$ min.

et ſemidiam. apert. tertiae lentis $= \frac{x}{\sqrt{m}} = \frac{\sqrt{m}}{50}$ dig.

Scholion.

315. Simili modo, quo hic caſum $i = 0$ expediuimus, etiam quaeſtio in genere pro quouis valore

lore ipſius i reſolui poterit; ex aequatione enim

$$\mathfrak{C}i = -(1 + Pk)M + \zeta$$ quum deducatur

$$Pk = \frac{\zeta - \mathfrak{C}i}{M} - 1$$ et quia eſt

$$M = \frac{1+i-\zeta}{m-1}$$ fiet $$Pk = -\frac{\mathfrak{C}i(m-1) + m\zeta - i - 1}{1+i-\zeta}.$$

Verum ob $k' = \frac{1}{i+k\zeta}$ erit etiam

$$Pk = \frac{m}{k'} = m(i + k\zeta);$$

vnde colligimus

$$-\frac{\mathfrak{C}i}{M} + \frac{m\zeta - i - 1}{1+i-\zeta} = mi + mk\zeta$$

hincque

$$k = -\frac{\mathfrak{C}i}{Mm\zeta} + \frac{m\zeta + mi\zeta - mii - mi - i - 1}{(1+i-\zeta)m\zeta}$$

et quia eſt

$$i + k\zeta = -\frac{\mathfrak{C}i}{Mm} + \frac{m\zeta - i - 1}{(1+i-\zeta)m}$$ erit

$$k' = \frac{m(1+i-\zeta)}{m\zeta - \mathfrak{C}i(m-1) - i - 1}$$ ideoque

$$Pk = \frac{m\zeta - \mathfrak{C}i(m-1) - i - 1}{1+i-\zeta}$$ et $$P = \frac{m}{kk'};$$

quia nunc k' debet eſſe quantitas poſitiua, neceſſe eſt, vt ſit $m\zeta > \mathfrak{C}i(m-1) + i + 1$; vnde facto calculo ſemper reperietur eſſe $P > 1$, ita, vt etiamſi non ſit $i = 0$ tamen ſolus caſus ſecundus ſupra memoratus locum habeat. Quia autem hypotheſis $i = 0$ tam commodam et concinnam ſupeditauit reſolutionem; nulla plane eſt ratio, cur litteram i ſiue poſitiuam ſiue negatiuam aſſumere vellemus, cum pro commodo nullum inde lucrum ſit exſpectandum. Praeter

ter concinnitatem calculi autem duo commoda, quae nobis iſta hypotheſis $i = 0$ largitur, maximi ſunt momenti quorum alterum, vti vidimus, in hoc conſiſtit, vt litterae $\mathfrak{C}$ et C arbitrio noſtro permittantur, hocque modo nimia lentis ocularis paruitas euitari queat: alterum vero commodum huic nihil cedere eſt cenſendum, propterea quod tam exigua apertura lenti tertiae ſine vllo ſiue campi ſiue claritatis detrimento tribui poſſit, vt omne lumen peregrinum tutius, quam per diaphragmata ordinaria excludatur.

Problema 3.

316. *Si teleſcopium huius generis ita ex quatuor lentibus ſit componendum, vt binae mediae ambae inter imaginem priorem et poſteriorem conſtituantur, indolem eius indagare eiusque conſtructionem deſcribere.*

Solutio.

Poſitis igitur, vt ante, noſtris fractionibus

$$\frac{\alpha}{b} = -P; \; \frac{\beta}{c} = -Q; \; \frac{\gamma}{d} = -R;$$

hic litterae P et R debent eſſe negatiuae manente Q poſitiua: quare ſi ponatur $P = -k$ et $R = -k'$, vt ſit $m = Qkk'$ elementa noſtra ita ſe habebunt:

$$b = \frac{\alpha}{k}; \; \beta = \frac{B\alpha}{k}; \; c = \frac{-B\alpha}{Qk};$$

$$\gamma = \frac{-BC\alpha}{Qk}; \; d = \frac{-BC\alpha}{Qkk'} = \frac{-BC\alpha}{m}$$

hincque

hincque interualla

$$\alpha + b = \alpha (1 + \tfrac{1}{k}); \text{ ideoque } \alpha > 0$$

$$\beta + c = \tfrac{B\alpha}{k}(1 - \tfrac{1}{Q}); \text{ hinc } B(1 - \tfrac{1}{Q}) > 0$$

$$\gamma + d = \tfrac{-BC\alpha}{Qk}(1 + \tfrac{1}{k'}); \text{ hinc } BC < 0$$

Pro campo apparente ſtatuamus $\pi = -\omega\xi$; $\pi' = +i\xi$ et $\pi'' = -\xi$ vt fiat

$$\Phi = \tfrac{\omega + i + 1}{m - 1}.\xi = M\xi, \text{ exiſtente } M = \tfrac{\omega + i + 1}{m - 1}$$

atque hinc primo erit diſtantia oculi

$$O = \tfrac{-\pi''}{\Phi}.\tfrac{d}{m} = \tfrac{d}{Mm}.$$

deinde margo coloratus euaneſcet, ſi fuerit

$$0 = \tfrac{\omega}{P} + \tfrac{i}{PQ} + \tfrac{1}{PQR} \text{ ſeu}$$

$$0 = -\tfrac{\omega}{k} - \tfrac{i}{Qk} + \tfrac{1}{Qkk'}$$

vnde concludimus

$$k' = \tfrac{1}{i + Q\omega} \text{ et } m = \tfrac{Qk}{i + Q\omega}$$

tum vero conſiderari oportet ſequentes aequationes:

$$-\tfrac{\mathfrak{B}\omega}{M} = 1 + k; \quad \tfrac{\mathfrak{C}i}{M} + \tfrac{\omega}{M} = -1 - Qk \text{ ſeu}$$

$$\mathfrak{C}i = -(1 + Qk)M - \omega \text{ et}$$

$$\mathfrak{B}\omega = -(1 + k)M$$

quarum euolutio commode generaliter inſtitui non poteſt, ſed caſus magis particulares contemplari conueniet. Verum caſus extremi duo habentur; alter, quo

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quo lens in ipſam imaginem priorem, alter vero, quo in imaginem poſteriorem cadit. Illo ſcilicet fit $Q=0$; hoc vero $Q=\infty$. Inter hos autem quaſi medius quidam praecipue perpendi meretur oriundus ex valore $Q=1$, quos caſus deinceps ſeorſim euoluamus. Hic igitur tantum ſupereſt formulam adiungere pro confuſione deſtruenda; ex qua ſcilicet diſtantia α determinatur

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right) - \frac{1}{B^3\mathfrak{C}QK}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right) - \frac{\lambda'''}{B^3C^3m}\right)}$$

Coroll. 1.

317. Quoniam inuenimus $k'=\frac{1}{i+Q\omega}$, ob $Q>0$ euidens eſt, ambas litteras i et ω ſimul negatiuas eſſe non poſſe. Neque vero etiam ambae poſſunt eſſe poſitiuae, ſi enim ω eſſet poſitiuum, foret $\mathfrak{B}$ ideoque et B negatiuum; hincque ob $BC<0$ deberet eſſe C poſitiuum, ideoque et $\mathfrak{C}$ poſitiuum, ac proinde $\mathfrak{C}i$ poſitiuum, id quod fieri non poſſe ex valore pro $\mathfrak{C}i$ ſupra dato manifeſtum eſt.

Coroll. 2.

318. Cum igitur ambae litterae ω et i nec poſitiuae nec negatiuae eſſe queant; neceſſe eſt, alteram eſſe poſitiuam, alteram negatiuam. Si ſit $\omega>0$, modo vidimus, eſſe debere $\mathfrak{B}<0$ et $B<0$ hincque $C>0$. Sin autem ſit $\omega<0$, erit $\mathfrak{B}>0$; de B vero hinc nihil definitur. Ex altera vero aequatione poſito

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ſito $\omega = -\zeta$, erit $\mathfrak{C}\, i = -(1+Qk)M + \zeta$ vnde intelligitur, ſi fuerit $\zeta > (1+Qk)M$ fore $\mathfrak{C} > o$; ſin autem ſit $\zeta < (1+Qk)M$ fore $\mathfrak{C} < o$. Prius autem euenit, ſi fuerit $1+k > (1+Qk)\mathfrak{B}$, ſeu $\mathfrak{B} < \frac{1+k}{1+Qk}$. Poſterius vero ſi $\mathfrak{B} > \frac{1+k}{1+Qk}$; hoc ipſo autem poſteriori caſu cum ſint $\mathfrak{C}$ et C negatiua, debet eſſe B poſitiuum; ideoque $\mathfrak{B} < 1$ ex quo ſequitur fore $Q > 1$.

Euolutio caſus primi, quo $Q = o$.

319. Quia eſt $Q = o$ erit ſecundum interuallum $= -\frac{B\alpha}{Qk} = c$; ideoque $\beta = o$. ergo vel $B = o$ vel $k = \infty$. At prius fieri nequit, foret enim $\mathfrak{B} = o$ et q ſeu diſtantia focalis ſecundae lentis $= o$, quod eſt abſurdum. Reſtat ergo, vt ſit $k = \infty$ et cum ſit $q = \frac{\mathfrak{B}\alpha}{k}$ erit $\mathfrak{B} = \frac{kq}{\alpha} = \infty$, atque hinc $B = -1$. Ex quo ſequitur ob $BC < o$ fore $C > o$ et $\mathfrak{C} < 1$. Cum vero ſit $Q = o$ et $K = \infty$, productum QK debet eſſe finitum, quare ſtatuatur $QK = l$, vt ſit

$$b = o;\ \beta = o;\ c = \frac{+\alpha}{l};\ \gamma = \frac{C\alpha}{l};$$

$$d = \frac{C\alpha}{m};\ \text{porroque}\ O = \frac{C\alpha}{Mm^2}.$$

Deſtructio vero marginis colorati poſtulat $k' = \frac{1}{i}$; ita, vt iam i certe ſit fractio poſitiua, et $m = \frac{l}{i}$. Ambae autem aequationes noſtrae fundamentales dabunt, prior $\mathfrak{B}\,\omega = -kM$ ſiue $\frac{kq\omega}{\alpha} = -kM$; ideoque $\omega = -\frac{M\alpha}{q}$; poſterior vero $\mathfrak{C}\, i = -(1+l)M + \frac{M\alpha}{q}$;

 quod

quod cum debeat eſſe poſitiuum, oportet eſſe $\frac{\alpha}{q} > l + 1$; ſiue $q < \frac{\alpha}{1+l}$. Quia $\omega < 0$, ſcribatur $\omega = -\zeta$ et litteras i et ζ in calculo retineamus eritque $l = mi$; $q = \frac{M\alpha}{\zeta}$ ac proinde $\mathfrak{C}\, i = -(1 + mi)\, M + \zeta$. Vnde cum ſit $\mathfrak{C} > 0$, ſimulque $\mathfrak{C} < 1$ nanciſcimur hos limites:

$$1^\circ.\ \zeta > (1 + mi)\, M;\quad 2^\circ.\ \zeta < (1 + mi)\, M + i.$$

cum iam ſit $M = \frac{1+i-\zeta}{m-1}$, hoc valore ſubſtituto ex iſtis limitibus colliguntur ſequentes,

$$1^\circ.\ \zeta > \frac{1+mi}{m} \text{ et } 2^\circ.\ \zeta < \frac{1+mi}{m} + \frac{(m-1)i}{m(1+i)}$$

$$\text{ſiue } \zeta < \frac{1+2mi+mii}{m(1+i)}$$

ex quibus ſi littera i pro lubitu capiatur indeque ζ debite aſſumatur, omnia pro teleſcopio erunt determinata, quo autem melius de campo iudicare poſſimus, loco ζ ſeorſim vtrumque limitem ſubſtituamus ac prior quidem limes dabit $M = \frac{1}{m}$; alter vero limes minor $M = \frac{1}{m(1+i)}$; inter quos valores littera M ideoque et campus apparens continebitur.

Pro definienda autem diſtantia α formula ſuperior hanc induet formam.

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + * + \frac{1}{\mathfrak{C}i}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{\mathfrak{C}}\right) + \frac{\lambda'''}{\mathfrak{C}^3 m}\right)}.$$

De hoc autem caſu iterum valet, quod ſupra commemorauimus, ſcilicet ob impuritates minimas lentis

in

in loco imaginis conſtitutae repraeſentationem obiectorum inquinari.

De cetero autem campus ſemper maior eſt ſemiſſi campi ſimplicis, quem vero defectum noua lente adjicienda facile ſupplere licet.

Euolutio caſus, quo $Q = \infty$.

320. Hoc ergo caſu fit ſecundum interuallum $\beta + c = \frac{B\alpha}{k}$; vnde ſequitur B poſitiuum ideoque C negatiuum. Tum vero quia $c = -\frac{\beta}{Q}$ erit $c = 0$ et $\gamma = 0$. Cum autem huius lentis diſtantia focalis ſit $r = \mathfrak{C} c$, erit $\mathfrak{C} = \infty$ hincque $C = -1$ et quia $B > 0$, fiet $\mathfrak{B} > 0$, at < 1.

Cum porro ſit $m = Q k k'$, neque vero $k = 0$, neceſſe eſt, vt ſit $k' = 0$, ex quo ponatur $Q k' = l$, vt fiat $m = k l$. Iam vero ex margine colorato habemus $k' = \frac{1}{1 + Q\omega} = \frac{l}{Q}$; vnde ſequitur $\omega = \frac{1}{l}$ hincque poſitiuum. Cum autem $\mathfrak{B}$ ſit poſitiuum, ex prima aequatione fundamentali ſequitur $\omega = \frac{-(1+k)M}{\mathfrak{B}}$ vnde oporteret eſſe ω quantitatem negatiuam, quod cum illi concluſioni aduerſetur, manifeſtum eſt, hunc caſum eſſe impoſſibilem ſeu potius hoc caſu marginem coloratum deſtrui non poſſe. Ceterum hoc caſu lens tertia in ipſo loco ſecundae imaginis foret conſtituta, quod cum contradictionem inuoluat, hinc facile intelligitur, tertiam lentem notabili interuallo ante imaginem poſteriorem conſtitutam eſſe debere.

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Euolutio caſus prorſus ſingularis, quo $Q = 1$ et radii per binas lentes priores tranſmiſſi iterum fiunt paralleli.

321. Hoc ergo caſu teleſcopium erit quaſi ex duobus tubis aſtronomicis compoſitum, certo quodam interuallo ab eodem axe a ſe inuicem remotis, ad quod genus vulgaria teleſcopia terreſtria dicta, ſunt referenda. Cum igitur ſit $Q = 1$, ne interuallum ſecundum $\beta + c$ ob $\beta = -Qc$ euaneſcat, debet eſſe tam β, quam c, infinitum, id quod eueniret, tam ſi $k = 0$, quam ſi $B = \infty$. prius autem hic locum habere nequit, quia interuallum primum etiam fieret infinitum; ex quo neceſſe eſt, vt ſit $B = \infty$ et $\mathfrak{B} = 1$. Ne autem tertium interuallum euadat $= \infty$; productum $B\,C$ debet eſſe quantitas finita et negatiua; quare ſtatuatur $B\,C = -\vartheta$; ideoque $C = -\frac{\theta}{B} = 0$. Vt autem interuallum medium valorem finitum, puta $= \eta\,\alpha$, obtineat, quantitas B non tanquam vere infinita, ſed tantum praegrandis conſiderari debet, donec ſcilicet conditionibus praeſcriptis ſatisfecerimus, vnde etiam valor ipſius Q aliquantillum ab vnitate diſcrepare reperietur, quoniam enim eſſe debet

$$\frac{B\alpha}{k}\left(1 - \frac{1}{Q}\right) = \eta\,\alpha; \text{ inde fit}$$

$$Q = \frac{B}{B - \eta k} = 1 + \frac{\eta k}{B};$$

tum vero etiam erit

$$\mathfrak{B} = \frac{B}{1 + B} \text{ et } C = -\frac{\theta}{B} \text{ et } \mathfrak{C} = \frac{-\theta}{B - \theta} = \frac{-\theta}{B}.$$

His

His notatis noſtrae aequationes fundamentales erunt

$$\omega = \frac{-(1+k)M(1+B)}{B} \text{ et}$$

$$+\frac{\theta i}{B} = +(1+k)M + \frac{\eta k^2 . M}{B} + \omega$$

in qua ſi loco ω ex priore ſubſtituatur valor inuentus obtinebitur

$$\frac{\theta i}{B} = \frac{\eta k^2 M - (1+k)M}{B} \text{ hincque } i = \frac{(\eta k^2 - k - 1)M}{\theta}$$

et nunc licebit ponere $B = \infty$, $\mathfrak{B} = 1$, $C = \mathfrak{C} = 0$, ita tamen, vt ſit $BC = -\vartheta$. Deſtructio autem marginis colorati praebet $k' = \frac{1}{i+\omega}$ et ob $kk' = m$ colligetur $i + \omega = \frac{k}{m}$; quia deinde eſt $M = \frac{1+i+\omega}{m-1}$ fiet nunc $M = \frac{m+k}{m(m-1)}$ et ſi valores pro i et ω inuenti ſubſtituantur in formula $i + \omega = \frac{k}{m}$ orietur haec aequatio

$$\frac{k}{m} = \frac{(\eta k^2 - (1+k)(1+\theta))M}{\theta}$$

et pro M ſubſtituto valore

$$\vartheta(m-1)k = (\eta k^2 - (1+k)(1+\vartheta))(m+k)$$

vnde colligitur

$$\vartheta = \frac{(\eta k^2 - k - 1)(m+k)}{k^2 + 2mk + m}$$

et quia ϑ debet eſſe numerus poſitiuus neceſſe eſt, vt ſit $\eta > \frac{k+1}{k^2}$ et quidem ita, vt ϑ non fiat nimis exiguum, quandoquidem nunc elementa noſtra ita exprimentur

$b =$

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$$b = \frac{\alpha}{k};\ \beta = \infty;\ c = \infty;\ \gamma = \frac{\theta\alpha}{k};\ d = \frac{\theta\alpha}{m};$$

$$\alpha + b = \alpha\left(1 + \frac{1}{k}\right);\ \beta + c = \eta\alpha;\ \gamma + d = \theta\alpha\left(\frac{1}{k} + \frac{1}{m}\right)$$

indeque diſtantia

$$O = \frac{\theta\alpha}{Mm^2} = \frac{(m-1)\theta\alpha}{m(m+k)}$$

atque diſtantiae focales

$$p = \alpha;\ q = \frac{\alpha}{k};\ r = \frac{\theta\alpha}{k};\ s = \frac{\theta\alpha}{m}.$$

Diſtantia autem α definiri debet ex aequatione ſequente:

$$\alpha = k x \sqrt[3]{\mu m \left(\lambda + \frac{\lambda'}{k} + \frac{\lambda''}{\theta^3 k} + \frac{\lambda'''}{\theta^3 m}\right)}$$

quare ne valor ipſius α nimis fiat magnus, conuenit k magnum aſſumi, tum vero ϑ non multo minus vnitate; quod ad prius attinet, etiam campus apparens ſuadet, litterae k quam maximum valorem dare, quia tum M continuo magis creſcit; verum probe notandum eſt, in formula $\Phi = M\xi$ pro littera ξ eatenus tantum valorem $\frac{1}{4}$ aſſumi poſſe, quatenus litterae i et ω vnitatem non ſuperant; ita, vt ſi vel i vel ω vnitatem ſuperaret, tum ξ in eadem ratione diminui deberet. Quam ob cauſſam maximi momenti eſt, in eum valorem ipſius k inquirere, vnde prodeat $i = 1$. Poſito autem $i = 1$ reperimus

$$1 + \omega = \frac{k}{m};\ \text{ſeu}\ m(m-2) = k^2 + 2mk$$

cuius aequationis reſolutio praebet

$$k = -m + \sqrt{2m(m-1)}$$

Hic

Hic ſcilicet valor ipſius k nobis praebet $i = 1$ et

$$\omega = \frac{k-m}{m} = \frac{-2m + \sqrt{2m(m-1)}}{m}$$

qui valor eſt negatiuus et vnitate minor, vnde pro campo apparente habebitur

$$\Phi = \frac{\sqrt{2m.(m-1)}}{m(m-1)}.\xi = \sqrt{\frac{2}{m(m-1)}}.\xi$$

ſin autem k adhuc maiorem adipiſceretur valorem, prodiret quidem i maius vnitate, ſed tum ξ ita ſumi deberet, vt fieret $i\xi = \frac{1}{4}$ ſeu $\xi = \frac{1}{4i}$, ſicque pro campo prodiret $\Phi = \frac{1+i+\omega}{m-1}.\frac{1}{4i}$ vnde calculum inſtituenti innoteſcit campum continuo diminui eo magis quo valor ipſius k illum terminum ſuperaverit. Maxime igitur hic caſus lucroſus eſt, ſi capiatur

$$k = -m + \sqrt{2m(m-1)};\ \text{vnde fit}$$

$$k' = \frac{m+\sqrt{2m(m-1)}}{m-2}.$$

Scholion.

322. Quia in antecedente problemate caſus maxime memorabilis eſt deductus, ponendo $i = 0$, ſuſpicari quis poſſet, etiam hic talem poſitionem inſtitui conuenire. Quamobrem hic oſtendamus, in hoc problemate neque poſitionem $i = 0$ neque $\omega = 0$ locum habere poſſe. Primo enim ſi eſſet $\omega = 0$, ob $k' = \frac{1}{i+Q\omega}$ deberet eſſe $i > 0$, at ob $\omega = 0$ prima aequatio $\mathfrak{B}\omega = -(1+k)M$ ſubſiſtere nequit, niſi ſit $\mathfrak{B} = \infty$, ideoque $B = -1$; iam ob $BC < 0$ debet eſſe C poſitiuum ideoque $\mathfrak{C}$ etiam > 0, ex quo

 patet,

patet, alteram aequationem $\mathfrak{C}\, i = -(1+Qk)\,M$ plane ſubſiſtere non poſſe; ſicque euictum eſt, ſumi non poſſe $\omega = 0$. Simili modo oſtendetur, numerum i euaneſcere non poſſe; tum enim ob $k' = \frac{1}{1+Q\omega}$ deberet eſſe $\omega > 0$ hincque poſterior aequatio

$$\mathfrak{C}\, i = -(1+Qk)\,M - \omega$$

ſubſiſtere nequit, niſi ſit $\mathfrak{C}\, i$ quantitas finita negatiua ideoque $\mathfrak{C} = \infty$; vnde fit $C = -1$, et hinc ob $BC < 0$ fiet $B > 0$ ſimulque $\mathfrak{B} > 0$ id quod primae aequationi $\mathfrak{B}\,\omega = -(1+k)\,M$ manifeſto contradicit; ex quo perſpicuum eſt, etiam numerum i non poſſe capi $= 0$. Neque ergo praeter tres caſus hic commemoratos vllus alius hic perpendi meretur atque poſtremus adeo tantis commodis reliquos omnes antecedit, vt is ſolus dignus videatur, qui in praxin deducatur; non ſolum enim maximum campum aperit, ſed etiam pro α valorem non nimis magnum largitur, quoniam in illa formula radicali cubica termini poſt λ ſequentes omnes fiunt valde parui eoque minores, quo maior fuerit multiplicatio, quoniam proxime fit $k = m(\sqrt{2}-1) = \frac{2}{5}m$. Tum vero hic etiam numerus ϑ arbitrio noſtro permittitur, quo efficere poſſumus, vt lentes poſtremae non fiant nimis exiguae, ſumto autem ϑ pro lubitu quantitas η ſequenti aequatione definietur, quia enim ſupra inuenimus

$$\vartheta = \frac{(\eta k^2 - k - 1)(m+k)}{k^2 + 2mk + m} \text{ ob } m(m-2) = 2mk + k^2$$

et

et $m + k = \sqrt{2m(m-1)}$ erit

$\vartheta = \frac{(\eta k^2 - k - 1)\cdot\sqrt{2}}{\sqrt{m(m-1)}}$ hincque

$\eta = \frac{k+1}{k^2} + \frac{\vartheta\sqrt{m(m-1)}}{k^2\sqrt{2}}$

ex quo valore interuallum ſecundae et tertiae lentis innoteſcit.

Problema 4.

323. Si teleſcopium huius generis ita ex quatuor lentibus ſit componendum, vt vna lens inter imaginem ſecundam et ocularem conſtituatur, indolem eius indagare eiusque conſtructionem deſcribere.

Solutio.

Quia igitur hic prima imago inter lentem primam et ſecundam, ſecunda vero imago inter lentem ſecundam et tertiam cadit, litterae P et Q erunt negatiuae, manente ſola R poſitiua. Quare ſi ſtatuatur $P = -k$ et $Q = -k'$ erunt elementa noſtra

$b = \frac{\alpha}{k}$; $\beta = \frac{B\alpha}{k}$; $c = \frac{B\alpha}{kk'}$; $\gamma = \frac{BC\alpha}{kk'}$

et $d = \frac{-BC\alpha}{kk'R} = \frac{-BC\alpha}{m}$.

Hincque interualla

$\alpha + b = \alpha(1 + \frac{1}{k})$; ideoque α poſitiuum

$\beta + c = \frac{B\alpha}{k}(1 + \frac{1}{k'})$

ergo $B > 0$. et $\mathfrak{B} > 0$ et ſimul $\mathfrak{B} < 1$

$\gamma + d = \frac{BC\alpha}{kk'}(1 - \frac{1}{R})$ ergo $C(1 - \frac{1}{R}) > 0$.

 Pro

Pro loco autem oculi erit $O = \frac{d}{Mm}$ quae vt ſit poſitiua debet eſſe $d > 0$, vnde haec noua reſultat conditio, vt ſit $C < 0$ quae conditio cum antecedente coniuncta dat $1 - \frac{1}{R} < 0$ ideoque $R < 1$. Quodſi iam ponamus $\pi = -\omega\xi$, $\pi' = i\xi$ et $\pi'' = -\xi$, vt fiat

$$\Phi = \frac{\omega + i + 1}{m - 1}\xi = M\xi, \text{ exiſtente } M = \frac{\omega + i + 1}{m - 1};$$

aequationes noſtrae fundamentales erunt $\mathfrak{B}\omega = -(1 + k)M$ et $\mathfrak{C}i = -(1 - kk')M - \omega$; ex quarum priore ſtatim ob $\mathfrak{B} > 0$ liquet fore $\omega < 0$.

Deſtructio autem marginis colorati poſtulat, vt ſit

$$0 = \frac{\omega}{P} + \frac{1}{PQ} + \frac{1}{PQR}; \text{ ideoque}$$

$$0 = -\frac{\omega}{k} + \frac{i}{kk'} + \frac{1}{kk'R} \text{ vnde } R = \frac{1}{\omega k' - i},$$

vt ergo R prodeat poſitiuum, i neceſſario debet eſſe numerus negatiuus. Statuamus ergo $\omega = -\zeta$ et $i = -y$, vt iam ſit pro campo apparente $M = \frac{1 - y - \zeta}{m - 1}$ ideoque $y + \zeta < 1$. Cum igitur ſit $R = \frac{1}{y - k'\zeta}$ atque hinc $m = \frac{kk'}{y - k'\zeta}$ notandum eſt, ob $R < 1$ et $R = \frac{m}{kk'}$ eſſe debere $kk' > m$; hinc quia eſt

$$y = k'\zeta + \frac{kk'}{m}; \text{ erit } y > 1,$$

ideoque multo magis $y + \zeta > 1$, quod cum ſit abſurdum patet, huius problematis caſum locum habere non poſſe.

Scho-

Scholion.

324. Cum igitur hoc problema penitus ſit excludendum, cum aeque parum conditioni marginis colorati ſatisfacere poſſit atque primum tribus tantum lentibus adhibitis, relinquuntur nobis tantum problema ſecundum ac tertium. Quia autem ex ſecundo caſus prorſus ſingularis ibi annotatus maxime reliquis omnibus antecellit, quemadmodum etiam ex tertio caſus vltimus prae ceteris maximam attentionem meretur, hinc conſtituemus duas praecipuas ſpecies teleſcopiorum tertii generis easque ſeorſim ita pertractabimus, vt primo oſtendamus, quemadmodum vtraque vna vel pluribus lentibus ex eodem vitro adiiciendis, deinde etiam ex diuerſo vitro ad maiorem perfectionis gradum euehi queant. Harum duarum vero ſpecierum poſterior ideo potiſſimum eſt notanda, quia teleſcopia communia terreſtria dicta quaſi in ſe complectitur, reuera enim ab iis differt plurimum, quatenus a vitiis, quibus haec inſtrumenta, vti vulgo fabricari ſolent, laborant, eſt liberata; vnde ſi etiam plures lentes in ſubſidium vocare nolimus, hinc regulae dari poterunt, haec teleſcopia terreſtria ita perficiendi, vt maior perfectio exſpectari nequeat. Prior autem ſpecies, quae longe aliam lentium ocularium diſpoſitionem poſtulat, olim prorſus fuit ignota ac nuper demum a ſolertiſſimo Dollondo in praxin introduci eſt coepta. Quatenus ſcilicet lentibus minima apertura praeditis eſt vſus; neque tamen a ſola experientia ſum-

mus perfectionis gradus, cuius haec ſpecies eſt capax, ſperari poterat. Hoc tamen facile eſt animaduerſum, niſi inſuper vna lens adiungatur, campum nimis fore paruum, quam vt ii acquieſcere queamus. Vidimus enim campum ſemper aliquanto eſſe minorem, quam in tubis aſtronomicis vulgaribus, ad quod remedium etiam in ſequentibus recurremus. Denique circa hanc ſpeciem annotari conuenit, nos in poſterum iis menſuris eſſe vſuros, quae in paragrapho § 314 ſunt ſtatutae, vbi ſcilicet poſuimus $\zeta = \frac{1}{\sqrt{n}}$, cum inde aptiſſimae ad praxin determinationes obtineri videantur.

SECTIO-

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SECTIONIS TERTIAE.

CAPVT II.

DE TELESCOPIIS TERRESTRIBVS COMMVNIBVS EORVMQVE PERFECTIONE.

Definitio.

325.

Character huiusmodi teleſcopiorum in hoc conſiſtit, quod radii per duas priores lentes transmiſſi iterum inter ſe fiant paralleli, ita, vt haec teleſcopia ex duobus tubis aſtronomicis ſint compoſita.

Coroll. I.

326. Cum haec teleſcopia ex quatuor lentibus conſtent, quarum tam binae priores, quam binae poſteriores ſecundum rationem tuborum aſtronomicorum ſibi ſunt iunctae; multiplicatio teleſcopii eſt in ratione compoſita ambarum multiplicationum, quas ambo iſti tubi aſtronomici producerent.

Coroll.

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Coroll. 2.

327. Scilicet ſi lentis primae ponatur diſtantia focalis $= p$; ſecundae $= q$; tertiae $= r$ et quartae $= s$ binae priores lentes ad interuallum $= p + q$ diſpoſitae multiplicationem praebent $= \frac{p}{q}$; binae poſteriores vero ad interuallum $= r + s$ diſpoſitae multiplicationem $= \frac{r}{s}$; teleſcopium compoſitum multiplicationem producet $= \frac{pr}{qs}$.

Scholion 1.

328. Statim ab initio binae lentes poſteriores inter ſe factae ſunt aequales et quidem eiusdem diſtantiae focalis, ac lens ſecunda, quae tres lentes oculares vocari ſolent, ita, vt tum tubus poſterior nullam plane multiplicationem producat ob $r = s = q$. Quanto autem interuallo hi duo tubi ſiue lentes ſecunda et tertia a ſe inuicem debeant eſſe remotae, auctores non ſatis definiunt; plerumque autem hoc ſpatium fieri iubent $= 2q$ ita, vt cum etiam ſit $r = s = q$ tota longitudo futura ſit $= p + 5q$. Deinde autem artifices obſeruarunt, haec teleſcopia meliorem effectum producere, ſi tres lentes poſteriores continuo certa ratione diminuantur, id quod egregie conuenit cum iis, quae ſupra de hoc teleſcopiorum genere annotauimus, vbi non ſolum multo maiorem campum, iis conciliauimus, quam vulgaris conſtructio ſuppeditat, ſed etiam id inprimis effecimus, vt margo colo-

coloratus penitus euaneſceret. Quocirca praecepta pro conſtructione ante inuenta hic ordine proponi conueniet:

Conſtructio Teleſcopiorum terreſtrium ex quatuor lentibus compoſitorum pro quauis multiplicatione m.

Quanta ſtatui debeat lentis obiectiuae diſtantia focalis deinceps definiemus, quando pro ſingulis lentibus ſequentibus numeros λ, λ', λ'', λ''' aſſignauerimus.

I. Si igitur $p = a$ denotet diſtantiam focalem lentis obiectiuae, eius figuram vtique ex numero $\lambda = 1$ peti conueniet, ita, vt ſi ratio refractionis ſit $n = 1,55$ habeatur

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{p}{\sigma} = 0,6144 \cdot p. \\ \text{poſter.} = \frac{p}{\rho} = 5,2439 \cdot p. \end{cases}$$

pro eius apertura ſemidiameter hactenus poſita eſt $\frac{m}{50}$ dig. Sin autem vel maior claritas deſideretur vel minor ſufficiat, loco 50 vel numerus maior vel minor aſſumi poterit.

Interuallum lentis ſecundae a prima debet eſſe $= p + q$, vbi valor ipſius q mox indicabitur.

II. Pro lente ſecunda ſi eius diſtantia focalis ponatur $= q$, in ſuperiore capite vidimus, ſumi conuenire $q = \frac{p}{k}$ exiſtente $k = -m + \sqrt{2m(m-1)}$ et quia pro eius apertura debet eſſe $\omega = \frac{-(1+k)(m+k)}{m(m-1)}$ qui valor pro maioribus multiplicationibus erit circi-

ter $\omega = -\frac{5}{5}$, vnde cum haec apertura non ſit maxima, etiam non opus eſt, vt haec lens fiat vtrinque aeque conuexa ſed ſufficiet, vt pro ea ſumatur $\lambda' = 1$ vnde huius lentis conſtructio erit

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{q}{\varrho} = 5,2439. \\ \text{poſter.} = \frac{q}{\sigma} = 0,6144. \end{cases}$$

Et aperturae ſemidiameter ſi capiatur $= \frac{1}{4}\frac{q}{\sigma}$ conditioni praeſcriptae ſatisfaciet.

Diſtantia autem tertiae lentis a ſecunda, quae ſupra eſt poſita $= \eta\, a$, definita eſt

$$\eta = \frac{k+1}{k^2} + \frac{\theta \sqrt{m}(m-1)}{k^2 \sqrt{2}}$$

vbi numerus ϑ arbitrio noſtro relinquitur, quem autem neque multo maiorem neque minorem vnitate ſumi conueniet.

III. Pro tertia lente quoniam ea maximam aperturam recipere debet ob $i = 1$, ideoque vtrinque aeque conuexa confici debet, erit $\lambda'' = 1,6299$ et cum eius diſtantia focalis ſit $r = \frac{\theta b}{k}$ erit radius vtriusque faciei $= 1,10\, r$ cuius pars quarta dabit ſemidiametrum aperturae.

Ab hac lente diſtantia ad quartam eſt

$$= r + s = \vartheta\, a\left(\frac{1}{k} + \frac{1}{m}\right).$$

IV. Quia quarta lens etiam maximam aperturam admittere ideoque etiam vtrinque aequaliter convexa

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vexa eſſe debet, pro ea etiam erit $\lambda''' = 1,6299$; vnde cum eius diſtantia focalis ſit $s = \frac{\theta a}{m}$ erit radius vtriusque faciei $= 1, 10. s$ et $\frac{1}{4} s$ dabit ſemidiametrum eius aperturae, tum vero diſtantia ab hac lente ad oculum erit

$$= \frac{s}{Mm} = \frac{s. m-1}{\sqrt{2m(m-1)}} = \frac{s\sqrt{m-1}}{\sqrt{2m}}.$$

V. Hocque teleſcopium campum oſtendet, cuius ſemidiameter eſt $\Phi = \frac{\sqrt{2}}{\sqrt{m(m-1)}}. \xi$ ſeu in menſura angulorum $\Phi = \frac{1217}{\sqrt{m(m-1)}}$ min.

VI. Tota autem huius inſtrumenti longitudo ad oculum vsque erit

$$= \left(\frac{(k+1)^2}{k^2} + \frac{\theta. m-1. \sqrt{2}(m-1)}{k^2\sqrt{m}}\right) p.$$

VII. Pro diſtantia autem focali p, ſi deſideretur claritas $y = \frac{1}{30}$ dig. et pro gradu diſtinctionis $k = 50$, vt ſit $kx = m$ ob litteram μ parum ab vnitate deficientem debebit ſumi in digitis

$$p = m\sqrt[3]{m}\left(1 + \frac{1}{k} + \frac{1,6290}{\theta^3}\left(\frac{1}{k} + \frac{1}{m}\right)\right)$$

ac ſi tam minore claritate, puta $y = \frac{1}{70}$ et minore gradu diſtinctionis puta $k = 35$ acquieſcere velimus, iſte valor ipſius p ad ſemiſſem redigi poterit.

Exemplum.

329. Si huiusmodi teleſcopium tantum nouies multiplicare debeat, vt ſit $m = 9$, reperietur $k = 3$

 hinc-

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hincque $q = \frac{p}{3}$; $r = \frac{\vartheta p}{3}$ et $s = \frac{\vartheta p}{9}$; vnde erit totius teleſcopii longitudo $= (\frac{16}{9} + \frac{32}{27}\vartheta)p$ et ſemidiameter campi $= 2^\circ\, 23'$.

Tum vero diſtantia focalis p ita aſſumi debebit

$$p = 9\sqrt[3]{9\left(1 + \frac{1}{3} + \frac{4}{9}\cdot\frac{1,6299}{\vartheta^3}\right)}$$

ſumto ergo $\vartheta = 1$, vt longitudo fiat $= \frac{80}{27}p$, ſeu propemodum $= 3p$ colligetur $p = 9\sqrt[3]{18,5196}$. ſeu propemodum $p = 24$ dig. vnde longitudo tota $= 72$ dig. $= 6$ ped.; quae longitudo, vti animaduertimus, ad ſemiſſem reduci poſſet.

Scholion 2.

330. Verum etiam longitudo trium pedum pro tam exigua multiplicatione enormis videbitur, praecipue cum vulgo eiusmodi teleſcopia circumferantur multo breuiora magisque amplificantia. At praecipua cauſa huius longitudinis in campo apparente eſt ſita, quem maximum producere ſumus conati, qui ſine dubio multo maior eſt, quam in vulgaribus eiusmodi inſtrumentis deprehenditur. Interim tamen deſtructio marginis colorati non parum ad longitudinem confert perinde ac inſignis claritatis et diſtinctionis gradus, qui nobis erat propoſitus, ex quo inſtrumenta ſecundum haec praecepta parata plurimum antecellent iis, quae vulgo circumferuntur et quae plerumque tot tantisque vitiis laborant, vt in praxi vix tolerari queant. Non mediocriter autem eorum longitudo

gitudo diminui poſſet, ſi loco lentis obiectiuae ſiue lens duplicata ſiue etiam triplicata, quales ſupra ex principio minimi ſunt inuentae, ſubſtituantur, ſiquidem tum valor ipſius λ priori caſu ad $\frac{1}{5}$, poſteriore vero ad $\frac{1}{24}$ reduceretur; ita, ſi in noſtro exemplo λ fuiſſet $= \frac{1}{5}$ inueniſſemus $p = 20$ dig. et teleſcopii longitudo adhuc ad 5 pedes excreuiſſet. Sin autem lente obiectiua triplicata vſi eſſemus, vt fuiſſet $\lambda = \frac{1}{24}$ prodiiſſet $p = 19\frac{1}{5}$ dig. vnde patet, a lentibus illis duplicatis et triplicatis, quales ſupra ſunt deſcriptae atque adeo a lentibus perfectis, vbi foret $\lambda = 0$, haud notabile decrementum longitudinis exſpectari poſſe, ſaltem pro minoribus multiplicationibus, vbi poſt ſignum radicale cubicum termini λ ſequentes admodum ſunt notabiles pro maioribus autem multiplicationibus maius lucrum eſſet futurum, quod vix tamen ad ſemiſſem redire poſſet. Quare pro hac ſpecie teleſcopiorum praecipue in id eſt incumbendum, vt lens obiectiua ita duplicetur vel triplicetur vt non ſolum confuſio ab ipſa oriunda, ſed et ea, quae a ſequentibus lentibus omnibus naſcitur, ad nihilum redigatur, tum enim diſtantiam p maiorem ſtatui non erit neceſſe, quam apertura ob claritatem requiſita poſtulat; quem caſum in ſequente problemate ita euoluamus, vt exiguum ſpatium intra lentes priores admittamus.

 Proble-

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Problema 2.

331. In hac teleſcopiorum ſpecie loco lentis obiectiuae eiusmodi binas lentes ex eodem vitro parandas ſubſtituere, vt omnis confuſio etiam a reliquis lentibus oriunda ad nihilum redigatur, ſicque his teleſcopiis minima longitudo concilietur.

Solutio.

Cum igitur hic habeantur quinque lentes, ſtatuamus noſtras fractiones $\frac{\alpha}{b} = -P$; $\frac{\beta}{c} = -Q$; $\frac{\gamma}{d} = -R$ et $\frac{\delta}{e} = -S$. Quarum litterarum prima P proxime erit $= 1$; ſecunda Q erit negatiua $= -k$; tertia R etiam erit $= 1$, ſed ita tamen, vt interuallum tertium $\gamma + d$ fiat quantitas finita, ſcilicet $\eta\,\alpha$; denique vero erit $S = -k'$, ita, vt noſtra elementa futura ſint

$$b = -\frac{\alpha}{P};\ c = -\frac{B\alpha}{Pk};\ d = \frac{BC\alpha}{PkR} = -\infty$$

$$\beta = -\frac{B\alpha}{P};\ \gamma = -\frac{BC\alpha}{Pk} = \infty;$$

$$\delta = \frac{BCD\alpha}{PkR};\ e = \frac{BCD\alpha}{PkRk'} = \frac{BCD\alpha}{m}.$$

Hincque interualla

1°. $\alpha + b = \alpha(1 - \frac{1}{P}) = \frac{\alpha}{50}$ vti ſupra iam aſſumſimus, ita vt ſit $P = 1\frac{1}{49}$.

2°. $\beta + c = -\frac{B\alpha}{P}(1 + \frac{1}{k})$;

3° $\gamma + d = -\frac{BC\alpha}{Pk}(1 - \frac{1}{R}) = \eta\,\alpha$ vbi ſcilicet eſt $C = \infty$, hincque $\frac{1}{R} = 1 + \frac{\eta Pk}{BC}$.

4°. quia

4°. quia erat $C = \infty$ debet eſſe D infinite parvum, ita, vt ſit $CD = -\vartheta$ eritque hoc interuallum $\delta + e = -\frac{B\theta\alpha}{PkR}(1 + \frac{1}{k'})$; exiſtente multiplicatione $m = PkRk'$ ſeu proxime $m = kk'$. Quia autem fieri poſſet, vt diſtantiam α negatiuam capi expediret, ſtatuamus primum interuallum $\alpha + b = \zeta\alpha$ fietque $P = \frac{1}{1-\zeta}$, vbi notandum, ſi α eſſet quantitas negatiua, tam ζ, quam η negatiue accipi debere, ſemper autem neceſſe erit, vt ſit $-B\alpha > 0$ ſeu $B\alpha < 0$ et $\vartheta > 0$, vti initio iam aſſumſimus, vbi poſuimus $CD = -\vartheta$. Cum nunc pro campo apparente ſit $\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{m-1}$ ſtatuamus $\pi = -v\xi$; $\pi' = \omega\xi$; $\pi'' = -\xi$ et $\pi''' = \xi$, vt ſit $\Phi = \frac{v + \omega + 2}{m-1}\xi = M\xi$ exiſtente $M = \frac{v + \omega + 2}{m-1}$; ex quibus pro loco oculi colligimus $O = \frac{e}{Mm}$, exiſtente $e = \frac{-B\theta\alpha}{m}$; conſideremus nunc noſtras formulas fundamentales

1°. $\mathfrak{B}v = (P - 1)M$

2°. $\mathfrak{C}\omega = -(1 + Pk)M - v$

3°. $\mathfrak{D} = -(1 + PkR)M - v - \omega$

de quibus obſeruari oportet fore primam $\mathfrak{B}v = \frac{\zeta}{1-\zeta}M$; ſicque valor v ob duplicem cauſam fiet quantitas minima, ita, vt etiam mv adhuc ſit valde paruum. Pro ſecunda autem, quia eſt $C = \infty$ erit $\mathfrak{C} = \frac{C}{C+1} = 1 - \frac{1}{C}$; pro tertia autem, quia eſt $D = 0$ ſeu potius $D = -\frac{\vartheta}{C}$ erit $\mathfrak{D} = \frac{-\vartheta}{C-\vartheta} = \frac{-\vartheta}{C}$ deinde etiam hic recordar. oportet eſſe $R =$

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$$R = \frac{BC}{BC + \eta P k} = 1 - \frac{\eta P k}{BC}$$

quia igitur ex ſecunda aequatione ob

$$\mathfrak{C} = \frac{C}{1+C} \text{ eſt } \omega = -(1 + Pk)\,M(1 + \tfrac{k}{C}) - v(1 + \tfrac{k}{C}),$$

ſi hic valor in tertia aequatione ſubſtituatur, erit

$$-\frac{\vartheta}{C} = -(1 + Pk)\,M + \frac{\eta P^2 k^2 M}{BC}$$
$$-v + (1 + Pk)\,M(1 + \tfrac{k}{C}) + v(1 + \tfrac{k}{C})$$

vbi cum termini finiti ſe mutuo deſtruant, ex infinite paruis concluditur fore

$$\vartheta = -\frac{\eta P^2 k^2 . M}{B} - (1 + Pk)\,M - v$$

vnde fit

$$\eta = -\frac{(1 + Pk)B}{P^2 k^2} - \frac{B\vartheta}{P^2 k^2 M} - \frac{Bv}{P^2 k^2 M}$$

vbi terminus vltimus tuto omitti poteſt.

Deſtructio porro marginis colorati poſtulat hanc aequationem

$$0 = \frac{v}{P} + \frac{\omega}{PQ} + \frac{1}{PQR} + \frac{1}{PQRS}$$

quae pro noſtro caſu fit

$$0 = v - \frac{\omega}{k} - \frac{1}{k} + \frac{1}{kk'}$$

vnde neglecto termino primo deducitur $k' = \frac{1}{\omega + 1}$; et ob $m = P\,k\,k'$ erit $m = \frac{Pk}{\omega + 1}$.

Cum autem ſit $\omega = -(1 + Pk)\,M$ et $M = \frac{\omega + 2}{m - 1}$ neglecto termino v fiet $(m - 1)\omega = -(1 + Pk)(\omega + 2)$ hincque $\omega = \frac{-2(1 + Pk)}{m + Pk}$, atque $M = \frac{2}{m + Pk}$. Quare

cum

cum ſit $m = \frac{Pk}{\omega + 1}$ ſubſtituto valore ipſius ω obtinemus $m - \frac{2m(1 + Pk)}{m + Pk} = Pk$; hincque

$$mm - 2m = P^2 k^2 + 2Pkm,$$

quae m^2 vtrinque addito praebet

$$2m(m - 1) = (Pk + m)^2 \text{ ideoque}$$

$$Pk = -m + \sqrt{2m(m - 1)}.$$

Hoc ergo valore pro Pk aſſumto, pro campo apparente adipiſcemur maximum valorem, qui erit $\Phi = \frac{2}{\sqrt{2m(m-1)}} . \xi$ et in menſura angulorum ob $\xi = \frac{1}{4}$ erit $\Phi = \frac{1718}{\sqrt{2m(m-1)}}$ min. Nunc autem praecipuum opus ſupereſt in eo conſiſtens, vt binae priores lentes ita definiantur, vt formula pro ſemidiametro confuſionis inuenta penitus euaneſcat, vnde ſequens aequatio erit reſoluenda:

$$0 = \lambda - \frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{\lambda''}{B^3 Pk} - \frac{\lambda'''}{B^3 \theta^3 Pk} - \frac{\lambda''''}{B^3 \theta^3 m} \quad \text{ſeu}$$

$$0 = \lambda - \frac{\lambda'}{\mathfrak{B}^3 P} - \frac{\lambda''}{B^3 Pk} - \frac{\lambda'''}{B^3 \theta^3 Pk} - \frac{\lambda''''}{B^3 \theta^3 m} - \frac{\nu}{\mathfrak{B} B P}$$

in qua aequatione vt ante iam vidimus ſumi poteſt $\lambda'' = 1$ et quia duae poſtremae lentes debent eſſe vtrinque aequaliter conuexae, erit pro vitro communi $\lambda''' = \lambda'''' = 1,6299$. Ex hac vero aequatione vel λ vel λ' definiri debet, prouti coefficiens ipſius λ' maior eſt vnitate, ſiue minor. Ceterum notandum eſt, omnes quantitate hic praeter litteras B et $\mathfrak{B}$ ſatis eſſe determinatas, ita, vt in hoc negotio tantum litterae B et $\mathfrak{B}$ arbitrio noſtro permittantur; in quo duo caſus ſunt perpendendi, alter, quo $\mathfrak{B}$ eſt fractio vnitate maior,

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ior, puta $\frac{1+i}{i}$; alter vero, quo eſt vnitate minor, puta $= \frac{i}{1+i}$.

Primo ſit $\mathfrak{B} = \frac{1+i}{i}$ erit $B = -1 - i$ ideoque numerus negatiuus, quo ergo caſu α debet eſſe poſitiuum ſeu prima lens conuexa, ſecunda vero concaua, pro qua valor λ' determinari debet, et quidem ex hac aequatione

$$\lambda' = \frac{(1+i)^3 P\lambda}{i^3} + \frac{\lambda''}{i^3 k} + \frac{\lambda'''}{i^3 \theta^3 k} + \frac{P\lambda''''}{i^3 \theta^3 . m} + \frac{(1+i)\nu}{i^2}$$

vbi ſumto $\lambda = 1$ euidens eſt λ' fieri vnitate maius.

At ſecundo ſi ſit $\mathfrak{B} = \frac{i}{1+i}$ erit $B = i$ ideoque poſitiuum; vnde diſtantia α fiet negatiua ſiue prima lens concaua, ſecunda vero conuexa, quo caſu numerus λ definiri oportet per hanc aequationem:

$$\lambda = \frac{(1+i)^3 \lambda'}{i^3 P} + \frac{\lambda''}{i^3 Pk} + \frac{\lambda'''}{i^3 \theta^3 Pk} + \frac{\lambda''''}{i^3 \theta^3 m} + \frac{(1+i)\nu}{i^2 P}$$

atque hic ſumi poterit $\lambda' = 1$; λ vero vnitate maius fiet.

Perſpicuum igitur eſt, ſimili fere modo, quo in priore caſu λ' definitur, in ſecundo caſu litteram λ definiri, propterea quod proxime eſt $P = 1$; quandoquidem inuenimus $P = \frac{1}{1-\zeta}$, vbi notetur priore caſu, quo α eſt poſitiuum, ſumi poſſe $\zeta = \frac{1}{50}$, vt ſit $P = \frac{50}{49}$, eodemque modo etiam η erit poſitiuum, quemadmodum etiam noſtra formula poſito $B = -1 - i$ declarat, ſcilicet $\eta = \frac{(1+Pk)(1+i)}{P^2 k^2} + \frac{(1+i)\theta}{P^2 k^2 M}$.

Pro

Pro altero autem caſu, quo α eſt quantitas negatiua, ſumi debet $\zeta = -\frac{1}{30}$, vt ſit $P = \frac{50}{31}$; ob eandemque rationem etiam η fiet negatiuum, ſcilicet

$$\eta = -\frac{i(1+Pk)}{P^2k^2} - \frac{\theta i}{P^2k^2M}.$$

Coroll. 1.

332. Cum tollendo marginem coloratum peruenerimus ad hanc aequationem

$$Pk = -m + \sqrt{2m(m-1)},$$

qua ob P datum, valor ipſius k determinatur, hinc habebimus

$$M = \frac{2}{\sqrt{2m(m-1)}} \text{ et } \omega = \frac{-2(1+Pk)}{\sqrt{2m(m-1)}} \text{ atque hinc}$$

$$\eta = \frac{-(1+Pk)B}{P^2k^2} - \frac{B\theta\sqrt{2m(m-1)}}{2P^2k^2}.$$

Coroll. 2.

333. Cum ſit $C = \infty$, $D = 0$ et $CD = -\vartheta$, fient noſtra elementa

$$b = \frac{-\alpha}{P};\ c = \frac{-B\alpha}{Pk};\ d = \infty;\ e = \frac{-B\theta\alpha}{m};$$

$$\beta = \frac{-B\alpha}{P};\ \gamma = \infty;\ \delta = \frac{-B\theta\alpha}{Pk};$$

hincque diſtantiae focales

$$p = \alpha;\ q = \frac{-B\alpha}{P};\ r = \frac{-B\alpha}{Pk};\ s = \delta = \frac{-B\theta\alpha}{Pk};\ t = \frac{-B\theta\alpha}{m}$$

tum vero lentium interualla

$$\alpha + b = \alpha\left(1 - \frac{1}{P}\right) = \zeta\alpha;\ \beta + c = -\frac{B\alpha}{P}\left(1 + \frac{1}{k}\right)$$

$$\gamma + d = \eta\alpha;\ \delta + e = -B\vartheta\alpha\left(\frac{1}{Pk} + \frac{1}{m}\right)$$

ac denique diſtantia oculi $O = \frac{-B\vartheta\alpha\sqrt{2m(m-1)}}{2m^2}$ vbi notetur, litteram ϑ arbitrio noſtro permitti, quo caueri poterit, ne vltimae lentes fiant nimis paruae.

Coroll. 3.

334. Ex his perſpicitur, quo maior capiatur littera B, eo maius prodire ſecundum interuallum cum ſequentibus, hincque longitudinem teleſcopii eo magis augeri; at littera B eo maior euadit, quo propius littera $\mathfrak{B}$ ad vnitatem accedit; ſiue enim ſit $\mathfrak{B} = \frac{1+i}{i}$ ſiue $\mathfrak{B} = \frac{i}{1+i}$, aucto numero i augetur numerus B; quare cum littera $\mathfrak{B}$ etiam nunc arbitrio noſtro permittatur, neutiquam expediet, eam vnitati nimis propinquam ſtatui, neque tamen etiam conueniet pro i numerum valde paruum aſſumi, veluti dimidium vel fractionem adhuc minorem; tum enim ex vltima aequatione numerus vel λ vel λ' prodiret nimis magnus, ſcilicet adeo maior, quam 27. Vnde concluditur, numerum i ad minimum vnitate maiorem capi debere.

Coroll. 4.

335. Hic igitur commodum cum incommodo compenſatur; ſi enim i vnitate minus caperetur, obtineremus commodum breuitatis tubi; contra vero nimis magnus valor numeri λ vel λ' inſigne eſſet incommodum; ſin autem numerum i vnitate multo maio-

maiorem ſumeremus; obtineremus quidem commodum, vt λ vel λ' parum vnitatem excederent, contra vero tubus fieret nimis longus.

Coroll. 5.

336. Sin autem optio detur inter valores $\frac{1+i}{i}$ et $\frac{i}{1+i}$ pro $\mathfrak{B}$ aſſumendos, retinente i in vtroque eundem valorem; tunc λ vel λ' eundem fere valorem nanciſceretur. Verum priore caſu cum fiat $B = -1 - i$ longitudo tubi maior prodiret, quam altero caſu, quo eſſet $B = i$, quam ob rem ſemper conſultius eſt, poſteriorem caſum eligere, quo lens prima eſt concaua et ſecunda conuexa; quam priorem, vbi viciſſim lens prima eſſet conuexa, ſecunda vero concaua.

Scholion 1.

337. Quae quo clarius perſpiciantur, ponamus $i = 2$ et $\mathfrak{B} = \frac{2}{3}$, vt fiat $B = 2$ tum igitur erit $P = \frac{50}{51}$ et elementa noſtra ſequenti modo ſe habebunt; exiſtente α quantitate negatiua:

$$b = \frac{-51}{50}\alpha; \; c = \frac{-2\alpha}{Pk}; \; d = -\infty; \; e = \frac{-2\theta\alpha}{m};$$

$$\beta = \frac{-2\alpha}{P} = \frac{-51\alpha}{25}; \; \gamma = \infty; \; \delta = \frac{-2\theta\alpha}{Pk}$$

exiſtente $Pk = -m + \sqrt{2m(m-1)}$ tum vero diſtantiae focales erunt

$$p = \alpha; \; q = \frac{-51\alpha}{75}; \; r = \frac{-2\alpha}{Pk}; \; s = \frac{-2\theta\alpha}{Pk} \text{ et } t = \frac{-2\theta\alpha}{m}.$$

at interualla lentium

$$\alpha + b = -\tfrac{1}{50}\cdot\alpha;\ \beta + c = -\tfrac{51}{25}\alpha - \tfrac{8\alpha}{Pk}$$

$$\gamma + d = \eta\alpha = \tfrac{-2(1+Pk)\alpha}{P^2k^2} - \tfrac{\theta\sqrt{2m(m-1)}}{P^2k^2}$$

$$\delta + e = \tfrac{-2\theta\alpha}{m} - \tfrac{2\theta\alpha}{Pk} = \tfrac{-2\theta\alpha}{Pkm}\sqrt{2m(m-1)}$$

et diſtantia oculi $O = \frac{-\theta\alpha\sqrt{2m(m-1)}}{mm}$ quibus factis campi ſemidiameter erit $\Phi = \frac{1718}{\sqrt{2m(m-1)}}$ min.

Pro apertura autem tertiae lentis notandum eſt, eſſe $\omega = \frac{-2(1+Pk)}{\sqrt{2m(m-1)}}$, ita, vt ſi m ſit numerus ſatis magnus fiat $\omega = -\frac{10}{17}$; vnde cum haec lens non maximam aperturam, ſed minorem, quae ſit ad maximam, vt 10 : 17, requirat, ſufficiet pro hac lente ſumſiſſe $\lambda'' = 1$; quare ſi et $\lambda' = 1$ at $\lambda''' = \lambda'''' = 1,6299$, pro lente obiectiua inueniemus

$$\lambda = \tfrac{27.51}{8.50} + \tfrac{1}{8Pk} + \tfrac{1,6299}{8\theta^3 Pk} + \tfrac{1,6299}{8\theta^3 m} + \tfrac{153\nu}{200}$$

exiſtente $\nu = 0,2326$ pro refractione ſcilicet $n = 1,55$.

Hinc autem inuento numero λ, prima lens obiectiua concaua ita conſtrui debet, vt fiat radius faciei anterioris $= \frac{\alpha}{\sigma - \tau\sqrt{\lambda - 1}}$ poſterioris vero $= \frac{\alpha}{\varrho + \tau\sqrt{\lambda - 1}}$; exiſtente $\varrho = 0,1907$; $\sigma = 1,6274$; $\tau = 0,9051$.

Pro ſecunda autem lente capi debebit radius faciei anter. $= \frac{2b}{2\varrho + \sigma}$ et poſterioris $= \frac{2b}{2\sigma + \varrho}$; exiſtente $b = -\frac{59}{50}\cdot\alpha$.

Pro

Pro tertia lente erit radius faciei anterioris $= \frac{c}{\rho}$ et poſterioris $= \frac{c}{\sigma}$, exiſtente $\sigma = \frac{-2\alpha}{Pk}$.

Pro quarta vero lente radius vtriusque faciei $= 1,10\, s$ et pro quinta lente radius faciei vtriusque $= 1,10\, t$.

Ad menſuras vero abſolutas inueniendas conſideretur in conſtructione lentium primae et ſecundae minimus radius, qui ſit $= m\alpha$, cuius pars quarta $\frac{1}{4} m\alpha$ aequetur ſemidiametro aperturae ob claritatem requiſitae, qui ſit $\frac{m}{50}$ dig. hincque fit $\alpha = \frac{-2m}{25m}$ dig. quae menſura ſi forte det vltimas lentes nimis exiguas, vt ſupra vſu venit, tantum litterae θ tribuatur valor vnitate pro lubitu maior; cum hinc longitudo teleſcopii vix augeatur. Colligitur autem tota haec longitudo ad oculum vsque

$$= -\alpha\left(2\tfrac{3}{10} + \frac{2(1+2Pk)}{P^2 k^2} + \frac{(m+Pk)^3\theta}{m^2 P^2 k^2}\right).$$

Exempl. I.

338. Si fuerit $m = 9$, erit $Pk = 3$ et $k = \frac{153}{50}$ ob $P = \frac{50}{51}$; vnde elementa teleſcopii erunt

$$b = -\tfrac{51}{50}\alpha;\ \beta = -\tfrac{51}{33}\cdot\alpha;\ c = -\tfrac{2\alpha}{3};$$

$$\gamma = \infty;\ d = -\infty;\ \delta = -\tfrac{2\theta\alpha}{3};\ e = -\tfrac{2\theta\alpha}{9};$$

et diſtantiae focales

$$p = \alpha;\ q = -\tfrac{17}{33}\alpha;\ r = -\tfrac{2}{3}\alpha;\ s = -\tfrac{2\theta\alpha}{3};\ t = -\tfrac{2\theta\alpha}{9};$$

et

et interualla

$$\alpha + b = -\tfrac{8}{70}\alpha;\ \beta + c = -\tfrac{207}{72}\alpha;$$

$$\gamma + d = -\tfrac{(1+12\theta)}{9}\alpha;\ \delta + e = -\tfrac{18\alpha}{9}$$

et diſtantia Oculi $O = -\frac{40\alpha}{27}$.

Tum vero campi apparentis ſemidiameter

$$\Phi = 143 \text{ min.} = 2^{\circ}\, 23'.$$

Nunc vero habebimus

$$\lambda = 3,4425 + 0,04166 + \frac{0,09055}{\vartheta^3}$$

$$+ 0,1779$$

$$3,6204$$

$$0,0416$$

$$\lambda = 3,6620 + \frac{0,09055}{\vartheta^3}.$$

Sumamus nunc $\vartheta = 1$. vt fiat $\lambda = 3,75255$;

$$\lambda - 1 = 2,75255 \text{ et } \tau.\sqrt{\lambda - 1} = 1,50162$$

quare conſtructio lentis primae ita ſe habebit

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{\alpha}{0,1258} = 7,9491.\alpha \\ \text{poſter.} = \frac{\alpha}{1,6923} = 0,5909.\alpha \end{cases}$$

Pro ſecunda autem lente erit

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{2b}{0,5088} = -1,0155.\alpha \\ \text{poſter.} = \frac{2b}{1,4455} = -0,5921.\alpha \end{cases}$$

Pro tertia autem lente erit

$$\text{radius faciei} \begin{cases} \text{anter.} = \frac{c}{0,1607} = -3,4959.\alpha \\ \text{poſter.} = \frac{c}{1,6274} = -0,4097.\alpha \end{cases}$$

Pro

Pro lente quarta

radius faciei vtriusque $= - 0,7333. \alpha$

Pro lente denique quinta

radius faciei vtriusque $= - 0,2444. \alpha$

Iam in duabus prioribus lentibus occurrit

radius minimus $= 0,5909\, \alpha$, vt ſit

$m = 0,5909$, adeoque $\alpha = - \frac{72}{58,59}$ dig.

ſeu $\alpha = - 1\frac{1}{4}$ dig.

Vnde ſequens prodibit

Conſtructio huius Teleſcopii pro multiplicatione $m = 9$. lentibus ex vitro communi factis.

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = - 9,93 \text{ dig.} \\ \text{poſter.} = - 0,73 \text{ dig.} \end{cases}$

cuius diſtantia focalis $= - 1\frac{1}{4}$ dig.

ſemidiameter aperturae $= 0,18$. dig.

diſtantia ad lentem ſecund. $= 0,025$. dig.

II. Pro ſecunda lente

radius faciei $\begin{cases} \text{anter.} = 1,27. \text{ dig.} \\ \text{poſter.} = 0,74. \text{ dig.} \end{cases}$

cuius diſtantia focalis $= 0,85$ dig.

ſemidiameter aperturae vt ante $= 0,18$ dig.

diſtantia ad lentem tertiam $= 3,38$ dig.

III. Pro tertia lente

radius faciei { anter. $= +$, 37 dig.
poſter. $=$ 0, 51 dig.

cuius diſtantia focalis $= +$ 0, 83 dig.

ſemidiameter aperturae $=$ 0, 13 dig.

diſtantia ad quartam $=$ 2, 78 dig.

IV. Pro quarta lente

radius vtriusque faciei $=$ 0, 92 dig.

cuius diſtantia focalis $= +$ 0, 83. dig.

ſemidiameter aperturae $=$ 0, 23 dig.

interuallum ad quintam $=$ 1, 11 dig.

V. Pro quinta lente

radius vtriusque faciei $=$ 0, 30 dig.

cuius diſtantia focalis $=$ 0, 28 dig.

ſemidiameter aperturae $=$ 0, 07 dig.

et diſtantia ad oculum $=$ 0, 19 dig.

ſicque tota inſtrumenti longit. $=$ 7, 49 dig.

et ſemidiameter campi $= 2° 23'$.

Hac ergo perfectione adhibita teleſcopium, quod ante erat 6 ped. reductum eſt ad $7\frac{1}{2}$ dig.

Exeml. II.

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Exempl. II.

339. Si multiplicatio ſit $m = 50$, erit $Pk = 20$ et $k = \frac{102}{5}$ vnde elementa noſtra erunt

$$b = -\tfrac{51}{50}\alpha;\ \beta = -\tfrac{51}{25}\alpha;\ c = -\tfrac{\alpha}{10};\ \gamma = \infty;$$
$$d = -\infty;\ \delta = -\tfrac{\theta\alpha}{10};\ e = -\tfrac{\theta\alpha}{25};$$

et diſtantiae focales

$$p = \alpha;\ q = -\tfrac{17}{25}\alpha;\ r = -\tfrac{\alpha}{10};\ s = -\tfrac{\theta\alpha}{16}\ \text{et}\ t = -\tfrac{\theta\alpha}{25};$$

et interualla lentium

$$\alpha + b = -\tfrac{1}{50}\alpha;\ \beta + c = -\tfrac{107}{50}\alpha;$$
$$\gamma + d = \tfrac{-(21 + 35\theta)\alpha}{200};\ \delta + e = \tfrac{-7\theta\alpha}{50};$$

atque diſtantia oculi $O = -\frac{7\theta\alpha}{250}$ et campi apparentis ſemidiameter erit $= 24\frac{1}{2}$ min.

Nunc vero prodibit

$$\lambda = 3,4425 + \tfrac{0,0143}{\theta^2}$$
$$+\ 0,0063$$
$$0,1779$$
$$\lambda = 3,6267 + \tfrac{0,0143}{\theta^2}.$$

Sumatur nunc $\vartheta = 2$, eritque $\lambda = 3,6285$;

$$\lambda - 1 = 2,6285 \text{ et } \tau\sqrt{(\lambda - 1)} = 1,4674;$$

vnde fiet

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = \frac{\alpha}{0,1600} = 6,2500.\ \alpha. \\ \text{poſter.} = \frac{\alpha}{1,6581} = 0,6031.\ \alpha. \end{cases}$

II. Pro ſecunda lente, vti ante

radius faciei $\begin{cases} \text{anter.} = -1,0155.\alpha \\ \text{poſter.} = -0,5921.\alpha \end{cases}$

III. Pro tertia lente

radius faciei $\begin{cases} \text{anter.} = \frac{c}{8,1967} = -0,5244.\alpha \\ \text{poſter.} = \frac{c}{1,7474} = -0,0615.\alpha \end{cases}$

IV. Pro quarta lente

radius faciei vtriusque $= -0,2200.\alpha$

V. Pro quinta lente

radius faciei vtriusque $= -0,0880.\alpha$

Iam cum ſit in duabus prioribus lentibus radius minimus $0,5921.\alpha$, erit

$m = 0,5921$, adeoque $\alpha = -\frac{400}{59,21}$ dig.

ita, vt capi poſſet $= -7$ dig.

Vnde ſequens prodibit

Conſtructio huius Teleſcopii pro multiplicatione $m = 50$.

I. Pro prima lente

radius faciei $\begin{cases} \text{anter.} = -43,75 \text{ dig.} \\ \text{poſter.} = -4,22 \text{ dig.} \end{cases}$

cuius diſtantia focalis $= -7$. dig.

ſemidiameter aperturae $= 1,05$. dig.

diſtantia ad lentem ſecundam $= 0,14$. dig.

II. Pro

II. Pro ſecunda lente

radius faciei { anter. = 7, 11. dig.
poſter. = 4, 14. dig.

cuius diſtantia focalis eſt 4, 76 dig.

ſemidiameter aperturae, vt ante, = 1 dig.

interuallum ad tertiam lentem 14, 98 dig.

III. Pro tertia lente

radius faciei { anter. = 3, 67. dig.
poſter. = 0, 43. dig.

diſtantia focalis eſt 0, 7. dig.

ſemidiameter aperturae = 0, 11 dig.

interuallum ad quartam = 3, 18 dig.

IV. Pro quarta lente

radius vtriusque faciei = 1, 54 dig.

cuius diſtantia focalis eſt 1, 40 dig.

ſemidiameter aperturae = 0, 38 dig.

interuallum ad quintam lentem = 1, 96 dig.

V. Pro quinta lente

radius vtriusque faciei = 0, 61 dig.

cuius diſtantia focalis = 0, 56 dig.

ſemidiameter aperturae = 0, 14 dig.

diſtantia ad oculum = 0, 39 dig.

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ſicque longitudo tota $= 20\frac{2}{3}$ dig. propemodum et ſemidiameter campi $= 24\frac{1}{2}$ min.

Scholion 2.

340. Hoc ergo etiam poſtremum teleſcopium facile per tubos ductitios ita parari poteſt, vt commode quis ſecum id portare poſſit, cum lente illa concaua omiſſa hoc teleſcopium vltra viginti pedes excreviſſet. Circa tubos autem ductitios hic notari oportet, dum ductus ad oculum accommodatur, ſolam lentem ocularem mobilem eſſe debere, reliquas vero lentes omnes in locis hic aſſignatis perpetuo conſiſtere debere, id quod in perpetuum de omnibus teleſcopiis, quae hic tractantur, eſt tenendum ceterum non opus eſt, vt perfectioni quam variae vitri ſpecies largiuntur, caput peculiare tribuamus, vt hactenus fecimus, ſed ſolutio praecedentis problematis paucis mutandis ad hunc ſcopum accommodari poteſt, vti in problemate ſequente oſtendemus.

Problema 3.

341. Si prima lens obiectiua concaua ex vitro chryſtallino paretur, dum reliquae ex vitro coronario conficiuntur, conſtructionem teleſcopii deſcribere, in quo non margo ſolum coloratus, ſed etiam tota confuſio a diuerſa radiorum refrangibilitate oriunda penitus deſtruatur.

Solutio

Solutio.

Hoc problema, vt hactenus fecimus, ex principiis ſupra ſtabilitis ſi reſoluere vellemus, omnia plane eodem modo ſe eſſent habitura, vti in problemate praecedente vsque ad eum locum, vbi marginem coloratum ſuſtulimus, atque etiam haec ipſa aequatio non eſſet diſcrepatura ab ea, quam in praecedenti problemate tractauimus, quoniam in ea prima lens non in computum venit, ita, vt hinc etiam eaedem determinationes obtinerentur atque hucusque litterae $\mathfrak{B}$ et B etiam nunc manſurae eſſent indeterminatae; iam autem demum vltimae aequationis, qua confuſio penitus e medio tollitur, ratio erit habenda et aequatio eo pertinens ſi pro prima lente formulam differentialem $\frac{dn}{n-1}$ littera N, pro ſequentibus autem lentibus litteris N′ denotemus per hasque aequationem diuidamus, habebimus

$$0 = \frac{N}{N'} - \frac{1}{\mathfrak{B}P} - \frac{1}{B\mathfrak{C}^2 Pk} - \frac{1}{BP\theta k} - \frac{1}{B\theta m}$$

in qua aequatione terminus tertius cum ſequentibus prae duobus primis tam ſunt exigui, vt ſine errore negligi queant, praecipue cum vti iam ſaepius notauimus, natura rei non permittat, vt haec aequatio adcurate reſoluatur, neque id etiam ſcopus noſter poſtulet. Quare ſumtis tantum duobus terminis prioribus colligemus $\mathfrak{B} = \frac{N'}{NP}$, ſcilicet ob hanc conditionem lentis primae e vitro chryſtallino parandae totum diſcrimen in reſolutione, in hoc tantum conſiſtit, vt nunc

nunc cum littera $\mathfrak{B}$ ante arbitrio noſtro manſiſſet relicta, definiatur; quocirca quia ex Dollondi experimentis habemus $N : N' = 10 : 7$ ac praeterea ſit $P = \frac{50}{31}$ conſequimur nunc $\mathfrak{B} = \frac{357}{500}$ qui valor proxime reducitur ad hanc $\mathfrak{B} = \frac{5}{7}$; ſiue etiam $\mathfrak{B} = \frac{2}{3}$, qui eſt ipſe valor, quem in praecedentibus iam exemplis ipſi $\mathfrak{B}$ tribuimus; quicunque autem valor ipſi $\mathfrak{B}$ tribuatur, in aequationem vltimam, ex qua numerus λ definitur, leue quoddam diſcrimen ingreditur, cum enim nunc primus terminus per μ, ſequentes vero per μ' ſint multiplicandi, diuiſione per μ' facta haec aequatio fiet

$$\frac{\mu}{\mu'}\lambda = \frac{\lambda'}{\mathfrak{B}^3 P} + \frac{\lambda''}{B^3 P k} + \frac{\lambda'''}{B^3 \theta^3 P k} + \frac{\lambda''''}{B^3 \theta^3 . m} + \frac{\nu}{\mathfrak{B} B P}$$

vbi, vt ante, ſumi poteſt $\lambda' = 1$ et $\lambda'' = 1$, at quia lentes poſteriores ex vitro coronario, quo $n = 1,53$ conficiuntur, pro duabus poſtremis lentibus, quae vtrinque aequaliter conuexae eſſe debent, erit $\lambda''' = \lambda'''' = 1,60006$, litterae autem eo pertinentes erunt

$\mu' = 0,9875$; $\nu' = 0,2196$; $\varrho' = 0,2267$ et
$\sigma' = 1,6601$; $\tau' = 0,9252$.

Pro prima autem lente chryſtallina erit

$\mu = 0,8724$; $\nu = 0,2529$; $\varrho = 0,1414$;
$\sigma = 1,5827$ et $\tau = 0,8775$.

Coroll. I.

342. Nunc igitur demum intelligitur, cur praeſtet, primam lentem ex vitro chryſtallino parare, quam ſecun-

ſecundam, ſi enim prima eſt chryſtallina ſit $\mathfrak{B} = \frac{5}{7}$ et $B = \frac{5}{2}$. Sin autem ſecundam chryſtallinam faceremus, foret $\mathfrak{B} = \frac{7}{5}$ et $B = -\frac{7}{2}$. Quare cum omnes ſequentes diſtantiae multiplicatae ſint per B, eae ac propterea tota longitudo tubi prodiret poſteriore caſu maior, quam primo, idque in ratione 7 : 5.

Coroll. 2.

343. Si diſcrimen diſperſionis ambarum vitri ſpecierum minus eſſet, quam hic ſecundum Dollondi experimenta aſſumimus; tunc fractio pro $\mathfrak{B}$ aſſumenda propius ad vnitatem accederet, indeque B maiorem nanciſceretur valorem ſicque inſtrumentum longius euaderet; ex quo ad praxin plurimum expedit, vt duae vitri ſpecies ratione diſperſionis maxime inter ſe differentes eligantur ſiquidem hoc modo teleſcopia multo breuiora redderentur.

Scholion.

344. Quoniam igitur hic primam lentem ex vitro chryſtallino, reliquas ex coronario fieri aſſumimus, experimentis Dollondianis innixi ſtatuamus $\mathfrak{B} = \frac{5}{7}$, vt ſit $B = \frac{5}{2}$ ac poſito $\vartheta = 2$, ne lens ocularis fiat nimis parua, elementa noſtra ſequenti modo ſe habebunt:

$$b = -\tfrac{51}{50}a;\ c = -\tfrac{5\alpha}{2Pk};\ d = -\infty;\ e = -\tfrac{5\alpha}{m};$$
$$\beta = -\tfrac{51}{50}a;\ \gamma = \infty;\ \delta = -\tfrac{5\alpha}{Pk};$$

et diſtantiae focales

$$p = \alpha;\ q = -\tfrac{51}{75}\alpha;\ r = -\tfrac{5\alpha}{2Pk};\ s = +\tfrac{5\alpha}{Pk};\ t = -\tfrac{5\alpha}{m};$$

hincque interualla

$$a + b = -\tfrac{1}{50}\alpha;\ \beta + c = -\tfrac{51}{25}\alpha - \tfrac{5\alpha}{2Pk}$$

$$\gamma + d = \eta\alpha = -\tfrac{5(1+Pk)\alpha}{2P^2k^2} - \tfrac{5\cdot\sqrt{2m(m-1)}\,\alpha}{2P^2k^2}$$

$$\delta + e = -\tfrac{5\alpha\sqrt{2m(m-1)}}{mPk}$$

et diſtantia oculi $O = -\frac{5\sqrt{2m(m-1)}}{2m^2}\alpha$

exiſtente $Pk = -m + \sqrt{2m(m-1)}$

tum autem ſemidiameter campi $\Phi = \frac{1718}{\sqrt{2m(m-1)}}$ min.

Vt igitur hinc conſtructionem pro quauis multiplicatione m inueſtigemus, methodo iam ſaepius adhibita vtentes primo euoluamus caſum, quo $m = 25$ tum vero caſum, quo $m = \infty$.

Exemplum I.

345. Sit multiplicatio $m = 25$ ac reperietur $\sqrt{2m(m-1)} = 34,64101$; hincque $Pk = 9,64101$; vnde interualla ita ſe habebunt:

$a + b = -0,02\,\alpha$; $\beta + c = -2,80930.\,\alpha$

$\gamma + d = -1,21770.\,\alpha$; $\delta + e = -0,71860.\,\alpha$

et diſtantia Oculi $= -0,13844.\,\alpha$

His

His praemiſſis quaeratur λ ex aequatione ſupra data et inuenietur

$$\lambda = 3,16815 + 0,007514 + 0,001502$$
$$+ 0,000579 + 0,14198 \text{ ſeu}$$

$$\lambda = 3,31972; \text{ vnde fit } \tau \sqrt{(\lambda - 1)} = 1,33648.$$

Hinc igitur ſi F et G denotent radios anterioris et poſterioris faciei, habebimus

I. Pro prima lente chryſtallina

$$F = \frac{\alpha}{\sigma - 1,3363} = \frac{\alpha}{0,2462} = 4,0617\,\alpha$$
$$G = \frac{\alpha}{\rho + 1,3363} = \frac{\alpha}{1,4779} = 0,6766.\,\alpha$$

II. Pro ſecunda autem lente coronaria erit

$$F = \frac{sb}{s\rho' + 2\sigma'} = \frac{sb}{4,3337} = -1,1451\,\alpha$$
$$G = \frac{sb}{s\sigma' + 2\rho'} = \frac{sb}{1,7559} = -0,5826\,\alpha$$

quae conſtructio pro omni multiplicatione valet.

III. Pro tertia lente coronaria habebimus

$$F = \frac{c}{\rho'} = \frac{c}{0,2267} = -\frac{11,0278.\alpha}{Pk} = -1,1438.\,\alpha$$
$$G = \frac{c}{\sigma'} = \frac{c}{1,6606} = -\frac{1,5055.\alpha}{Pk} = -0,1562.\,\alpha$$

vbi valores penultimi pro omni multiplicatione valent.

IV. Pro quarta lente itidem coronaria, cuius diſtantia focalis $= s = -\frac{5\alpha}{Pk}$, erit

$$F = G = 1,06.\,s = -\frac{5,30.\alpha}{Pk} = -0,5497.\,\alpha$$

vbi valor penultimus pro omni multiplicatione valet.

V. Pro quinta lente etiam coronaria cuius diſtantia focalis eſt $t = -\frac{5a}{m}$, erit

$$F = G = 1,06.\, t = -\frac{5,3a}{m} = -0,212.\, a$$

vbi iterum forma penultima pro omni multiplicatione valet.

Exemplum II.

346. Si ſit multiplicatio m infinita ſeu praegrandis, erit $\sqrt{2m(m-1)} = m\sqrt{2} = 1,41421.\, m$ hincque $Pk = 0,41421\, m$; vnde interualla erunt

$$\alpha + b = -0,02\, a;$$

$$\beta + c = -2,55\, a - 6,0356.\, \frac{a}{m};$$

$$\gamma + d = -26,6425.\frac{a}{m};\quad \delta + e = -17,0712.\frac{a}{m};$$

et diſtantia oculi $O = -3,5355.\, \frac{a}{m}$.

His praemiſſis quaeratur λ ex aequatione data et habebitur

$$\lambda = 3,16815 + 0,14198 = 3,31013$$

vnde fit $\tau\sqrt{(\lambda - 1)} = 1,3337$; quare habebitur

I. Pro prima lente

$$F = \frac{a}{6 - 1,3337} = \frac{a}{0,2490} = 4,0160.\, a$$

$$G = \frac{a}{\varrho + 1,3337} = \frac{a}{1,4751} = 0,6779.\, a$$

II. Secunda lens conuenit cum exemplo praecedente.

III. Pro tertia lente erit

$$F = \frac{-11,0278.a}{Pk} = -26,6237.\, \frac{a}{m}$$

$$G = \frac{-1,5058.a}{Pk} = -3,6357.\, \frac{a}{m}$$

IV. Pro

IV. Pro quarta lente erit

$$F = G = \frac{-5,9\alpha}{9n} = -12,7955 \cdot \frac{\alpha}{m}$$

V. Pro quinta denique lente

$$F = G = -5,3 \cdot \frac{\alpha}{m}$$

Elementa autem ſequenti modo ſe habebunt:

$$b = -1,02\,\alpha;\; c = -6,0355 \cdot \frac{\alpha}{m};\; \delta = -12,0710 \cdot \frac{\alpha}{m}$$

$$\beta = -2,55\,\alpha;\; \gamma = \infty;\; d = -\infty;\; e = -5 \cdot \frac{\alpha}{m}$$

hincque diſtantiae focales

$$p = \alpha;\; q = -0,72857 \cdot \alpha;\; r = -6,0355 \cdot \frac{\alpha}{m}$$

$$s = -12,0710 \cdot \frac{\alpha}{m};\; t = -5\,\frac{\alpha}{m}.$$

Exemplum III.

347. Ex collatione praecedentium exemplorum pro quauis multiplicatione maiore m conſtructionem huiusmodi teleſcopiorum deſcribere.

Primo elementa ſequenti modo expreſſa reperientur:

$$b = -1,02\,\alpha;\; \beta = -2,55 \cdot \alpha;$$

$$c = -\left(6,0355 + \frac{11,1750}{m}\right)\frac{\alpha}{m};\; \gamma = \infty;\; d = -\infty;$$

$$\delta = -\left(12,0710 + \frac{22,3500}{m}\right)\frac{\alpha}{m};\; e = -5 \cdot \frac{\alpha}{m};$$

Hincque diſtantiae focales:

$$p = \alpha;\; q = -0,72857 \cdot \alpha;\; r = -\left(6,0355 + \frac{11,1750}{m}\right)\frac{\alpha}{m};$$

$$s = -\left(12,0710 + \frac{22,3500}{m}\right)\frac{\alpha}{m};\; t = -5 \cdot \frac{\alpha}{m};$$

 et

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et interualla lentium

$$\alpha+b=-0{,}02\,\alpha;\quad \beta+c=-2{,}55\,\alpha-\left(6{,}0355+\frac{11{,}1750}{m}\right)\frac{\alpha}{m};$$

$$\gamma+d=-\left(26{,}6425+\frac{25}{m}\right)\frac{\alpha}{m};$$

$$\delta+e=-\left(17{,}0710+\frac{22{,}5500}{m}\right)\frac{\alpha}{m};$$

et diſtantia oculi $O=-\left(3{,}5355-\frac{1{,}8625}{m}\right)\frac{\alpha}{m}$;
et tandem ſemidiameter campi ſemper eſt

$$\Phi=\frac{1718}{\sqrt{2m(m-1)}}\ \text{min.}$$

Lentium vero conſtructio ipſa ita ſe habebit:

I. Pro prima lente chryſtallina

radius faciei $\begin{cases}\text{anter.} = \left(4{,}0160+\frac{1{,}14}{m}\right)\alpha\\ \text{poſter.} = \left(0{,}6779-\frac{0{,}2525}{m}\right)\alpha\end{cases}$

II. Pro ſecunda lente coronaria

radius faciei $\begin{cases}\text{anter.} = -1{,}1451.\,\alpha\\ \text{poſter.} = -0{,}5826.\,\alpha\end{cases}$

III. Pro tertia lente coronaria

radius faciei $\begin{cases}\text{anter.} = -\left(26{,}6237+\frac{49{,}28}{m}\right)\frac{\alpha}{m}\\ \text{poſter.} = -\left(3{,}6357+\frac{6{,}73}{m}\right)\frac{\alpha}{m}\end{cases}$

IV. Pro quarta lente coronaria

radius vtriusque faciei $=-\left(12{,}7953+\frac{23{,}68}{m}\right)\frac{\alpha}{m}$

V. Pro quinta lente coronaria

radius vtriusque faciei $=-5{,}30.\frac{\alpha}{m}$.

Nunc

Nunc denique iudicandum reſtat, quantum valorem ipſi α tribui conueniat. Hunc in finem conſideretur duarum priorum lentium radius minimus, qui eſt $-0,5826.\alpha$ cuius pars quarta $-0,1456.\alpha$ ponatur aequalis ſemidiametro aperturae $\frac{m}{50}$; indeque reperietur $\alpha = -\frac{m}{7,28}$; quo quidem valore quantitas α minor accipi non debet; quocirca ſumatur $\alpha = -\frac{m}{7}$; atque obtinebitur ſequens

Conſtructio huiusmodi Teleſcopiorum pro quauis multiplicatione *m*.

Poſita igitur diſtantia focali $\alpha = -\frac{m}{7}$ dig. impetrabimus pro conſtructione quaeſita ſequentes menſuras.

I. Pro prima lente chryſtallina

radius faciei $\begin{cases} \text{anter.} = (-0,5737.m - 0,16)\,\text{dig.} \\ \text{poſter.} = (-0,0968\,m + 0,004)\,\text{dig.} \end{cases}$

cuius diſtantia focalis $= -\frac{m}{7}$ dig.

ſemidiameter aperturae $= \frac{m}{50}$ dig.

interuallum ad lentem ſecundam $= 0,00286.m$ dig.

II. Pro ſecunda lente coronaria

radius faciei $\begin{cases} \text{anter.} = 0,1636.\,m\ \text{dig.} \\ \text{poſter.} = 0,0832.\,m\ \text{dig.} \end{cases}$

cuius diſtantia focalis eſt $= 0,10408.\,m$ dig.

ſemidiameter aperturae $= \frac{m}{35}$ dig.

interuall. ad lentem tertiam $= (0,3643.m + 0,86 + \frac{1,6}{m})$ dig.

III. Pro

III. Pro tertia lente coronaria

radius faciei $\begin{cases} \text{anter.} = (3,80 + \frac{7,24}{m})\ \text{dig.} \\ \text{poſter.} = (0,52 + \frac{0,9}{m})\ \text{dig.} \end{cases}$

cuius diſtantia focalis eſt $(0,86 + \frac{1,6}{m})$ dig.

ſemidiameter aperturae $= 0,13$ dig.

interuallum ad quartam $= (3,80 + \frac{14}{m})$ dig.

IV. Pro quarta lente coronaria

radius faciei vtriusque $= (1,82 + \frac{3,4}{m})$ dig.

cuius diſtantia focalis eſt $(1,72 + \frac{3,2}{m})$ dig.

ſemidiameter aperturae $= 0,43$ dig.

interuallum ad quintam $= (2,44 + \frac{3,2}{m})$ dig

V. Pro quinta lente coronaria

radius vtriusque faciei $= 0,76$. dig.
cuius diſtantia focalis $= 0,71$. dig.
ſemidiameter aperturae $= 0,18$ dig.
interuallum ad oculum $= (0,50 - \frac{0,2}{m})$ dig.

VI. Tota ergo, teleſcopii longitudo inde colligitur haec: $(0,3672.\, m + 7,60 + \frac{15,6}{m})$ dig. vnde patet, ſi $m = 100$, longitudinem inſtrumenti non eſſe ſuperaturam $44\frac{1}{2}$ dig.

VII. Semidiameter denique campi apparentis erit $\Phi = \frac{1718}{\sqrt{2,m\,(m-1)}}$ min., qui ergo pro $m = 100$ fiet 12 minut.

Scho-

Scholion.

348. Haec ergo teleſcopia adhuc ſatis breuia forent, ſi modo in praxi lentes quam exactiſſime ſecundum menſuras praeſcriptas liceret elaborare et ſi etiam vtraque vitri ſpecies praeciſe eandem refractionem admitteret, quam hic ſuppoſuimus; perpetuo autem tenendum eſt, ſi vitri refractio diſcrepet ab ea, quam aſſumſimus, tunc totum calculum de nouo eſſe inſtituendum, qui ſcilicet ad formationem lentium ſpectat; deinde vero etiam haec regula probe eſt obſeruanda, vt, quo minus feliciſſimum ſucceſſum ab artifice exſpectare queamus, menſurae hic praeſcriptae augeri atque adeo duplicari vel triplicari debeant; id quod commodiſſime fiet, ſi digiti menſuram multo maiorem accipiamus. Semper autem etiamſi artifex ſummam induſtriam adhibeat, vix vnquam ſperandum erit, vt primum ſtatim, quod produxerit, inſtrumentum voto reſpondeat; quin potius ſemper neceſſe erit, vt lentis primae concauae praeſertim plura exempla elaborentur, vt ex iis optimum per experientiam eligi poſſit; quamuis enim eaedem menſurae retineantur; tamen ſemper vſu veniet, vt plura exempla omnia inter ſe al quantillum diſcrepent. Quin etiam ſaepe conſultum erit, ipſam menſuram pro conſtructione huius lentis aliquantillum immutare, ita tamen, vt eadem diſtantia focalis conſeruetur, et pro quauis menſura aliquot exempla conficere, ſcilicet ſi ex theoria radii facierum anterioris et poſterioris iſtius lentis in-

uenti fuerint F et G, haic figuram ſaepe ita immutari conueniet, vt capiatur radius faciei anterioris $= F \mp F^2 \omega$ poſterioris vero $= G \pm G^2 . \omega$, ſumendo pro ω tantilla fractione, quae adhuc in praxi ſentiri queat; tum enim in diſtantia focali nihil mutabitur. Denique etiam quaedam monenda reſtant circa diaphragmata in huiusmodi teleſcopiis vſurpanda, quia enim in iis duae imagines reales reperiuntur in vtriusque loco etiam diaphragma conſtitui poterit, cuius apertura ipſam illam imaginem capere debet. Primae autem imaginis ſemidiameter eſt

$$= \alpha \Phi B = B \alpha M \xi = \tfrac{1}{4} M B \alpha.$$

eſt vero M in noſtro caſu $= \frac{2}{\sqrt{2m(m-1)}}$ et $B = \frac{5}{8}$ adeoque iſte ſemidiameter erit $= \frac{5 . \alpha}{4\sqrt{2m(m-1)}}$ ſumtoqe $\alpha = \frac{m}{7}$, vt ante, ſemidiameter iſte erit

$$= \frac{5m}{28\sqrt{2m(m-1)}} = \frac{5}{28\sqrt{2}} = \tfrac{1}{8} \text{ dig.}$$

niſi m ſit numerus paruus. Secundae autem imaginis ſemidiameter eſt $= \alpha \Phi . BCD = \alpha \Phi B \vartheta$; quarecum ſumſerimus $\vartheta = 2$ poſterius diaphragma aperturam habere debet cuius ſemidiameter ſit duplo maior, quam antecedens, ſcilicet $\frac{1}{4}$ dig. a quo vero nullus vſus exſpectari poterit, cum poſtremae lentes ipſae multo minorem aperturam poſtulent, ita, vt ſolum diaphragma prius vtilitatem habere poſſit, cui etiam, ſi libuerit, micrometrum adplicari poterit.

SECTIO-

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SECTIONIS TERTIAE.

CAPVT III.

DE
ALTERA TERTII GENERIS TELESCOPIORVM SPECIE PRINCIPALI, EORVMQVE PERFECTIONE.

Definitio.

349.

Ad alteram hanc ſpeciem referimus ea teleſcopia, quae ſupra §. 310. et quidem ſpeciatim in ſubnexo Corollario 2. §. 314. ſunt explicata, in quibus ſcilicet lens ſecunda adhuc ante primam imaginem realem collocatur; tertia vero lens poſt hanc imaginem in eo loco, vbi lentis primae inſtar obiecti conſideratae imago per ſecundam lentem projiceretur, qui locus cum ante imaginem ſecundam cadat, lens quarta ocularis in debito loco conſtituitur. Speciatim autem ſi primae lentis diſtantia focalis ponatur $= \alpha$, ſecunda lens ita ſtatuitur, vt ſit $b = -\frac{\alpha}{\sqrt{n}}$ ſiue interuallum primae et ſecundae lentis $= \alpha\left(1 - \frac{1}{\sqrt{n}}\right)$.

 Coroll.

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Coroll. 1.

350. Cum igitur haec teleſcopia quatuor conſtent lentibus, pro iis elementa ita ſe habebunt:

$$b = \frac{-\alpha}{\sqrt{m}};\ \beta = \frac{\sqrt{m}-1}{2m}\alpha;\ c = \frac{\sqrt{m}-1}{2m}.\alpha;$$

$$\gamma = \frac{\sqrt{m}-1}{2m}C\alpha;\ d = \frac{\sqrt{m}-1}{2m\sqrt{m}}.C\alpha;$$

ita, vt ſit $B = \frac{1-\sqrt{m}}{2\sqrt{m}}$; $\mathfrak{B} = \frac{1-\sqrt{m}}{1+\sqrt{m}}$; et C arbitrio noſtro relinquatur.

Coroll. 2.

351. Ex his elementis erunt lentium diſtantiae focales

$$p = \alpha;\ q = \frac{\sqrt{m}-1}{(1+\sqrt{m})\sqrt{m}}.\alpha;\ r = \frac{\sqrt{m}-1}{2m}\mathfrak{C}\alpha;$$

et $s = \frac{\sqrt{m}-1}{2m\sqrt{m}}.C\alpha$. et lentium interualla

$$\alpha + b = \left(1-\frac{1}{\sqrt{m}}\right)\alpha;\ \beta + c = \frac{\sqrt{m}-1}{m}.\alpha;\ \gamma + d = \frac{m-1}{2m\sqrt{m}}C\alpha$$

et diſtantia oculi $O = \frac{m-1}{2mm}.\alpha$; ita, vt tota longitudo futura ſit $= \frac{m-1}{m}\left(1+\frac{1+\sqrt{m}.C}{2m}\right).\alpha$ vbi tantum monendum eſt, pro C numerum poſitiuum accipi debere.

Coroll. 3.

352. Litterae autem maiuſculae P, Q, R pro hac ſpecie fient $P = \sqrt{m}$; $Q = -1$ et $R = -\sqrt{m}$ ita, vt hinc prodeat $PQR = m$, vti rei natura poſtulat.

Scho-

Scholion.

353. Hic autem inprimis rationem reddere oportet conditionis in definitione commemoratae, qua diximus, lentem tertiam ibi eſſe collocandam, vbi primae lentis inſtar obiecti conſideratae imago per ſecundam lentem proiecta eſſet caſura. Cum enim ſecundae lentis diſtantia focalis ſit $q = \frac{\sqrt{m}-1}{(1+\sqrt{m})\sqrt{m}} \cdot \alpha$, eius autem diſtantia a prima lente $= (1 - \frac{1}{\sqrt{m}})\alpha$, quae vocetur y, ſi prima lens vti obiectum conſideretur, eius imago poſt ſecundam lentem cadet ad diſtantiam $\zeta = \frac{yq}{y-q}$; eſt vero $y - q = \frac{\sqrt{m}-1}{\sqrt{m}+1}\alpha$ hincque $\zeta = \frac{\sqrt{m}-1}{m}\alpha$, cui praeciſe diſtantia tertiae lentis a ſecunda aequatur. Hanc autem conditionem ideo in definitionem introduximus, quoniam eius ope locus tertiae lentis facillime per praxin aſſignatur. Ceterum ſupra iam notauimus, ſemidiametrum campi apparentis fore $\Phi = \frac{850}{m+\sqrt{m}}$ min. qui vtique augmentatione indiget, cum has lentes perficere conabimur. Denique ibidem quoque eſt oſtenſum, ſemidiametrum aperturae tertiae lentis ſtatui debere $= \frac{\sqrt{n}}{50}$ dig.

Pro ſecunda autem lente, quia poſuimus, $\pi = \omega\xi$ et $\omega = -\zeta = -\frac{1}{\sqrt{m}}$, ſemidiameter eius aperturae eſſe debet $= \frac{q}{4\sqrt{m}} = \frac{\sqrt{m}-1}{4m(1+\sqrt{m})} \cdot \alpha$.

Problema 1.

354. Inter binas poſtremas lentes huius teleſcopiorum ſpeciei nouam lentem interere, qua campus apparens magis amplificetur.

Solutio.

Cum igitur hic occurrant quinque lentes ſtatuantur noſtrae quaternae fractiones:

$$\frac{\alpha}{b} = -\mathrm{P};\ \frac{\beta}{c} = -\mathrm{Q};\ \frac{\gamma}{d} = -\mathrm{R};\ \frac{\delta}{e} = -\mathrm{S}.$$

quarum litterarum duae debent eſſe negatiuae, quarum prior erit Q ſtatuaturque $\mathrm{Q} = -k$; altera vero erit R vel S; vtram autem negatiuam ſtatui conueniat, nondum definiamus. Hinc igitur elementa noſtra erunt

$$b = \frac{-\alpha}{\mathrm{P}};\ c = \frac{-\mathrm{B}\alpha}{\mathrm{P}k};\ d = \frac{\mathrm{BC}\alpha}{\mathrm{P}k\mathrm{R}};\ e = \frac{-\mathrm{BCD}\alpha}{\mathrm{P}k\mathrm{RS}};$$

$$\beta = \frac{-\mathrm{B}\alpha}{\mathrm{P}};\ \gamma = \frac{-\mathrm{BC}\alpha}{\mathrm{P}k};\ \delta = \frac{\mathrm{BCD}\alpha}{\mathrm{P}k\mathrm{R}};$$

diſtantiae autem focales:

$$p = \alpha;\ q = \frac{-\mathfrak{B}\alpha}{\mathrm{P}};\ r = \frac{-\mathrm{B}\mathfrak{C}\alpha}{\mathrm{P}k};\ s = \frac{\mathrm{BC}\mathfrak{D}\alpha}{\mathrm{P}k\mathrm{R}};\ t = \frac{-\mathrm{BCD}\alpha}{\mathrm{P}k\mathrm{RS}};$$

hincque lentium interualla

$$\alpha + b = \alpha\left(1 - \frac{1}{\mathrm{P}}\right);\ \beta + c = \frac{-\mathrm{B}\alpha}{\mathrm{P}}\left(1 + \frac{1}{k}\right)$$

$$\gamma + d = \frac{-\mathrm{BC}\alpha}{\mathrm{P}k}\left(1 - \frac{1}{\mathrm{R}}\right);\ \delta + e = \frac{+\mathrm{BCD}\alpha}{\mathrm{P}k\mathrm{R}}\left(1 - \frac{1}{\mathrm{S}}\right)$$

quae cum eſſe debeant poſitiua et α iam ſit poſitiuum, neceſſe eſt, vt ſit 1°. $\mathrm{P} > 1$; 2°. $\mathrm{B} < 0$; 3°. quod ad bina reliqua interualla attinet, duos caſus diſtingui conuenit.

Caſus prior, quo $\mathrm{R} > 0$ et $\mathrm{S} = -k'$, hocque caſu debet eſſe $\mathrm{C}\left(1 - \frac{1}{\mathrm{R}}\right) > 0$ et $\mathrm{CD} < 0$; quo ipſo etiam ſit e poſitiuum.

Caſus poſterior, quo $\mathrm{R} < 0$ ſeu $\mathrm{R} = -k'$ et $\mathrm{S} > 0$. Hoc ergo caſu eſſe debet $\mathrm{C} > 0$ ideoque etiam

$\mathfrak{C} > 0.$

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$\mathfrak{C} > 0$, at < 1 et $D(1 - \frac{1}{S}) > 0$. Vt autem etiam fiat $e > 0$, debet eſſe $D < 0$, ideoque $S < 1$.

Nunc igitur conſideremus campum apparentem, cuius ſemidiameter eſt $\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{m -}$ ac ſtatuamus, vt hactenus, $\pi = -\omega \xi$; $\pi' = 0$ ex natura huius ſpeciei; $\pi'' = -\xi$; et $\pi''' = \xi$ vt fiat

$$\Phi = \frac{\omega + 2}{m - 1} \xi = M \xi, \text{ exiſtente } M = \frac{\omega + 2}{m - 1};$$

atque hinc iam ſtatim pro loco oculi prodit

$$O = \frac{e}{Mm} = \frac{(m-1)e}{m(\omega + 2)}.$$

Aequationes porro fundamentales erunt:

1°. $\frac{\mathfrak{B}\pi}{\Phi} = 1 - P$; ſeu $\mathfrak{B}\omega = -(1 - P)M$

2°. $0 = -(1 + Pk)M - \omega$

3°. $\mathfrak{D} = -(1 + PkR)M - \omega$

vbi cum ex prima ſit $\omega = \frac{-(-P)M}{\mathfrak{B}}$ hic valor in ſecunda ſubſtitutus dat $0 = (1 + Pk)\mathfrak{B} + P - 1$; vnde ſequitur $\mathfrak{B} = -\frac{(P-1)}{1+Pk}$; ita, vt $\mathfrak{B}$ ac proinde etiam B ſit numerus negatiuus; fit autem $B = \frac{-(P-1)}{P(1+k)}$ et $\omega = -(1 + Pk)M$; tum vero ex tertia erit $\mathfrak{D} = Pk(1 - R)M$, litterae vero C et $\mathfrak{C}$ arbitrio noſtro manent relictae. Pro binis ergo caſibus memoratis erit

Pro priore, quo $S = -k'$, $\mathfrak{D} = Pk(1 - R)M$. Si ergo *fuerit* $R > 1$ debet eſſe $C > 0$ et $D < 0$ at cum fiat $\mathfrak{D} < 0$, ſponte illa conditio $D < 0$ impletur. *Sin autem ſit* $R < 1$ erit $\mathfrak{D} > 0$;

debet

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debet autem eſſe $C < 0$ et $D > 0$, conſequenter $\mathfrak{D} < 1$, ideoque $Pk(1-R)M < 1$.

Pro poſteriore caſu, quo $R = -k'$, erit $\mathfrak{D} = Pk(1+k')M$ ideoque $\mathfrak{D} > 0$ ante autem vidimus, hoc caſu eſſe debere $C > 0$ adeoque $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$. Tum vero $D(1-\frac{1}{S}) > 0$. Quare cum eſſe debeat $S < 1$, erit $D < 0$ vnde ob $\mathfrak{D} > 0$ colligitur $\mathfrak{D} > 1$.

Nunc pro tollendo margine colorato habebitur haec aequatio:

$$0 = \frac{\omega}{P} - \frac{1}{PkR} - \frac{1}{PkRS};$$

ex qua colligitur

$0 = \omega kRS - S - 1$; ſeu $0 = kRS(1+Pk)M + S + 1$ vbi ergo binos noſtros caſus diſtingui oportet.

I. Si $S = -k'$, habebitur $0 = -kk'.R(1+Pk)M - k' + 1$ vnde fit $R = \frac{1-k'}{kk'(1+Pk)M}$; vnde patet, eſſe debere $k' < 1$. vnde ſi prodeat $R > 1$, debet eſſe $C > 0$ et $D < 0$. Sin autem prodeat $R < 1$ debet eſſe $D > 0$, $C < 0$, $\mathfrak{D} > 0$ et $\mathfrak{D} < 1$. adeoque $Pk(1-R)M < 1$.

II. Si $R = -k'$, erit $0 = -kk'S(1+Pk)M + S + 1$ vnde colligitur $k' = \frac{S+1}{kS(1+Pk)M}$ quae expreſſio per ſe eſt poſitiua. Hoc autem caſu ſupra vidimus eſſe debere $C > 0$ adeoque $\mathfrak{C} < 0$ et $\mathfrak{C} < 1$ et $D < 0$, ita, vt hoc caſu ſumendum ſit $S < 1$.

Deni-

Denique hic meminiſſe oportet, eſſe $PkRS = -m$, quae conditio ſecundum binos caſus conſiderari debet.

I. caſu, quo $S = -k'$, ob $R = \frac{m}{Pkk'}$ noſtra aequatio dat $0 = -\frac{m}{P}(1 + Pk)M - k' + 1$ vnde colligitur $k' = 1 - \frac{m}{P}(1 + Pk)M$; ita, vt eſſe debeat $m(1 + Pk)M < P$ vbi notetur, ſi prodeat $R > 1$, eſſe debere $C > 0$ et $D < 0$; ſi autem prodeat $R < 1$, debere eſſe $C < 0$ et $D > 0$, $\mathfrak{D} > 0$ et $\mathfrak{D} < 1$.

II. caſu, ſi $R = -k'$, vt ſit $m = Pkk'S$, noſtra aequatio dat $0 = -\frac{m}{P}(1 + Pk)M + S + 1$; vnde colligitur $S = \frac{m}{P}(1 + Pk)M - 1$ ita, vt eſſe debeat $m(1 + Pk)M > P$. Cum autem debeat eſſe $S < 1$, etiam eſſe debet $m(1 + Pk)M < 2P$; praeterea recordemur, eſſe debere $C > 0$, adeoque $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$, et $D < 0$.

Tandem circa has formulas probe obſeruandum eſt ob valorem ω inuentum litteram M per reliqua elementa commode exprimi poſſe. Cum enim ſit

$\omega = -(1 + Pk)M$, aequatio $\frac{\omega + 2}{m - 1} = M$ dabit

$M = \frac{2}{m + Pk}$ et $\omega = \frac{-2(1 + Pk)}{m + Pk}$

ita, vt pro campo apparente prodeat

$\Phi = \frac{2}{m + Pk} \cdot \xi$ ſeu $\Phi = \frac{859}{m + Pk}$ min.

 Tum

Tum vero etiam pro loco oculi $O = \frac{e(m+Pk)}{2m}$.
Quibus obſeruatis binos caſus ſeorſim euoluamus.

I. Euolutio caſus, quo $S = -k'$.

355. Hoc ergo caſu elementa noſtra ita ſe habebunt:

$$b = -\frac{\alpha}{P};\ c = \frac{-B\alpha}{Pk};\ d = \frac{BC\alpha}{PkR};\ e = \frac{BCD\alpha}{m};$$
$$\beta = \frac{-B\alpha}{P};\ \gamma = \frac{-BC\alpha}{Pk};\ \delta = \frac{BCD\alpha}{PkR};$$

hincque interualla

$$\alpha + b = \alpha(1 - \tfrac{1}{P});\ \beta + c = -\tfrac{B\alpha}{P}(1 + \tfrac{1}{k})$$
$$\gamma + d = -\tfrac{BC\alpha}{Pk}(1 - \tfrac{1}{R});\ \delta + e = \tfrac{BCD\alpha}{PkR}(1 + \tfrac{1}{k'})$$

vbi ergo eſſe debet $P > 1$, et $\mathfrak{B} = \frac{-(P-1)}{1+Pk}$ hincque $B = \frac{-(P-1)}{P(1+k)}$.

Tertium vero interuallum dat hanc conditionem $C(1 - \frac{1}{R}) > 0$ et vltimum $CD < 0$; eſt autem $\mathfrak{D} = Pk(1-R)M = \frac{2Pk(1-R)}{m+Pk}$ et $D = \frac{2Pk(1-R)}{m-Pk+2PkR}$.

Deſtructio autem marginis colorati poſtulat, vt ſit

$$k' = 1 - \tfrac{m}{P}(1 + Pk)M = 1 - \frac{2m(1+Pk)}{P(m+Pk)}$$ et
$$R = \frac{m(m+Pk)}{k(P(m+Pk) - 2m(1+Pk))};$$

quamobrem debet eſſe $P(m+Pk) > 2m(1+Pk)$ ideoque $k < \frac{m(P-2)}{P(2m-P)}$; quare cum illa quantitas maior debeat eſſe, quam k, ob $2m > P$, debet eſſe $P > 2$; ex qua etiam conditione patet, ſemper eſſe debere

$R > 1$

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$R > 1$ adeoque $C > 0$ et $D < 0$, vti ex valore ipſius D manifeſtum eſt. Quo his conditionibus ſatisfiat formulaeque euadant ſimpliciores, ſtatuamus $Pk = \sqrt{m}$ vt fiat $M = \frac{2}{m+\sqrt{m}}$ ideoque $\Phi = \frac{2}{m+\sqrt{m}} \cdot \xi = \frac{1719}{m+\sqrt{m}}$ min. qui valor duplo maior eſt, quam ante. Tum vero erit $\omega = \frac{-2(1+\sqrt{m})}{m+\sqrt{m}}$; porro ſi capiatur $P = 4\sqrt{m}$, prodit $k = \frac{1}{4}$; $R = 2\sqrt{m}$ et $k' = \frac{1}{2}$ hincque $\mathfrak{D} = \frac{2(1-2\sqrt{m})}{1+\sqrt{m}}$ et $D = \frac{2(1-2\sqrt{m})}{5\sqrt{m}-1}$. Praeterea vero $\mathfrak{B} = -\frac{(4\sqrt{m}-1)}{1+\sqrt{m}}$ et $B = -\frac{(4\sqrt{m}-1)}{5\sqrt{m}}$; vnde omnia interualla prodibunt poſitiua, dummodo pro C ſumatur quantitas poſitiua.

II. Euolutio caſus, quo $R = -k'$.

356. Pro hoc ergo caſu deſtructio marginis colorati praebet

$$0 = -\frac{2m(1+Pk)}{P(m+Pk)} + S + 1$$

vnde concluditur

$$S = \frac{2m(1+Pk)}{P(m+Pk)} - 1$$

ita, vt eſſe debeat

$$2m(1+Pk) > P(m+Pk)$$

tum vero ob $S < 1$, debet eſſe

$$2m(1+Pk) < 2P(m+Pk)$$

ſtatuamus nunc iterum, vt ante, $Pk = \sqrt{m}$ fietque $S = \frac{2\sqrt{m}}{P} - 1$, ita, vt nunc capi debeat $P < 2\sqrt{m}$ et $P > \sqrt{m}$; littera autem k cadet intra limites 1 et $\frac{1}{2}$.

 Tum

Tum vero ob $S = \frac{2\sqrt{m}}{P} - 1$ erit $k' = \frac{m}{S\sqrt{m}} = \frac{P.\sqrt{m}}{2\sqrt{m} - P}$.
Definito autem P erit

$$\mathfrak{B} = -\frac{(P-1)}{1+\sqrt{m}} \text{ et } B = -\frac{(P-1)}{P+\sqrt{m}} \text{ et}$$

$$\mathfrak{D} = \frac{2(1+k')}{1+\sqrt{m}} \text{ et } D = \frac{2(1+k')}{\sqrt{m}-2k'-1} \text{ ſiue}$$

$$\mathfrak{D} = \frac{2(2\sqrt{m}-P+P\sqrt{m})}{(1+\sqrt{m})(2\sqrt{m}-P)},$$

qui valor cum ſit poſitiuus et vnitate maior, littera D ſponte fit negatiua, quemadmodum conditiones poſtulant, dummodo C capiatur poſitiuum. Quo autem omnia plene determinentur, ſtatuamus inſuper $P = \frac{3}{2}\sqrt{m}$ ac fiet $k = \frac{2}{3}$; $k' = 3\sqrt{m}$, et $S = \frac{1}{3}$,

$$\mathfrak{B} = -\frac{(3\sqrt{m}-2)}{2(1+\sqrt{m})} \text{ et } B = -\frac{(3\sqrt{m}-2)}{5\sqrt{m}}$$

$$\mathfrak{D} = \frac{2(3\sqrt{m}+1)}{\sqrt{m}+1} \text{ et } D = \frac{-2(3\sqrt{m}+1)}{5\sqrt{m}+1}$$

quibus valoribus omnibus conditionibus ſatisfit.

Scholion.

357. En ergo duos caſus huiusmodi teleſcopiorum penitus determinatos pro data multiplicatione m, quorum effectus in praxi idem eſſe debet. Cum autem poſteriore caſu longitudo inſtrumenti minor euadat, quam priore, eum merito hic praeferimus; quam obrem operae pretium erit, in conſtructionem iſtorum Teleſcopiorum adcuratius inquirere. Notatis igitur praecipuarum litterarum valoribus, ſcilicet

$$P = \tfrac{3}{2}\sqrt{m};\ k = \tfrac{2}{3};\ k' = 3\sqrt{m} = -R;\ S = \tfrac{1}{3};$$

$\mathfrak{B} =$

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$$\mathfrak{B} = -\frac{(3\sqrt{m}-2)}{2(1+\sqrt{m})};\quad B = -\frac{(3\sqrt{m}-2)}{5\sqrt{m}}$$

$$\mathfrak{D} = \frac{2(3\sqrt{m}+1)}{\sqrt{m}+1};\quad D = -\frac{2(3\sqrt{m}+1)}{5\sqrt{m}+1};$$

et quia C debet eſſe poſitiuum, ponatur

I. $C = \vartheta$ vt ſit $\mathfrak{C} = \frac{\theta}{1+\theta}$;

elementa noſtra ita erunt expreſſa:

$$b = -\frac{2\alpha}{3\sqrt{m}};\quad \beta = \frac{2(3\sqrt{m}-2)\alpha}{15m};$$

$$c = \frac{3\sqrt{m}-2}{5m};\quad \gamma = \frac{\theta(3\sqrt{m}-2)}{5m};$$

$$d = \frac{\theta(3\sqrt{m}-2)}{15m\sqrt{m}};\quad \delta = \frac{-2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{15(5\sqrt{m}+1)m\sqrt{m}}\alpha;$$

$$e = \frac{2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{5(5\sqrt{m}+1)m\sqrt{m}}\alpha;$$

hinc diſtantiae focales

$$p = \alpha;\quad q = \frac{3\sqrt{m}-2}{3(1+\sqrt{m})\sqrt{m}}\alpha;\quad r = \frac{\theta}{1+\theta}\cdot\frac{3\sqrt{m}-2}{5m}\alpha;$$

$$s = \frac{2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{15(\sqrt{m}+1)m\sqrt{m}}\alpha \text{ et}$$

$$t = \frac{2\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{5(5\sqrt{m}+1)m\sqrt{m}};$$

et lentium interualla

$$\alpha + b = \alpha\left(1 - \frac{2}{3\sqrt{m}}\right);\quad \beta + c = \frac{3\sqrt{m}-2}{5m}\alpha;$$

$$\gamma + d = \frac{\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{15m\sqrt{m}}\alpha;$$

$$\delta + e = \frac{4\theta(3\sqrt{m}-2)(3\sqrt{m}+1)}{15(5\sqrt{m}+1)m\sqrt{m}}\alpha;$$

et diſtantia oculi

$$O = \frac{e(1+\sqrt{m})}{2\sqrt{m}} = \frac{\theta(1+\sqrt{m})(3\sqrt{m}-2)(3\sqrt{m}+1)}{5m^2(5\sqrt{m}+1)}.\alpha$$

vnde tota oritur longitudo teleſcopii

$$= \alpha\frac{(3\sqrt{m}-2)(1+\sqrt{m})}{3m} + \frac{\theta(\sqrt{m}+1)(3\sqrt{m}+1)(3\sqrt{m}-2)(5\sqrt{m}+3)}{15m^2(5\sqrt{m}+1)};$$

 ita,

ita, vt ſi m ſit numerus praemagnus, haec longitudo fiat $(1 + \frac{1}{8\sqrt{m}} + \frac{3\theta}{5\sqrt{m}})\alpha$ et quia hoc caſu fit $e = \frac{18\theta\alpha}{25m}$, ſi liceret capere $\alpha = \frac{m}{7}$ dig.; ſtatui conueniret $\vartheta = 5$, vt vltimae lentis diſtantia focalis fieret circiter $\frac{1}{2}$ dig.; quando autem α multo maiorem obtinet valorem, facile capi poterit $\vartheta = 1$.

II. Adcuratius etiam inquirere debemus, quantam aperturam cuique lenti tribui oporteat, ac pro prima quidem lente ſemper ſumi ſolet ſemidiameter aperturae $x = \frac{m}{50}$ dig. pro reliquis lentibus ex formulis ſupra expoſitis colligitur:

Semidiameter aperturae ſecundae lentis

$$= \pi q \pm \frac{qx}{\mathfrak{B}p} = \frac{1}{4}\omega q + \frac{qx}{\mathfrak{B}z}$$

$$= \frac{1}{4} q \left(\frac{2}{\sqrt{m}} + \frac{8(1+\sqrt{m})}{8\sqrt{m}-2} \cdot \frac{x}{\alpha}\right)$$

Semidiameter aperturae tertiae lentis

$$= \frac{rx}{B\mathfrak{C}p} = \frac{x}{\sqrt{m}} = \frac{\sqrt{m}}{50} \text{ dig.}$$

Quarta autem et quinta lens maximam aperturam capere debent; vnde eas vtrinque conuexas effici oportet.

III. Quod nunc ad litteras λ attinet, pro prima lente ſemper ſumi conuenit $\lambda = 1$, qui valor etiam pro ſecunda lente ſumi poſſe videtur, ſiquidem numerus m non ſit admodum paruus, de quo autem quouis caſu ſeorſim erit diſpiciendum. Pro tertia enim lente ob minimam aperturam nullum eſt dubium,

um, quin ſumi poſſit $\lambda'' = 1$. Quoniam vero quarta lens debet eſſe vtrinque aequaliter conuexa, pro ea ſumi debet

$$\lambda''' = 1 + \left(\frac{\sigma - \varrho}{2\,\tau}\right)^2 (1 + 2\,\mathfrak{D})^2$$
$$= 1 + \left(\frac{\sigma - \varrho}{2\,\tau}\right)^2 \left(\frac{11\sqrt{m} + 1}{\sqrt{m} + 1}\right)^2.$$

Pro quinta autem lente erit $\lambda'''' = 1 + \left(\frac{\sigma - \varrho}{2\,\tau}\right)^2$.

IV. His igitur valoribus pro λ, λ' ... ſtabilitis quantitas α ex ſequente formula definiri debet:

$$\alpha = k x \sqrt[3]{\mu m} \left\{ \begin{array}{l} \lambda - \frac{1}{\mathfrak{B} P}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{1}{B^3 \mathfrak{C} P k}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) \\ - \frac{1}{B^3 C^3 \mathfrak{D} P k k'}\left(\frac{\lambda'''}{\mathfrak{D}^2} + \frac{\nu}{D}\right) + \frac{\lambda''''}{B^3 C^3 D^3 . m} \end{array} \right\}$$

vbi meminiſſe iuvabit ſumi ſolere $x = \frac{m}{50}$ et $k = 50$, vt ſit $k x = m$. Interim tamen ſi vel maiore claritatis vel diſtinctionis gradu contenti eſſe velimus, pro $k x$ ſumi poterit $\frac{4}{5} m$. Deinde etiam hinc euidens eſt, ob illum praegrandem valorem ipſius λ''', qui ſcilicet quadratum $(2\,\mathfrak{D} - 1)^2$ inuoluebat, terminum inde hic oriundum iterum ſatis fieri paruum, cum is diuiſus ſit per $\mathfrak{D}^3$; praeterquam quod eius denominator ob $P k k' = 3\,m$ per ſe ſit ſatis magnus. Denique adhuc notari debet, numerum λ'' multiplicari per quantitatem ſatis notabilem, cum ſit $-\frac{1}{B^3}$ propemodum $\frac{125}{27}$ et $\frac{1}{\mathfrak{C}^3} > 1$; ideoque $-\frac{1}{B^3 \mathfrak{C}^3}$ vltra 5 aſſurgat atque adeo ad 40 vsque, ſi ſumeretur $\theta = 1$; ita vt $P k = \sqrt{m}$ in denominatore, hunc terminum vix infra

ſra vnitatem diminuere poſſit. Cui incommodo remedium afferri poſſet, hanc lentem ſecundum praecepta in Libr. I. de lentibus compoſitis tradita duplicando. Hoc autem neceſſe non erit, quando ipſam lentem obiectiuam ita duplicabimus, vt omnis confuſio a reliquis etiam lentibus oriunda tollatur.

Exemplum.

358. Sumto $m = 25$, conſtructionem huiusmodi teleſcopii deſcribere.

I. Cum ſit $m = 25$, erit $\sqrt{m} = 5$, indeque

$$P = \tfrac{15}{2};\ k = \tfrac{2}{3};\ k' = 15;\ S = \tfrac{1}{3};$$

$$\mathfrak{B} = -\tfrac{13}{12};\ B = -\tfrac{13}{25};\ \mathfrak{D} = \tfrac{16}{3};\ D = -\tfrac{16}{13};$$

vnde elementa noſtra erunt

$$b = -\tfrac{2\alpha}{15};\ \beta = \tfrac{26\alpha}{375};\ c = \tfrac{13\alpha}{125};\ \gamma = \tfrac{13\theta\alpha}{125};$$

$$d = \tfrac{13\theta\alpha}{1875};\ \delta = \tfrac{-16\theta}{1875}\alpha;\ e = \tfrac{16\theta\alpha}{625};$$

et diſtantiae focales

$$p = \alpha;\ q = \tfrac{13}{95}\cdot\alpha;\ r = \tfrac{\theta}{1+\theta}\cdot\tfrac{13}{125}\cdot\alpha;$$

$$s = \tfrac{208}{5625}\alpha \text{ et } t = \tfrac{16\theta}{625}\cdot\alpha;$$

et interualla lentium

$$\alpha + b = \tfrac{13}{15}\alpha;\ \beta + c = \tfrac{13}{75}\alpha;$$

$$\gamma + d = \tfrac{208\theta}{1875}\alpha;\ \delta + e = \tfrac{32\theta}{1875}\cdot\alpha;$$

ac diſtantia oculi $O = \tfrac{48\theta}{3125}\cdot\alpha$;

ita, vt tota longitudo futura ſit $\alpha\left(\tfrac{26}{25} + \tfrac{449\theta}{3125}\right)$

Campi

Campi autem apparentis ſemidiameter erit
$\frac{1718}{30}$ min. $= 57' \, 16''$.

II. Semidiameter aperturae lentis primae $= \frac{1}{2}$ dig.
- - - - ſecundae $= \frac{1}{4} q \left(\frac{2}{3} + \frac{48}{13} \cdot \frac{x}{\alpha} \right)$, vnde colligere licet, pro hac lente dimidiam aperturam ſufficere.
- - - - tertiae $= \frac{1}{10}$ dig.

III. Deinde porro erit $\lambda = 1$; $\lambda' = 1$ fortaſſe; $\lambda'' = 1$; $\lambda''' = 1 + \frac{841}{9} \left(\frac{\sigma - \varrho}{27} \right)^2$, vbi notandum, ſi vitrum commune adhibeatur, quo $n = 1,55$ fore $\lambda''' = 1, + 0,6299 \cdot \frac{841}{9} = 59,702$ et $\lambda'''' = 1,6299$.

Ex aequatione pro α colligere licet, numerum ſub ſigno radicali contentum circiter vltra $2 \mu m$. excreſcere, vnde eius loco tuto ſcribere poſſumus 64 ſicque obtinebimus $\alpha = 100$. dig. $= 8 \frac{1}{3}$ ped.

Pro maioribus autem multiplicationibus haec quantitas in ratione $m \sqrt[3]{m}$ creſcet neque haec longitudo ſatis magna imminui poterit, niſi formulam pro ſemidiametro confuſionis ad nihilum redigamus, id quod vti ex ſuperioribus liquet, facile praeſtabitur, ſi his quinque lentibus adhuc lentem concauam praefigamus, ſiue ex eodem ſiue ex vitro chryſtallino parandam.

Problema 2.

359. Hanc teleſcopiorum ſpeciem ante primam lentem praefigendo lentem concauam ita perficere, vt confu-

confuſio penitus tollatur ſicque haec teleſcopia breuiſſima reddantur, ſeruato campo ante inuento.

Solutio.

Cum igitur nunc ſex habeamus lentes, quinque litterae erunt conſiderandae P, Q, R, S, T, ad lentium interualla relatae, quarum prima P debet dare interuallum minimum, quod ob α negatiuum ſtatuamus $= -\frac{1}{50}.\alpha$, vt fiat $P = \frac{50}{51}$. Deinde cum ſequentia interualla reſpondeant litteris Q, R; S, T, quae ante erant P, Q, R, S, nunc ponamus $R = -k$ et $S = -k'$, eruntque elementa

$$b = -\frac{\alpha}{P};\; c = \frac{B\alpha}{PQ};\; d = +\frac{BC\alpha}{PQk};\; e = \frac{BCD\alpha}{PQkk'};$$

$$\beta = -\frac{B\alpha}{P};\; \gamma = \frac{BC\alpha}{PQ};\; \delta = \frac{BCD\alpha}{PQk};\; \varepsilon = \frac{BCDE\alpha}{PQkk'}$$

$$\text{et } f = \frac{-BCDE\alpha}{PQkk'T} = \frac{-BCDE\alpha}{m};$$

vnde interualla colliguntur

1°. $\alpha + b = \alpha(1 - \frac{1}{P})$; quod fit ſumto $P = \frac{50}{51}$.

2°. $\beta + c = -\frac{B\alpha}{P}(1 - \frac{1}{Q})$; vnde cum Q capi debeat > 1, debet eſſe B poſitiuum, ideoque $\mathfrak{B} > 0$ et < 1.

3°. $\gamma + d = \frac{BC\alpha}{PQ}(1 + \frac{1}{k})$; vnde C debet eſſe negatiuum.

4°. $\delta + e = \frac{BCD\alpha}{PQk}(1 + \frac{1}{k'})$; vnde D debet eſſe poſitiuum, ideoque $\mathfrak{D} > 0$ et $\mathfrak{D} < 1$.

5°. $\varepsilon +$

5°. $\varepsilon + f = \frac{BCDE\,\alpha}{PQkk'}(1 - \frac{1}{T})$; vnde debet eſſe $\varepsilon(1 - \frac{1}{T})$ poſitiuum, ſed cum et f debeat eſſe maius nihilo, debet eſſe E negatiuum, ergo $T < 1$.

Iam pro campo apparente ponamus
$\pi = -v\,\xi$; $\pi' = \omega\,\xi$; $\pi'' = 0$; $\pi''' = \xi$ et $\pi'''' = -\xi$
vt fiat $\Phi = \frac{v + \omega + 2}{m - 1}.\,\xi = M\,\xi$, exiſtente $M = \frac{v + \omega + 2}{m - 1}$; vnde pro loco oculi fit $O = \frac{f}{Mm}$. Ex his autem formabuntur ſequentes aequationes fundamentales:

1°. $\mathfrak{B}\,v = -(1 - P)\,M$.

2°. $\mathfrak{C}\,\omega = -(1 - PQ)\,M - v$.

3°. $\mathfrak{D}.\,0 = -(1 + PQk)\,M - v - \omega$.

4°. $\mathfrak{E} = -(1 - PQkk')\,M - v - \omega$.

Ex quarum tertia ſtatim habemus

$v + \omega = -(1 + PQk)\,M$ eſt vero etiam

$v + \omega = (m - 1)\,M - 2$; vnde

$M = \frac{2}{m + PQk}$ ſicque viciſſim $v + \omega = \frac{-2(1 + PQk)}{m + PQk}$.

Quia nunc prima aequatio dat

$v = \frac{-2(1 - P)}{\mathfrak{B}(m + PQk)}$; ſecunda praebebit

$\mathfrak{C}\,\omega = \frac{-2(1 - PQ)}{m + PQk} + \frac{2(1 - P)}{\mathfrak{B}(m + PQk)}$.

quare nunc fiet

$$v + \omega = \frac{2(1 - P)}{\mathfrak{B}\mathfrak{C}(m + PQk)} - \frac{2(1 - PQ)}{\mathfrak{C}(m + PQk)} = \frac{-2(1 + PQk)}{m + PQk}$$

 quae

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quae aequatio reducta dabit

$$(1-\mathfrak{B})(1-\mathfrak{C})-(1-\mathfrak{C})P+\mathfrak{B}PQ+\mathfrak{B}\mathfrak{C}PQk=0$$

quae ad formam hanc reducitur:

$$\frac{1-P}{BC}-\frac{P(1-Q)}{C}+PQ(1+k)=0$$

quae aequatio inſeruit relationi inter litteras B et C definiendae. Littera autem D arbitrio noſtro manet relicta, dummodo capiatur poſitiua. Tandem vero quarta aequatio dat

$$\mathfrak{E}=-\frac{2(1-PQkk')}{m+PQk}+\frac{2(1+PQk)}{m+PQk}=\frac{2PQk(1+k')}{m+PQk},$$

qui valor cum ſit poſitiuus, debet eſſe

$$2PQk(1+k')>m+PQk \text{ ſiue } PQk(1+2k')>m$$

Denique deſtructio marginis colorati poſtulat hanc aequationem:

$$0=\frac{v}{P}+\frac{\omega}{PQ}-\frac{0}{PQk}+\frac{1}{PQkk'}+\frac{1}{PQkk'T}$$

quae ſubſtitutis pro v et ω valoribus abit in hanc:

$$0=\frac{-2(1-P)}{\mathfrak{B}(m+PQk)}-\frac{2(1-PQ)}{\mathfrak{C}(m+PQk)Q}+\frac{2(1-P)}{\mathfrak{B}\mathfrak{C}(m+PQk)Q}$$
$$+\frac{1}{Qkk'}+\frac{1}{Qkk'T}.$$

ſiue

$$0=\frac{2}{Q(m+PQk)}\left(\frac{(1-P)(1-Q)}{\mathfrak{B}}-1-PQk\right)$$
$$+\frac{1}{Qkk'}+\frac{1}{Qkk'T}$$

Vt huic aequationi commodiſſime ſatisfaciamus primo terminos factore $(1-P)$ adfectos ob ſummam paruitatem reiiciamus, quandoquidem non opus eſt, vt in hac

hac reſolutione ſummum rigorem ſequamur, et habebimus

$$\frac{2(1+PQk)}{m+PQk} = \frac{1}{kk'}\left(1+\frac{1}{T}\right)$$

vbi ſtatim ſecundum naturam huius ſpeciei teleſcopiorum ſupra ſtabilitam ſtatuamus $PQk = \sqrt{m}$ et $T = \frac{1}{2}$; vnde fiet $\frac{4}{\sqrt{m}} = \frac{3}{kk'}$; hinc $kk' = \frac{3\sqrt{m}}{4}$. Quia nunc erit $kk'T = \frac{3\sqrt{m}}{8} = \frac{m}{PQ}$ ita, vt ſit $PQ = \frac{8}{3}.\sqrt{m}$, ob P datum etiam Q definietur. Quia porro eſt $PQk = \sqrt{m}$, erit $k = \frac{3}{8}$, hincque $k' = 2\sqrt{m}$, ſicque valores harum litterarum ita ſe habebunt:

$$P = \frac{50}{51};\ PQ = \frac{8}{3}\sqrt{m};\ k = \frac{3}{8};\ k' = 2\sqrt{m} \text{ et}$$

$$T = \frac{1}{2};\ \text{hincque } PQk = \sqrt{m};$$

$$PQkk' = 2m \text{ et } PQkk'T = m.$$

Quod nunc ad reliquas litteras B, C ... attinet, aequatio ſupra data, ſi etiam factor $1 - P$ reiiciatur, dabit:

$$-\frac{1+PQ}{C} + PQ(1+k) = 0$$

vnde inuenitur

$$C = \frac{1-PQ}{PQ(1+k)} = \frac{3-4\sqrt{m}}{7\sqrt{m}} \text{ et } \mathfrak{C} = \frac{3-4\sqrt{m}}{3(1+\sqrt{m})}.$$

Litterae autem B et $\mathfrak{B}$ arbitrio noſtro permittuntur, ita, vt ſi prima lens concaua ex vitro chryſtallino paretur, vt ſupra vidimus, poni conueniat $\mathfrak{B} = \frac{5}{7}$; porro vero litterae $\mathfrak{D}$ et D hinc plane non determinantur, niſi quod vtramque poſitiuam eſſe oportet,

ex quo ſtatuamus $D = \theta$, hincque $\mathfrak{D} = \frac{}{1+\theta}$; denique vero erit

$$\mathfrak{E} = \frac{2(1+2\surd m)}{1+\surd m}; \text{ hincque } E = \frac{-2(1+2\surd m)}{1+3\surd m};$$

qui valores vni conſpectui ita repraeſentantur:

$$\mathfrak{B} = \tfrac{5}{7};\ \mathfrak{C} = \frac{3-4\surd m}{3(1+\surd m)};$$

$$\mathfrak{D} = \frac{\theta}{1+\theta} \text{ et } \mathfrak{E} = \frac{2(1+2\surd m)}{1+\surd m};$$

$$B = \tfrac{5}{2};\ C = \frac{3-4\surd m}{7\surd m};\ D = \mathfrak{D} \text{ et } E = \frac{-2(1+2\surd m)}{1+3\surd m};$$

hincque

$$BC = \frac{5(3-4\surd m)}{14\surd m};\ BCD = \frac{5\theta(3-4\surd m)}{14\surd m};$$

$$BCDE = \frac{5\theta(4\surd m-3)(1+2\surd m)}{7\surd m(1+3\surd m)};$$

ex quibus elementa noſtra penitus determinantur. Nihil igitur aliud ſupereſt, niſi vt ſemidiameter confuſionis ad nihilum redigatur, id quod fit ſequente aequatione:

$$\lambda = \frac{1}{P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{1}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$$
$$- \frac{1}{B^3C^3 \cdot PQk}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right)$$
$$- \frac{1}{B^3C^3D^3PQkk'}\left(\frac{\lambda''''}{\mathfrak{E}^3} + \frac{\nu}{E\mathfrak{E}}\right) + \frac{\lambda'''''}{B^3C^3D^3E^3 \cdot m};$$

ſi ſcilicet omnes lentes ex eodem vitro ſint factae. Sin autem prima lens ſit chryſtallina; reliquae vero coronariae, valor ipſius λ hinc inuentus inſuper multiplicari debet per $\frac{9875}{8724}$, quae fractio eſt fere $\frac{17}{15}$, propius vero $\frac{163}{144}$.

Circa

Circa hanc vero aequationem obſeruandum eſt, ſumi debere $\lambda' = 1$; $\lambda'' = 1$; $\lambda''' = 1$. Pro quinta autem lente, vt vtrinque fiat aeque conuexa, ſumi debet

$$\lambda'''' = 1, + 0,60006.(1 - 2\mathfrak{C})^2 = 1 + \frac{0,60006(1 + 7\sqrt{m})^2}{(1 + \sqrt{m})^2}$$

Pro ſexta vero $\lambda''''' = 1,60006$.

Coroll. 1.

360. Pro his igitur teleſcopiis cum fiat $M = \frac{2}{m + \sqrt{m}}$ erit ſemidiameter campi apparentis $\Phi = \frac{1718}{m + \sqrt{m}}$. min.

Coroll. 2.

361. Semidiametri autem aperturae ſingularum lentium ita definiuntur: ex §. 21.

Pro prima $= x$.

Pro ſecunda $= \frac{x}{P}$.

Pro tertia $= \frac{r}{2\sqrt{m}} \pm \frac{x}{PQ}$.

Pro quarta $= 0\,s \pm \frac{x}{PQk}$.

Pro quinta $= \frac{t}{4} \pm \frac{x}{PQkk'}$.

Pro ſexta $= \frac{u}{4} \pm \frac{x}{PQkk'T} = \frac{u}{4} \pm \frac{x}{m}$.

Coroll. 3.

362. Si in locis imaginum realium velimus diaphragmata conſtituere, reperitur

Pro

Pro priori ſemidiameter aperturae $= \frac{2BC}{m+\sqrt{m}} \cdot \frac{\alpha}{4}$.

Pro poſteriore vero $= \frac{2BCD}{m+\sqrt{m}} \cdot \frac{\alpha}{4}$.

Scholion.

363. En ergo duplicem perfectionem huius generis teleſcopiorum; altera ſcilicet ſpectat ad campum apparentem; quem fere duplo maiorem reddidimus; altera vero conſiſtit in deſtructione confuſionis, qua efficitur, vt non opus ſit, quantitatem α maiorem accipere, quam apertura lentis obiectiuae ad claritatem requiſita poſtulat, ſicque longitudo teleſcopii tantopere contrahatur, quantum quidem fieri licet. Cum hic duae lentes poſt vltimam imaginem reperiantur, quibus campus duplo maior eſt factus, ita, ſi tres pluresue lentes adhibere velimus, campum, quouſque voluerimus, amplificare licebit. Quod cum vix maiorem calculum poſtulet, quam praecedens problema, operae pretium vtique erit, hanc inueſtigationem generatim ad quotcunque lentes extendere.

Problema 3.

364. Praefixa, vt ante, lente concaua, plures lentes poſt vltimam imaginem realem ita diſponere, vt campus apparens quantum libuerit amplificetur.

Solutio.

Hic omnia prorſus manent vt in problemate antecedente, quod ſcilicet ad elementa, diſtantias focales et

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et interualla lentium attinet, hoc tantum diſcrimine, vt ambae ſeries litterarum B, C, D etc. et P, Q, k, k', T etc. vlterius continuari debeant. Deinde littera M, qua campus apparens definitur, alium nanciſcetur valorem a numero lentium poſt vltimam imaginem inſerendarum. Sit igitur harum lentium numerus $= i$ eritque $M = \frac{v + \omega + i}{m - 1}$ tum vero aequationes fundamentales ſe habebunt, vt ante, niſi quod vlterius progrediantur, poſt tertiam autem, quamlibet ſequentium ope tertiae definiamus, vti ſequitur

1°. $\mathfrak{B} v = -(1 - P) M$

2°. $\mathfrak{C} \omega = -(1 - PQ) M - v$

3°. $0 = -(1 + PQk) M - v - \omega$ ſiue
$v + \omega = -(1 + PQk) M$ vnde
$M(m - 1) = -(1 + PQk) M + i$ et
$M = \frac{i}{m + PQk}$.

4°. $\mathfrak{E} = PQk(1 + k') M$

5°. $\mathfrak{F} = PQk(1 + k'T) M - 1$

6°. $\mathfrak{G} = PQk(1 + k'TU) M - 2$

7°. $\mathfrak{H} = PQk(1 + k'TUV) M - 3$

etc.

ex primis autem formulis colligetur, vt ante,

$$\frac{1 - P}{\mathfrak{B}} - \frac{P(1 - Q)}{\mathfrak{C}} + PQ(1 + k) = 0$$

vnde quia P proxime $= 1$, ideoque v pro nihilo haberi poteſt, erit ſatis exacte

$$\omega = -(1 + PQk)M = -\frac{(1 - PQ)M}{\mathfrak{C}}$$

vnde colligimus

$$\mathfrak{C} = \frac{1 - PQ}{1 + PQk} \text{ et } C = \frac{1 - PQ}{PQ(1 + k)}.$$

Hic autem ſufficit hunc valorem vero proxime definiuiſſe, quia aperturae lentium, vnde litterae v, ω etc. pendent, ſummam praeciſionem reſpuunt. Quod cum etiam valeat in aequatione, qua margo coloratus deſtruitur, habebitur, loco M ſubſtituto valore,

$$\frac{i(1 + PQk)}{m + PQk} = \frac{1}{kk'}\left(1 + \frac{1}{T} + \frac{1}{TU} + \frac{1}{TUV} \text{ etc.}\right)$$

quorum terminorum numerus cum ſit i et ſingulae litterae T, U, V vnitate debeant eſſe minores, ſtatuamus tam concinnitatis gratia, quam vt lentes poſtremae aequis fere interuallis diſtent,

$$T = \tfrac{1}{2};\ U = \tfrac{2}{3};\ V = \tfrac{3}{4};\ W = \tfrac{4}{5} \text{ etc.}$$

vt factor ipſius $\frac{1}{kk'}$ fiat

$$1 + 2 + 3 + 4 \ldots i = \frac{(1+i)i}{2}$$

deinde etiam, vt ante, ponamus $PQk = \sqrt{m}$, vt prodeat iſta aequatio

$$\frac{1}{\sqrt{m}} = \frac{1}{kk'} \cdot \frac{i(1+i)}{2} \text{ vnde elicitur } kk' = \frac{(1+i)\sqrt{m}}{2}.$$

Productum vero reliquarum litterarum

$$TUV\ldots = \tfrac{1}{i}, \text{ erit } kk'TUV\ldots$$

$$= \frac{(1+i)\sqrt{m}}{2i} = \frac{m}{PQ}; \text{ hincque ergo deducitur}$$

$PQ = \frac{2i\sqrt{m}}{1+i}$, et quia P per ſe datur, hinc Q definietur.

Deni-

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Denique ob $PQk = \sqrt{m}$; elicitur $k = \frac{1+i}{2i}$ et $k' = i\sqrt{m}$; hic ergo valores omnes ſequenti modo ſe habent:

$$PQ = \frac{2i\sqrt{m}}{1+i};\ k = \frac{1+i}{2i};\ k' = i\sqrt{m};$$

$$T = \tfrac{1}{2};\ U = \tfrac{2}{3};\ V = \tfrac{3}{4};\ W = \tfrac{4}{5} \text{ etc.}$$

$$PQk = \sqrt{m};\ PQkk' = im;$$

$$PQkk'T = \tfrac{im}{2};\ PQkk'TU = \tfrac{im}{3};\ \text{et}$$

$$PQkk'TUV \ldots = \tfrac{im}{i} = m.$$

Circa litteras B C D etc. prima B cum tertia D hinc non definitur; iam vero oſtendimus eſſe,

$$C = \frac{1-PQ}{PQ(1+k)} = \frac{1+i-2i\sqrt{m}}{(1+3i)\sqrt{m}} \text{ et}$$

$$\mathfrak{C} = \frac{1-PQ}{1+PQk} = \frac{1+i-2i\sqrt{m}}{(1+i)(1+\sqrt{m})}.$$

Ponamus igitur, vt ante, $D = \vartheta$ et $\mathfrak{D} = \frac{\vartheta}{1+\vartheta}$ ſequentes vero erunt

$$\mathfrak{E} = \frac{i(1+i\sqrt{m})}{1+\sqrt{m}};\ \mathfrak{F} = \frac{i(2+i\sqrt{m})}{2(1+\sqrt{m})} - 1;$$

$$\mathfrak{G} = \frac{i(3+i\sqrt{m})}{3(1+\sqrt{m})} - 2;\ \mathfrak{H} = \frac{i(4+i\sqrt{m})}{4(1+\sqrt{m})} - 3;$$

quarum litterarum penultima erit

$$\frac{2(i-1)+(3i-2)\sqrt{m}}{(i-1)(1+\sqrt{m})} \text{ et vltima } = 1.$$

Has igitur quoque litteras hic coniunctim aſpectui exponamus:

$\mathfrak{B} = \frac{5}{7}$ circiter
$\mathfrak{C} = \frac{-(2i\sqrt{m}-i-1)}{(1+i)(1+\sqrt{m})}$
$\mathfrak{D} = \frac{\theta}{1+\theta}$
$\mathfrak{E} = \frac{i+ii\sqrt{m}}{1+\sqrt{m}}$
$\mathfrak{F} = \frac{2(i-1)+(ii-2.1)\sqrt{m}}{2(1+\sqrt{m})}$
$\mathfrak{G} = \frac{3(i-2)+(ii-2.3)\sqrt{m}}{3(1+\sqrt{m})}$
$\mathfrak{H} = \frac{4(i-3)+(ii-3.4)\sqrt{m}}{4(1+\sqrt{m})}$

$B = \frac{5}{8}$ vel circiter
$C = \frac{-(2i\sqrt{m}-i-1)}{(1+3i)\sqrt{m}}$
$D = \vartheta$
$E = \frac{-(i+ii\sqrt{m})}{(i-1)(1+(i+1)\sqrt{m})}$
$F = \frac{-(2(i-1)+(ii-1.2)\sqrt{m})}{(i-2)(2+(i+2)\sqrt{m})}$
$G = \frac{-(3(i-2)+(ii-2.3)\sqrt{m})}{(i-3)(1+(i+3)\sqrt{m})}$
$H = \frac{-(4(i-3)+(ii-3.4)\sqrt{m})}{(i-4)(4+(i+4)\sqrt{m})}$

ex quibus valoribus omnia elementa ſecundum formulas ſatis cognitas definiri poſſunt. Deinde vero vt omnis confuſio tollatur, haec aequatio erit adimplenda:

$$\begin{aligned}\lambda = &\tfrac{1}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{v}{B\mathfrak{B}}\right) - \tfrac{1}{B^3PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{v}{C\mathfrak{C}}\right)\\ &- \tfrac{1}{B^3C^3PQk}\left(\tfrac{\lambda'''}{\mathfrak{D}^3} + \tfrac{v}{D\mathfrak{D}}\right)\\ &- \tfrac{1}{B^3C^3D^3PQkk'}\left(\tfrac{\lambda''''}{\mathfrak{E}^3} + \tfrac{v}{E\mathfrak{E}}\right)\\ &+ \tfrac{1}{B^3C^3D^3E^3PQkk'T}\left(\tfrac{\lambda'''''}{\mathfrak{F}^3} + \tfrac{v}{F\mathfrak{F}}\right)\\ &- \tfrac{1}{B^3C^3D^3E^3F^3PQkk'TU}\left(\tfrac{\lambda''''''}{\mathfrak{G}^3} + \tfrac{v}{G\mathfrak{G}}\right)\\ &+ \text{etc.}\end{aligned}$$

vbi, vt ante, notandum eſt, ſi lens prima concaua ex vitro chryſtallino paretur, reliquae autem omnes ex coronario; tum valorem hinc pro λ inuentum inſuper multiplicari debere per fractionem $\frac{5875}{8724}$; quo caſu ſiquidem ſtatuatur $\mathfrak{B} = \frac{5}{7}$, etiam omnis confuſio a diuerſa refrangibilitate radiorum oriunda tolli deberet,

ſc.li-

ſcilicet ſecundum Dollondi experimenta. Ceterum, vt iam monuimus, pro litteris λ', λ'' et λ''' vnitas poni poterit. Pro ſequentibus vero lentibus, quae omnes vtrinque aeque conuexae eſſe debent, ſtatui debet

$$\lambda'''' = 1 + 0,60006\,(2\,\mathfrak{E} - 1)^2;$$
$$\lambda''''' = 1 + 0,60006\,(2\,\mathfrak{F} - 1)^2;$$
$$\lambda'''''' = 1 + 0,60006\,(2\,\mathfrak{G} - 1)^2 \text{ etc.}$$

Coroll. 1.

365. Hoc igitur modo campi apparentis ſemidiameter erit

$$\Phi = \frac{i\xi}{m + \sqrt{m}} \text{ ſiue } \Phi = \frac{850\,i}{m + \sqrt{m}} \text{ minut.}$$

ac, ſi pro lente vltima fuerit diſtantia focalis $= \zeta$, pro loco oculi habebimus

$$O = \frac{\zeta}{Mm} = \frac{\zeta(m + \sqrt{m})}{im} = \frac{\zeta(1 + \sqrt{m})}{i\sqrt{m}}.$$

vnde ſi multiplicatio fuerit praemagna erit $O = \frac{\zeta}{i}$.

Coroll. 2.

366. Semidiametri aperturae ſingularum lentium ita definientur:

Pro Ima $= x$; IIda $= \frac{x}{P}$;

IIItia $= \frac{t}{\sqrt{m}} \cdot \frac{r}{4} \pm \frac{x(1+i)}{2i\sqrt{m}}$

IVta $= o\,\frac{s}{4} \pm \frac{x}{\sqrt{m}}$

Vta $= \frac{t}{4} \pm \frac{1}{im} \cdot x$

VIta $= \frac{u}{4} \pm \frac{2}{im} \cdot x$

VIIta $= \frac{v}{4} \pm \frac{1}{im}\, x.$

Coroll. 3.

367. Circa diaphragmata eadem eſt ratio, vt in problemate praecedente: ſcilicet pro diaphragmate in loco prioris imaginis collocando debet eſſe radius foraminis $= \frac{iBC}{m + \sqrt{m}} \cdot \frac{\alpha}{4}$; pro altero autem diaphragmate $= \frac{iBCD}{m + \sqrt{m}} \cdot \frac{\alpha}{4}$ vnde patet, haec foramina eo maiora fieri debere, quo magis campus amplificetur.

Scholion.

368. Hoc igitur problemate totum huncce de teleſcopiis tractatum finimus, quoniam cuncta praecepta pro illorum conſtructione ſatis ſunt expoſita, neque hic conſtructiones generales commode exhiberi queant, propterea quod hic non ſolum quantitates duplicis generis, vt ante, vbi ſcilicet vel numeri abſoluti vel per multiplicationem *m* diuiſi occurrebant, ſed triplicis adeo generis ſcilicet praeter numeros abſolutos quantitates primo per $\sqrt{m}$, vel etiam per *m* diuiſae in computum ſunt ducendae, ita, vt ex comparatione duorum caſuum nulla concluſio generalis colligi queat. Nihil igitur aliud hic reſtat, niſi vt pro qualibet multiplicatione, quam quis poſtulat, atque etiam pro quantitate campi, ſeu valore numeri *i* calculus ab initio inſtituatur, quem pro quouis caſu oblato ſuſcepiſſe ob rei dignitatem ſine dubio operae erit pretium: In quo quidem negotio etiam littera ϑ, quae arbitrio noſtro hactenus eſt permiſſa, determinari debet,

debet, quam commode vnitati aequalem vel maiorem aſſumere licet. Videtur autem aptiſſime poni poſſe $\vartheta = 2$; vnde poſteriora inſtrumenti interualla non nimis augentur, ſimul vero valor pro λ notabiliter minor prodit, quam ſi eſſet $\vartheta = 1$. Quo autem totus iſte calculus facilius ſuſcipi et abſolui queat; aliquot exempla hic ſubiungamus.

Exemplum I.

369. Si $m = 49$, vt ſit $\sqrt{m} = 7$ et pro campo apparente $i = 2$, ita, vt teleſcopium ex ſex lentibus ſit componendum et ſumatur praeterea $\vartheta = 2$. Primo colligantur litterae P, Q etc. vt ſequitur

$$P = \tfrac{50}{51};\ PQ = \tfrac{28}{3};\ k = \tfrac{3}{4};\ k' = 14;\ T = \tfrac{1}{2};$$

$$\text{Log.}\ \tfrac{1}{P} = 0,0086002;\ \text{Log.}\ \tfrac{1}{PQ} = 9.0299632;$$
$$\text{Log.}\ \tfrac{1}{PQk} = 9.1549019;\ \text{Log.}\ \tfrac{1}{PQkk'} = 8.0087738;$$
$$\text{Log.}\ \tfrac{1}{PQkk'T} = 8.3098038:$$

$\mathfrak{B} = \frac{5}{7}$; $l.\mathfrak{B} = 9,8538719$	$B = \frac{5}{2}$; $l.B = 0,3979399$
$\mathfrak{C} = -\frac{25}{24}$; $l.\mathfrak{C} = 0,0177287(-)$	$C = -\frac{25}{49}$; $l.C = 9,7077438(-)$
$\mathfrak{D} = \frac{2}{3}$; $l.\mathfrak{D} = 9,8239086$	$D = 2$; $l.D = 0,3010300$
$\mathfrak{E} = \frac{15}{4}$; $l.\mathfrak{E} = 0,5740313$	$E = -\frac{15}{11}$; $l.E = 0,1346984(-)$

ex his logarithmis formantur ſequentes:

$l.BC = 0,1056837(-)$; $l.BCD = 0,4067137(-)$
$l.BCDE = 0,5414121(+)$ $l.B\mathfrak{B} = 0,2518118(+)$
$l.C\mathfrak{C} = 9,7254725(+)$; $l.D\mathfrak{D} = 0,1249386(+)$
$l.E\mathfrak{E} = 0,7087297(-)$.

Hoc

Hoc quaſi primo labore confecto colligamus noſtra elementa, quae ita ſe habebunt:

$b = -1,02\,\alpha.$	$\beta = -2,55.\,\alpha$	$q = -0,72857.\alpha$
		$\text{Log.}\,q = 9,8624713(-)$
$c = +0,26785\,\alpha$	$\gamma = -0,13666.\alpha$	$r = -0,27901.\alpha$
		$\text{Log.}\,r = 9,4456318(-)$
$d = -0,18221\,\alpha$	$\delta = -0,36443\,\alpha$	$s = -0,12148.\alpha$
		$\text{Log.}\,s = 9,0844942(-)$
$e = -0,02603\,\alpha$	$\varepsilon = 0,03549.\,\alpha$	$t = -0,09762\,\alpha$
		$\text{Log.}\,t = 8,9895188(-)$
$f = -0,07099\,\alpha$		$u = -0,07099.\alpha.$

Pro oculo autem erit $O = \frac{4u}{7} = -0,04057.\,\alpha$

III. Hinc iam lentium interualla cognoſcuntur:

$1^\circ.\ \alpha + b = -0,02000.\,\alpha$
$2^\circ.\ \beta + c = -2,28215.\,\alpha$
$3^\circ.\ \gamma + d = -0,31887.\,\alpha$
$4^\circ.\ \delta + e = -0,39046.\,\alpha$
$5^\circ.\ \varepsilon + f = -0,03550.\,\alpha$
$6^\circ.\ \quad O = -0,04057.\,\alpha$

Tota longitudo $= -3,08755.\,\alpha$

Deinde etiam diaphragmata ita definiuntur:

Prius poſt lentem tertiam ad diſtantiam $\gamma = -0,13666.\,\alpha$, ponitur,

Eius ſemidiameter foraminis $= 0,0569.\,\alpha$

Poſte-

Poſterius ponitur poſt quartam lentem ad diſtantian
$\delta = -0,36443.\alpha$

Eius ſemidiameter foraminis $= 0,1138.\alpha$

Porro vero ſemidiameter campi apparentis erit $30\frac{2}{3}$ min.

IV. Nunc ſingulas lentes examinari conueniet, quarum non ſolum conſtructio, ſed etiam momentum confuſionis, quod quaelibet ad valorem λ confert; eſt definiendum, vbi quidem prima lens vltimo loco, poſtquam ſcilicet valor λ fuerit inuentus, tractari debebit. Quoniam igitur ſequentes lentes omnes ex vitro coronario fieri ſumuntur, valores eo pertinentes erunt:

$\nu = 0,2196$; Log. $\nu = 9.3416323$
$\sigma = 1,6601$
$\rho = 0,2267$

$\sigma - \rho = 1,4334$; Log. $\sigma - \rho = 0,1563674$
$\tau = 0,9252$;

Nunc igitur ſingulas lentes poſt primam ordine percurramus:

Pro lente ſecunda

$$1^{\circ}.\ \text{radius} \begin{cases} \text{anter.} \ \dfrac{q}{\sigma - \mathfrak{B}(\sigma-\rho) + \tau\nu\sqrt{(\lambda'-1)}} \\ \text{poſter.} \ \dfrac{q}{\rho + \mathfrak{B}(\sigma-\rho) - \tau\nu\sqrt{(\lambda'-1)}} \end{cases}$$

quae formulae ex ſuperioribus facile eliciuntur. Hic vero eſt $\lambda' = 1$ et calculus ita inſtituatur

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$l.\ \sigma - \varrho = 0,1563674$	$\sigma = 1,6601$
$L.\ \mathfrak{B} = 9,8538719$	ſubtr. $1,0239$
$0,0102393$	$0,6362$ den. rad. ant.
	$\varrho = 0,2267$
$\mathfrak{B}(\sigma - \varrho) = 1,02386$	add. $1,0239$
	$1,2506$ den. rad. poſt.

log. $q = 9,8624713$ (-)	$9,8624713$ (−)
log. den $= 9,8035937$	$0,0971184$
$0,0588776$ (-)	$9,7653529$ (−)
rad. anter. $= -1,14519 \cdot \alpha$	rad. poſt. $= -0,58257 \cdot \alpha$

2°. Semidiameter aperturae requiritur

$= \frac{51}{53} x = \frac{51}{53} \cdot \frac{m}{53}$ dig.

3°. Calculus pro momento confuſionis:

$l. \frac{1}{p} = 0,0086002$	$l. \lambda' = 0,0000000$	$l. \nu = 9,3416323$
	$l. \mathfrak{B}' = 9,5616157$	$l. B\mathfrak{B} = 0,2518118$
	$0,4383843$	$9,0898205$
adde log. coëffic. =	$0,0086002$	$0,0086002$
	$0,4469845$	$9,0984207$

Ergo pars prior $= 2,79888$
poſterior $= 0,12543$

Momentum confuſionis $= 2,92431$

Pro lente tertia

1°. radius ant. $= \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) + \tau \surd(\lambda'' - 1)}$

. . . poſter. $= \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) - \tau \surd(\lambda'' - 1)}$

vbi

vbi notetur, eſſe $\lambda'' = 1$.

$l.\ \sigma - \varrho = 0,1563674$	$\sigma = 1,6601$	$\varrho = 0,2267$
$l. - \mathfrak{C} = 0,0177287$	$+ 1,4931$	$- 1,4931$
$0,1740961$	$3,1532$	$- 1,2664$
$\mathfrak{C}(\sigma - \varrho) = - 1,49313$	denom. anter.	denom. poſter.

Log. $r = 9,4456318\ (-)$	$9,4456318\ (-)$
log. den. $= 0,4987515\ (+)$	$0,1025709\ (-)$
$8,9468803\ (-)$	$9,3430609\ (+)$

Ergo

radius anter. $= - 0,08848.\ \alpha$;

radius poſter. $= + 0,22032.\ \alpha$.

2°. Semidiameter aperturae requiſita $= \frac{x}{7} \cdot \frac{r}{4} + \frac{3}{28} \cdot x$. ſiue $= 0,02\ \alpha + \frac{3}{28}\ x$. quam aperturam haec lens vtique ſuſtinere poteſt.

3°. Calculus pro momento confuſionis:

$l.\ \frac{1}{PQ} = 9,0299632$	$l.\ \lambda'' = 0,0000000$	$l.\ \nu = 9,3416323$
$3\ l.\ B = 1,1938197$	$3\ l.\ \mathfrak{C} = 0,0531861\ (-)$	$l.\ C\mathfrak{C} = 9,7254725$
$7,8361435$	$9,9468139$	$9,6161598$
	$7,8361435$	$7,8361435$
	$7,7829574\ (-)$	$7,4523033$

Ergo pars prior $+ 0,00606$

poſter. $- 0,00283$

Momentum confuſ. $= 0,00323$

 Pro

Pro lente quarta

1°. radius anter. $= \frac{s}{\sigma - \mathfrak{D}(\sigma - \varrho) + \tau\surd(\lambda''' - 1)}$

poſter. $= \frac{s}{\varrho + \mathfrak{D}(\sigma - \varrho) - \tau\surd(\lambda''' - 1)}$

vbi iterum ſumatur $\lambda''' = 1$.

$l.\sigma - \varrho = 0{,}1563674$	$\sigma = 1{,}6601$	$\varrho = 0{,}2267$
$l.\mathfrak{D} = 9{,}8239086$	$0{,}9556$	$0{,}9556$
$l.\mathfrak{D}(\sigma - \varrho) = 9{,}9802760$	$0{,}7045$	$1{,}1823$
$\mathfrak{D}(\sigma - \varrho) = 0{,}95560$	denom. anter.	denom. poſter.

log. $s = 9{,}0844942\ (-)$;	$9{,}0844942\ (-)$
log. den. $= 9{,}8478810$	$0{,}0727277$
$9{,}2366132\ (-)$	$9{,}0117665\ (-)$

radius anter. $= -0{,}17243\,\alpha$;

radius poſter. $= -0{,}10273.\,\alpha$;

2°. Semidiameter aperturae requiſitus $= \frac{1}{7}.\,x$. quam aperturam lens commode ſuſtinebit, ſi enim minor radius lentis ſecundae, qui eſt $0{,}58257.\,\alpha$, ſuſtinet aperturam x; hic radius minor, qui eſt $0{,}10273.\alpha$, commode ſuſtinebit aperturam $\frac{1}{7}\,x$.

3°. Calculus pro momento confuſionis:

$l.\frac{1}{PQk} = 9{,}1549019$	$l.\lambda''' = 0{,}0000000$	$l.\ \nu = 9{,}3416323$
$3l.BC = 0{,}3170511\,(-)$	$3l.\mathfrak{D} = 9{,}4717258$	$l.\mathfrak{D}D = 0{,}1249386$
$8{,}8378508$	$0{,}5282742$	$9{,}2166937$
	$8{,}8378508$	$8{,}8378508$
	$9{,}3661250$	$8{,}0545445$

Ergo

Ergo pars prior $0,23234$
poſter. $0,01133$

Mom. confuſ. $= 0,24367$

Pro lente quinta

1°. Quia haec lens vtrinque debet eſſe aeque convexa, ob eius diſtantiam focalem $t = -0,09762.\alpha$ erit radius vtriusque faciei $= 1,06.t = -0,10348.\alpha$ nunc vero erit $\lambda'''' = 1 + 0,60006(2\mathfrak{E} - 1)^2$ at eſt $2\mathfrak{E} - 1 = 6,5$; ergo

$\log.(2\mathfrak{E} - 1) = 0,8129134$; et
$\log.(2\mathfrak{E} - 1)^2 = 1,6258268$
$\log. 0,60006 = 9,7781947$

$1,4040215$

adeoque $\lambda'''' = 26,352$.

2°. Semidiameter aperturae hic per hypotheſin eſt $\frac{1}{4}t. = -0,02440.\alpha$; altera enim pars $\frac{1}{32}x$, quam haec lens facillime patitur.

3°. Calculus pro momento confuſionis:

$l.\frac{1}{P\sqrt{kk'}} = 8,0087738$	$l.\lambda'''' = 1,4208136$	$l.\nu = 9,3416323$
$3l.BCD = 1,2201411$	$3l.\mathfrak{E} = 1,7220939$	$l.E\mathfrak{E} = 0,7087297$
$6,7886327$	$9,6987197$	$8,6329026$
	$6,7886327$	$6,7886327$
	$6,4873524$	$5,4215353$

 Ergo

Ergo pars prior 0, 00031
poſter. — 0, 00002

Momentum confuſ. $=$ 0, 00029

Pro lente ſexta

1°. Quia per hypotheſin haec lens vtrinque debet eſſe aeque conuexa, ob eius diſtantiam focalem
$u = - 0,07099.\alpha$, erit
radius vtriusque faciei $= 1,06.\,u = - 0,07525.\alpha$
tum vero erit $\lambda'''' = 1,60006$.

2°. Semidiameter aperturae $= \frac{1}{4}u = -0,01775.\alpha$

3°. Calculus pro momento confuſionis:

l. $\frac{1}{PQkk'T} = 8,3098038$	l. λ''''' 0, 2041363
3. l. BCDE $= 1,6242363$	6, 6855675
6, 6855675	6, 8897038

Ergo momentum confuſ. $=$ 0, 00077.

His inuentis, colligantur omnia momenta confuſionis in vnam ſummam, quae erit 3, 17227. Nunc autem duo caſus ſunt conſiderandi, prout primam lentem concauam vel ex vitro coronario vel ex chryſtallino parare voluerimus, quos ſeorſim euolui oportet.

I. Pro prima lente concaua ex vitro coronario paranda.

Pro hac ergo lente erit
$\lambda = 3,17227$ vnde $\lambda - 1 = 2,17227$;

hinc-

hincque fiat ſequens calculus.

$$\begin{array}{r|c}
\text{Log.}\,(\lambda-1) = 0,3369138 & \\
\text{Log.}\,\sqrt{(\lambda-1)} = 0,1684569 & \text{ergo} \\
\text{Log}\,\tau = 9,9662356 & \tau\sqrt{(\lambda-1)} = 1,3636 \\
\hline
0,1346925 &
\end{array}$$

Nunc cum ſit pro hac lente

rad. anter. $= \frac{\alpha}{\sigma - \tau\sqrt{\lambda-1}}$; rad. poſter. $= \frac{\alpha}{\varrho + \tau\sqrt{\lambda-1}}$

calculus ita ſe habebit:

$$\begin{array}{rr}
\sigma = 1,6601; & \varrho = 0,2267 \\
\tau\sqrt{(\lambda-1)} = 1,3636 & 1,3636 \\
\hline
0,2965 & 1,5903
\end{array}$$

$$\begin{array}{r|r}
\text{l. } 0,2965 = 9,4720247 & \text{l. } 1,5903 = 0,2014791 \\
\text{compl.} = 0,5279753 & \text{compl.} = 9,7985208
\end{array}$$

ſicque prodit

radius anter. $= 3,37268.\,\alpha$; poſter. $= 0,62881.\,\alpha$

ſemidiametro aperturae exiſtente $x = \frac{m}{33}$ dig. $= 1$ dig.

II. Pro prima lente concaua ex vitro chryſtallino paranda.

Pro hac igitur lente erit $\lambda = \frac{1875}{8724} \cdot 3,17227$ ſeu $\lambda = 3,59080$; et quia pro vitro chryſtallino eſt

$$\varrho = 0,1414;\ \sigma = 1,5827;\ \tau = 0,8775;$$

calculus ita ſe habebit.

Log.

Log. $(\lambda-1) = 0,4134339$	
Log. $\sqrt{(\lambda-1)} = 0,2067169$	ergo
Log. $\tau = 9,9432471$	$\tau \sqrt{(\lambda-1)} = 1,41242$
$0,1499640$	

$\sigma = 1,5827$;	$\varrho = 0,1414$
ſubtr. $1,4124$	add. $1,4124$
$0,1703$	$1,5538$
log. $9,2312146$	log. $0,1913951$
compl. $0,7687853$	compl. $9,8086048$

ſicque prodit

rad. anter. $= 5,87199.\alpha$; rad. poſt. $= 0,64358.\alpha$

ſemidiametro aperturae exiſtente $x = \frac{m}{50} = 1$ dig.

VI. Quia binae priores lentes coniunctim lentem obiectiuam conſtituunt, cuius ſemidiameter aperturae $= 1$ dig.; ſtatuatur earum minimus radius, qui eſt $-0,58257.\alpha > 4$ dig. hincque concludetur, ſumi debere $-\alpha > \frac{4}{0,58257}$ dig. hoc eſt $-\alpha > 7$ dig. vel ſaltim non minus, ita, vt, ſi optimus ſucceſſus ſperari poſſet, accipere liceret $-\alpha = 7$ dig. Sin autem aberratio quaedam ſit pertimeſcenda, tantum opus erit menſuram vnius digiti augere. Commoditatis autem gratia ſumamus $\alpha = -10$ dig.; vnde ſequens prodit.

Con-

Conſtructio huius teleſcopii determinata, pro multiplicatione $m = 49$.

I. Pro lente obiectiua,
quatenus ex vitro coronario paratur.

rad. fac. $\begin{cases} \text{anter.} = -33,73 \text{ dig.} \\ \text{poſter.} = -6,29 \text{ dig.} \end{cases}$ Crown Glaſſ.

(I) Pro lente obiectiua,
quatenus ex vitro chryſtallino paratur.

rad. fac. $\begin{cases} \text{anter.} = -58,72 \text{ dig.} \\ \text{poſter.} = -6,43 \text{ dig.} \end{cases}$ Flint Glaſſ.

cuius diſtantia focalis pro vtroque caſu $= -10$ dig.
ſemidiameter aperturae $= 1$ dig.
Interuallum ad ſecundam $= 0,2 = \frac{1}{5}$ dig.

II. Pro lente ſecunda

rad. fac. $\begin{cases} \text{anter.} = 11,45 \text{ dig.} \\ \text{poſter.} = 5,82 \text{ dig.} \end{cases}$ Crown Glaſſ.

cuius diſtantia focalis $= 7,28$ dig.
ſemidiameter aperturae $= 1$ dig.
Interuallum ad tertiam $= 22,82$ dig.

III. Pro lente tertia

rad fac. $\begin{cases} \text{anter.} = 0,884 \text{ dig.} \\ \text{poſter.} = -2,20 \text{ dig.} \end{cases}$ Crown Glaſſ.

cuius diſtantia focalis $= 2,79$. dig.
ſemidiameter aperturae $= 0,3$ dig.
Interuallum ad quartam $= 3,19$. dig.

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IV. Pro lente quarta

rad. fac. $\left\{ \begin{array}{l} \text{anter.} = 1,72 \text{ dig.} \\ \text{poſter.} = 1,03 \text{ dig.} \end{array} \right\}$ Crown Glaſſ.

cuius diſtantia focalis 1, 21 dig.
ſemidiameter aperturae $= \frac{1}{7}$ dig.
Interuallum ad quintam $= 3,90$ dig.

V. Pro lente quinta

radius vtriusque faciei $= 1,03$ dig. Crown Glaſſ.
cuius diſtantia focalis eſt 0, 97 dig.
ſemidiameter aperturae $= \frac{1}{4}$ dig.
Interuallum ad ſextam $= 0,35$ dig.

VI. Pro lente ſexta

radius faciei vtriusque $= 0,75$ dig. Crown Glaſſ.
cuius diſtantia focalis $= 0,70$ dig.
ſemidiameter aperturae $= 0,18 = \frac{1}{5}$ dig.
Diſtantia ad oculum vsque $= 0,40$ dig.

Huius igitur teleſcopii longitudo tota fiet $= 30,87$ dig. $= 2\frac{1}{2}$ ped.
et ſemidiameter campi apparentis $= 30\frac{2}{3}$ min.

APPEN-

APPENDIX
DE
CONSTRVCTIONE TELESCOPIORVM CATOPTRICO-DIOPTRICORVM.

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APPENDIX
DE
TELESCOPIIS CATOPTRICO DIOPTRICIS.

CAPVT I.
DE
IMAGINIBVS PER SPECVLA SPHAERICA FORMATIS EARVMQVE DIFFVSIONE.

Problema I.

§. 1.

Si a puncto lucido in axe ſpeculi conſtituto radii axi proximi in ſpeculum incidant, inuenire locum imaginis.

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Solutio.

Tab. I. Fig. 1. Sit P A P ſpeculum ſphaericum probe politum centro O radio $OA = f$ deſcriptum, cuius axis ſit recta A O E, in cuius puncto E conſtitutum ſit punctum lucidum et ponatur eius diſtantia $EA = a$, vnde radii in totam ſpeculi ſuperficiem incidant, e quibus autem eos tantum hic conſideramus, qui axi ſint proximi ſeu qui in puncta a medio puncto ſpeculi A proxima incidant, talis igitur radius incidens ſit E A et ad punctum a ex centro O ducatur radius $Oa = f$ qui cum in ſpeculum ſit normalis, erit EaO angulus incidentiae, cui ab altera parte rectae Oa capiatur angulus aequalis OaF, eritque recta aF radius reflexus cum axe occurrens in puncto F, in quo puncto adeo omnes radii axi proximi e puncto E emiſſi concurrent, ſiquidem etiam radius E A ſecundum ipſum axem emiſſus in punctum F reflectitur, ita, vt punctum F ſit imago puncti lucidi E per reflexionem formata, et cum a radiis axi proximis formetur, in hoc puncto erit imago principalis, vti eam in tractatu de lentibus vocauimus. Ad locum igitur iſtius puncti F inueniendum conſideretur triangulum EaF, cuius angulus EaF biſectus eſt recta Oa, vnde notum theorema Geometricum praebet hanc proportionem $Ea : EO = Fa : FO$ deinde quia in triangulo EaO anguli ad E et ad a ſunt infinite parui in triangulo autem OaF anguli ad O et a; erit $Ea = EO + f$; et $Fa = f - OF$ vnde

vnde illa proportio abit in hanc

$$EO + f : EO = f - OF : OF$$

et componendo

$$2EO + f : EO = f : OF$$

Cum iam ſit $EO = EA - AO = a - f$ fiet

$$2a - f : a - f = f : OF$$

hincque $OF = \frac{(a-f)f}{2a-f}$

ſicque locus puncti F innoteſcit, cuius diſtantia a puncto A erit $AF = f - FO = \frac{af}{2a-f}$. *q. e. i.*

Coroll. 1.

§. 2. Ex data ergo diſtantia puncti lucidi E a ſpeculo $EA = a$, inuenimus diſtantiam imaginis principalis ſuper axe A F, quam cum in lentibus littera α deſignauerimus, etiam hic eadem littera vtamur, ita, vt ſit $\alpha = \frac{af}{2a-f}$.

Coroll. 2.

§. 3. Speculum hic tanquam concauum ſpectavimus, cuius radius eſſet $AO = f$. vnde valores poſitiui huius litterae f ſpecula concaua; valores vero negatiui ſpecula conuexa denotabunt. Tum vero etiam diſtantia α, quatenus valorem habet poſitiuum, diſtantiam imaginis ante ſpeculum indicabit; ſin autem prodeat negatiua, id indicio erit imaginem poſt ſpecu-

ſpeculum cadere eamque fore fictam, cum praeſens ſit realis. Hinc autem intelligitur, imaginem fore realem, ſi fuerit $a > \frac{1}{2}f$, ſiquidem ſit $f > 0$; ſin autem ſit $f < 0$ ſeu ſpeculum conuexum; tum imago ſemper poſt ſpeculum cadet, eritque ficta, non realis.

Coroll. 3.

§. 4. Si puncti lucidi diſtantia $AE = a$ fuerit infinita; tum diſtantia imaginis principalis a ſpeculo erit $AF = \frac{1}{2}f$ ita, vt haec diſtantia $AF = \frac{1}{2}f$ pro diſtantia focali ſpeculi ſit habenda hinc ſi ſpeculi diſtantiam focalem ponamus $= p$, erit radius ſpeculi $f = 2p$. Tum vero in genere diſtantiae a et α ita a ſe inuicem pendebunt, vt ſit $\alpha = \frac{ap}{a-p}$ hincque $p = \frac{a\alpha}{\alpha+a}$ et $\frac{1}{p} = \frac{1}{a} + \frac{1}{\alpha}$ prorſus vti in lentibus vſu venire ſupra vidimus.

Scholion.

§. 5. Hic notatu inprimis dignum occurrit, quod tres iſtae diſtantiae a, α et p eodem prorſus modo a ſe inuicem pendent, vti in lentibus; ex quo euidens eſt ratione calculi ſpecula perinde tractari poſſe ac lentes, quae calculi conuenientia adhuc in ſequentibus magis illuſtrabitur. Hic tantum notaſſe iuuabit, lentibus conuexis reſpondere ſpecula concaua; vti enim lentibus conuexis diſtantias focales poſitiuas tribuimus, quippe quarum foci ſunt reales, ita etiam ſpecula concaua realem habent focum ibique aeque vi vrendi pollent

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polłent atque lentes conuexae in ſuis focis; diſcrimen tamen in eo ſitum eſt, quod in ſpeculis concauis focus ante ea cadat, cum in lentibus conuexis poſt eas formetur atque ſimili modo ſpecula conuexa ad lentes concauas referentur dum in vtrisque focus tantum fictus datur, in quo ſcilicet radii non reuera congregentur. Quando ergo de ſpeculis ſermo erit, diſtantia focalis poſitiua ſemper ſpeculum concauum; diſtantia vero focalis negatiua ſpeculum conuexum indicabit, ac ſi diſtantia focalis euadat infinita, ſpeculum erit planum, ſimili modo, quo lens diſtantiam focalem habens infinitam eſt plano plana. Praeterea vero etiam obſeruaſſe iuuabit, ſi vti in Dioptrica fecimus, ſtatuamus $\alpha = \mathrm{A}a$ et $\mathfrak{A} = \frac{\mathrm{A}}{\mathrm{A}+1}$, tum etiam fore $p = \mathfrak{A}a$.

Problema 2.

§. 6. Si non amplius lucidum punctum E, ſed obiectum E ε axi ſpeculi perpendiculariter inſiſtat, eius imaginem, quae in puncto F ſitu inuerſo repraeſentabitur, definire.

Solutio.

Ponatur iterum diſtantia huius obiecti a ſpeculo E A $= a$, ſitque eius magnitudo E ε $= \zeta$, quippe qua denominatione ſupra de lentibus ſumus vſi, ita, vt ζ ſemper ſit quantitas valde parua reſpectu diſtantiae E A $= a$, ſeu angulus E A ε quaſi infinite paruus. Deinde ſit vt ante radius ſpeculi O A $= f$, eius di- Tab. I. Fig 2.

 ſtantia

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ſtantia focalis $= p$, ita, vt ſit $f = 2p$, et diſtantia imaginis principalis a ſpeculo $AF = \alpha$ ita, vt ſit $\alpha = \frac{ap}{a-p}$. His poſitis facile intelligitur, imaginem quaeſitam in punctum F incidere atque ad contrariam partem axis fore directam; ducta enim recta ϵA referet radium incidentem, cui conuenit radius reflexus $A\zeta$, qui ergo per imaginis extremitatem tranſire debebit; vnde ſi in puncto F normaliter ad axem ducatur recta $F\zeta$, ad radium reflexum $A\zeta$ terminata, haec recta $F\zeta$ imaginem principalem obiecti exhibebit, cuius ergo magnitudo ex ſimilitudine triangulorum $AE\epsilon$ et $AF\zeta$ ita definietur, vt ſit

$$F\zeta = \frac{AF.E\epsilon}{AE} = \frac{\alpha.\zeta}{a}$$

quod idem etiam hoc modo oſtendi poteſt. Ex puncto ϵ per centrum ſpeculi O ducatur etiam radius incidens $\epsilon O \alpha$, qui cum ſit normalis, eius reflexus in ipſum cadet tranſibitque etiam per punctum ζ vnde ſimilitudo $\triangle$lorum $OE\epsilon$ et $OF\zeta$ dabit $F\zeta = \frac{OF.E\epsilon}{OE}$. Eſt vero $OF = f - \alpha$, et $OE = a - f$ ex quo ſit $F\zeta = \frac{(f-\alpha)\zeta}{a-f}$. Cum ex ſuperiori problemate ſit

$$\alpha = \frac{af}{2a-f} \text{ hincque } f = \frac{2a\alpha}{a+\alpha} \text{ erit}$$

$$f - \alpha = \frac{(a-\alpha)\alpha}{a+\alpha} \text{ et } a - f = \frac{(a-\alpha)a}{a+\alpha};$$

hincque ſubſtitutis his valoribus, fiet $F\zeta = \frac{\alpha\zeta}{a}$, prorſus, vt ante quo ipſo confirmatur rectam $F\zeta$ axi recte normalem eſſe ductam.

Coroll.

Coroll. 1.

§. 7. Hic ergo etiam magnitudo imaginis principalis eodem plane modo ex obiecti magnitudine determinatur, quo in Dioptrica id fieri ſupra oſtendimus vnde ſi vt ibi fecimus ſtatuamus $\alpha = A\, a$, habebimus etiam hic $F\,\zeta = A\,\zeta$.

Coroll. 2.

§. 8. Quia noſtra figura teleſcopium concauum refert, eius analogia cum lentibus conuexis etiam hic manifeſto cernitur, quemadmodum enim lentes convexae imagines inuerſas poſt ſe repraeſentant, ita ſpecula concaua imagines itidem inuerſas ante ſe referunt; iam enim obſeruauimus, quae poſt lentes contingunt, cum iis comparari debere, quae ante ſpecula contingunt.

Problema 3.

§. 9. Si a puncto lucido E in axe ſpeculi ſito radii incidant in extremitatem ſpeculi P, eorum cum axe concurſum in puncto ζ inueſtigare, indeque ſpatium diffuſionis determinare. Tab. I. Fig. 3.

Solutio.

Sit iterum diſtantia $E A = a$, radius ſpeculi $O A = O P = f = 2\,p$, denotante p diſtantiam ſpeculi focalem. Iam tantum ſit ſpeculum, vt ſit angulus $A O P = \omega$ et cum perpendiculum $P X$ denotet ſe-

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midiametrum aperturae ſpeculi, ſit haec linea $PX = x$ eritque $x = f.\ \text{ſin.}\ \omega$. Demiſſo iam ex puncto lucido E in radium PO productum perpendiculo ER ob $EO = a - f$ et angulum $EOR = \omega$ erit

$$ER = (a - f)\ \text{ſin.}\ \omega \text{ et } OR = (a - f)\ \text{coſ.}\ \omega.$$

hincque $PR = f + (a - f)\ \text{coſ.}\ \omega$;

vnde inuenitur

$$EP = \sqrt{(PR^2 + ER^2)}$$
$$= \sqrt{(a^2 - 2af + 2f^2 + 2f(a - f)\ \text{coſ.}\ \omega)}$$

quae breuitatis gratia ſit $= v$. atque hinc erit anguli incidentiae EPO, ideoque etiam anguli reflexionis

$$OPf \text{ ſinus } = \frac{ER}{EP}. = \frac{(a-f)\ \text{ſin.}\ \omega}{v}$$

et coſinus $= \frac{f + (a-f)\ \text{coſ.}\ \omega}{v}$.

Cum iam in $\triangle$lo OPf detur angulus OPf vna cum angulo $POf = \omega$ et latere $OP = f$; ſi vocetur angulus $AfP = \psi$ ob $\psi = \omega + OPf$ erit

$$\text{ſin.}\ \psi = \frac{f\ \text{ſin.}\ \omega + 2(a-f)\ \text{ſin.}\ \omega.\ \text{coſ.}\ \omega}{v}$$

atque hinc ex natura trianguli erit

$$\text{ſin.}\ \psi : OP = \text{ſin.}\ OPf : Of.$$

ex qua analogia colligitur

$$Of = \frac{f(a-f)}{f + 2(a-f)\ \text{coſ.}\ \omega}$$

hinc-

hincque interuallum

$$A f = \frac{f^2 + f(a-f)(2 \operatorname{coſ}.\omega - 1)}{f + 2(a-f)\operatorname{coſ}.\omega}$$

haecque eſt ſolutio generalis noſtri problematis.

Cum autem in praxi angulus A O P nunquam tantus aſſumatur, vt non liceat poteſtates anguli ω quadratica altiores negligere, expreſſio inuenta commode ad formam ſimpliciorem ſequenti modo reducetur. Cum ſit

$$\operatorname{coſ}.\ \omega = \sqrt{(1 - \operatorname{ſin}.\ \omega^2)} = 1 - \tfrac{1}{2} \operatorname{ſin}.\ \omega^2.$$

ob $\operatorname{ſin}.\ \omega = \frac{x}{f}$ erit $\operatorname{coſ}.\ \omega = 1 - \frac{x^2}{2f^2}$.

hincque ille denominator

$$f + 2(a-f)\operatorname{coſ}.\ \omega \text{ fiet } = 2a - f - \frac{(a-f)x^2}{f^2},$$

ex quo pariter proxime erit

$$\frac{1}{f + 2(a-f)\operatorname{coſ}.\omega} = \frac{1}{2a - f - \frac{(a-f)x^2}{f^2}}$$

$$= \frac{1}{2a-f} + \frac{(a-f)x^2}{f^2(2a-f)^2}.$$

Vnde interuallum modo inuentum fit

$$O f = \frac{f(a-f)}{2a-f} + \frac{(a-f)^2 x^2}{f(2a-f)^2}$$

atque hinc interuallum quod potiſſimum quaerimus,

$$A f = \frac{af}{2a-f} - \frac{(a-f)^2 x^2}{f(2a-f)^2}.$$

Quare cum ante locum imaginis principalis F ita inveniſſemus, vt eſſet

$$A F = \frac{af}{2a-f}$$

 nunc

nunc innoteſcit, ſpatium diffuſionis

$$Ff = \frac{(a-f)^2 \cdot x^2}{f(2a-f)^2}.$$

Praeterea cum etiam plurimum interſit angulum ψ noſſe, quo radii reflexi Pf ad axem inclinantur, ex formula ſupra inuenta colligemus itidem proxime $\psi = \frac{(2a-f)x}{af}$. Quoniam enim poteſtates ipſius x quadrato maiores negligimus, numerator ibi inuentus fit $\frac{(2a-f)x}{f}$ et in denominatore, vbi iam ipſum quadratum x^2 negligere licet, fit ſimpliciter $= a$.

Coroll. 1.

§. 10. Quo haec ad formulas pro lentibus datas accommodemus, vbi tantum binas diſtantias a et α in computum induximus, ob

$$\alpha = \frac{af}{2a-f} \text{ habebimus } f = \frac{2a\alpha}{a+\alpha};$$

vnde fit

$$a-f = \frac{(a-\alpha)a}{a+\alpha} \quad \text{et} \quad 2a-f = \frac{2a^2}{a+\alpha}$$

atque hinc ſpatium diffuſionis erit

$$Ff = \frac{(a+\alpha)(a-\alpha)^2 \cdot x^2}{4a^3 \cdot \alpha}.$$

quod ergo perinde ac in lentibus vſu venit quadrato ſemidiametri aperturae x^2 eſt proportionale; quin etiam ipſum hoc ſpatium Ff in eundem ſenſum cadit, ac in lentibus.

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Coroll. 2.

§. 11. Simili modo poterimus etiam angulum obliquitatis ψ per ſolas diſtantias a et α itemque x exprimere, prodibit enim $\psi = \frac{x}{\alpha}$. Hunc autem angulum ſupra in calculo circa lentes inſtituto ſollicite definiuimus.

Scholion.

§. 12. Cum quaeſtio eſſet de lentibus earumque apertura maxima, quam capere poſſent, ſumſimus x aequale parti quartae radii curuaturae; quodſi ergo hic idem inſtitutum ſequamur, et ſumamus $x = \frac{1}{4}f$ hinc reperietur angulus $\omega = 14^\circ\ 30'$. ita, vt totus arcus PAP infra 30° capi debeat. Quando autem hoc ſpeculum locum lentis obiectiuae ſuſtinet, eius apertura longe aliam determinationem poſtulat, quam ſcilicet ex menſura confuſionis definiri oportet, vnde huius ſpeculi apertura ad multo pauciores gradus reducetur, vti in ſequentibus docebitur. Nunc autem etiam opus eſt, vt oſtendamus, quemadmodum radii a noſtro ſpeculo reflexi et imaginem diffuſam formantes porro ab alio ſpeculo denuo reflectantur et qualem imaginis diffuſionem tum ſint producturi. Hunc in finem bina ſequentia lemmata perpendi conueniet.

Lemma I.

§. 13. Si diſtantia obiecti a ſpeculo $EA = a$. particula minima da vlterius a ſpeculo remoueatur; tum

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tum imago principalis, cuius diſtantia a ſpeculo erat $AF = \alpha$ ad ſpeculum propius accedet particula $d\alpha$, ita, vt ſit $d\alpha = \frac{-\alpha^2 . da}{a^2}$.

Demonſtratio.

Cum enim ſit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p} = \frac{2}{f}$ atque radius f idem maneat, vtcunque diſtantiae a et α inter ſe varientur; differentiatio dabit

$$\frac{da}{a^2} + \frac{d\alpha}{\alpha^2} = 0 \text{ vnde } d\alpha = -\frac{\alpha^2 da}{a^2}.$$

Lemma 2.

§. 14. Si radii in ſpeculum incidentes ad axem ſint inclinati angulo $= \Phi$, inuenire angulum ψ, ſub quo radii reflexi ad axem ſpeculi erunt inclinati.

Solutio.

Tab. I. Fig. 3. Sit igitur angulus $AEP = \Phi$, quo radii incidentes EP ad axem ſpeculi inclinantur eritque proxime $\Phi = \frac{x}{a}$; ideoque $x = a\Phi$. Tum vero vidimus, angulum, quo radii reflexi ad eundem axem inclinantur, fore $\psi = \frac{x}{\alpha}$; quocirca erit $\psi = \frac{a\Phi}{\alpha}$ ſeu erit $\Phi : \psi = \alpha : a$. ſeu reciproce vt diſtantiae a ſpeculo.

Problema 3.

§. 15. Si radii poſtquam a primo ſpeculo reflexi imaginem diffuſam formauerint in aliud ſpeculum ſuper eodem axe conſtitutum incidant, determinare

nare tam imaginem principalem, quam eius diffuſionem, quam radii a ſecundo ſpeculo reflexi exhibebunt.

Solutio.

Cum F ζ ſit imago principalis a primo ſpeculo formata, quam inuenimus $F\zeta = \frac{\alpha\zeta}{a}$, ſit eius diſtantia a ſecundo ſpeculo $FB = b$ atque ipſum hoc ſpeculum ita ſit comparatum, vt ab eius reflexione imago principalis formetur $G\eta$ ſitque diſtantia $BG = \beta$ atque vti iam vidimus reperietur $G\eta = \frac{\beta}{b}. F\zeta = \frac{\alpha\beta}{ab}\zeta$ quae imago iterum erit erecta atque a radiis axi proximis formata. Nunc etiam conſideremus in ſpatio diffuſionis dato extremitatem f, vnde radii emiſſi cum axe faciant angulum $= \psi = \frac{x}{\alpha}$; verum antequam huius obliquitatis rationem habeamus, fingamus punctum f etiam radios axi proximos emittere et cum id a ſpeculo B longius ſit remotum, quam F, eius radii concurrent in puncto huic ſpeculo propiore γ, ad quod inueniendum referet hic $db = Ff$ et $d\beta = -G\gamma$; vnde colligitur $G\gamma = \frac{\beta^2}{b^2}. Ff$. Quare ſi in f obiectum verum eſſet conſtitutum, eius imago principalis caderet in γ, quatenus autem ex f nulli alii radii emittuntur, niſi qui cum axe faciant angulum $= \psi$, ii denuo reflexi incident in axem in puncto g ipſi ſpeculo B adhuc propiore, quam γ, ita, vt hic caſus ſimilis ſit praecedenti problemati, quo punctum f reſpondet puncto E; punctum γ puncto

Tab. II. Fig. 4.

F et punctum g puncto f, hoc ſolo diſcrimine, vt quod ibi erat a et α hic ſit b et β licebit enim vtrique hic pro diſtantia Bf ſumere $BF = b$. et pro diſtantia $B\gamma$ ſumere β; hinc ergo per formulam ſupra inuentam ſi loco x hic ſcribatur y, fiet

$$\gamma g = \frac{(b+\beta)(b-\beta)^2 . y^2}{4 b^2 \beta}.$$

Quid autem nunc ſit y, ex angulo ψ facillime definitur. Ducto enim radio fQ ſub angulo $BfQ = \psi = \frac{x}{\alpha}$ erit y ſemidiameter aperturae huius ſpeculi QBQ ideoque $y = Bf . \psi = \frac{bx}{\alpha}$; quo valore ſubſtituto prodit

$$\gamma g = \frac{(b+\beta)(b-\beta)^2 . x^2}{4 \alpha^2 b \beta}.$$

Quocirca totum ſpatium diffuſionis iam erit

$$Gg = \frac{\beta^2}{b^2} . Ff + \gamma g; \text{ ſeu}$$

$$Gg = \frac{\beta^2}{b^2} . \frac{(a+\alpha)(a-\alpha)^2 . x^2}{4 a^3 . \alpha} + \frac{(b+\beta)(b-\beta)^2 x^2}{4 \alpha^2 b \beta}$$

Nunc autem poſt ſecundam reflexionem angulus, ſub quo radii extremi ad axem erunt inclinati, colligitur ex Lemmate $\omega = \frac{b\psi}{\beta} = \frac{b}{\alpha\beta} . x$.

Scholion I.

§. 16. Cum igitur ſpeculum, ad quod referuntur binae diſtantiae a et α et cuius ſemidiameter aperturae eſt $= x$, gignat ſpatium diffuſionis

$$Ff = \frac{(a+\alpha)(a-\alpha)^2 . x^2}{4 a^3 . \alpha}$$

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comparemus hoc ſpatium cum eo, quod lens ſub ſimilibus circumſtantiis producit, atque in primo libro vidimus, pro tali lente eſſe ſpatium diffuſionis §. 49.

$$Ff = \frac{n(4n-1)\alpha^2 x^2}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

quod quidem iam eſt minimum, quod a lente ad has diſtantias a et α relata cum apertura, cuius ſemidiameter eſt x, generari poteſt. Quo autem facilius hanc comparationem inſtituere valeamus, ponamus vtrinque diſtantiam obiecti a eſſe infinitam atque e ſpeculo naſcetur ſpatium diffuſionis $Ff = \frac{x^2}{8\alpha}$ quod autem a lente naſcitur, erit

$$Ff = \frac{n(4n-1)x^2}{8(n-1)^2(n+2)\alpha}$$ vbi $n : 1$

denotat rationem refractionis et ſumto $n = 1,55$, hoc ſpatium inuentum eſt $Ff = 0,938191 . \frac{x^2}{\alpha}$

Vnde patet, a ſpeculo multo minorem diffuſionem oriri, quam a lente, quandoquidem illa erit ad hanc, vt $\frac{1}{8} : 0,938191$; hoc eſt propemodum vt $1 : 7,505528$ ſeu vt $1 : 7\frac{1}{2}$ quae ergo proportio cum proprie in ſpeculis vel lentibus obiectiuis locum habeat, hinc praecipua cauſſa innoteſcit, cur ſpecula loco lentium obiectiuarum ſubſtituta multo breuiora teleſcopia ſuppeditauerint, quandoquidem ob minorem confuſionem diſtantiam focalem minorem accipere licet, ad quod accedit, quod in his teleſcopiis catoptricis radii in ſpeculum obiectiuum incidentes primo ad alterum ſpeculum reflectantur, vnde denuo per eandem viam re-

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vertuntur antequam per lentes oculares tranſeunt, ita, vt diſtantia amborum ſpeculorum bis ſit computanda, ſicque longitudo inſtrumenti denuo fere ad ſemiſſem reducatur. Hoc ergo commodum ſpecula praeſtarent, etiam ſine vllo reſpectu ad eorum qualitatem habito, qua radii diuerſorum colorum a reflexione non diſperguntur, vti fit in refractione. Verum tamen hic etiam inſigne ſpeculorum incommodum non eſt reticendum, in eo conſiſtens, quod ſpeculum etiam maxime politum ſemper multo pauciores radios reflectat, quam per lentem eiusdem magnitudinis transmittuntur. Atque haec cauſſa eſt, quod teleſcopia catoptrica plerumque multo minorem claritatis gradum largiantur.

Scholion 2.

§. 17. Quemadmodum hoc poſtremum problema reſoluimus, atque etiam diffuſionem imaginis a ſecundo ſpeculo natam definiuimus, ita eadem inueſtigatio ad plura ſpecula accommodari poſſet, niſi ipſa rei natura ſpeculorum vſum ad binarium reſtringeret. Quamobrem coacti ſumus radios a ſecundo ſpeculo reflexos ad lentes vitreas dirigere, per quas demum ad oculum propagentur atque ob hanc ipſam rationem ipſum ſpeculum obiectiuum circa medium perforatum eſſe debet, vt radiis a ſecundo ſpeculo reflexis tranſitus per hoc foramen concedatur, vbi ſimul a lentibus excipiantur. Quare cum hactenus ſpeculum

lum obiectiuum tanquam integrum ſimus contemplati, nunc ſupereſt, vt etiam foraminis, quo illud eſt pertuſum, in calculo rationem habeamus, vbi ſimul erit disquirendum, quomodo ſpeculum ſecundum reſpectu huius foraminis comparatum eſſe debeat, ne ſcilicet nimiam radiorum copiam intercipiat ac tamen ſufficiat omnibus radiis a primo ſpeculo reflexis excipiendis; haecque ergo momenta in ſequenti problemate accuratius perpendemus.

Problema 4.

§ 18. Si in teleſcopio loco lentis obiectiuae adhibeatur ſpeculum concauum $P\pi A\pi P$ in medio pertuſum foramine $\pi A \pi$, cuius centrum ſit in axe AB in quo ad diſtantiam quaſi infinitam obiectum ſeu punctum lucidum concipiatur, ex quo radii axi paralleli in iſtud ſpeculum $P\pi\pi P$ incidant indeque reflexi ad ſpeculum minus ſuper eodem axe normaliter poſitum QBQ dirigantur, vnde porro ad lentem vitream prope foramen $\pi\pi$ itidem ſuper eodem axe normaliter ſitam reflectantur; determinare imagines, per duplicem reflexionem formatas earumque diffuſionem. Tab. III. Fig. 5.

Solutio.

Sit ſemidiameter totius ſpeculi obiectiui $AP = x$ et ſemidiameter foraminis $A\pi = y$, radius vero curvaturae ſpeculi $= f$; ideoque diſtantia focalis $p = \frac{1}{2}f$ tum vero ſpeculi minoris QBQ ſit diſtantia focalis

$= q$ et diſtantia horum ſpeculorum $AB = k$. His poſitis cum obiectum in axe AB ad diſtantiam infinitam remotum concipiatur radii inde axi paralleli ad ſpeculum obiectiuum PP peruenient, qui ergo vt totam eius ſuperficiem reflectentem $P\pi$ quaquaverſus adimpleant, ſpeculum QBQ maius eſſe non debet, quam foramen $\pi\pi$ neque etiam id minus eſſe conueniet, quia alioquin radii ab obiecto directe in foramen lentemque ibi ſitam ingrederentur et repraeſentationem inquinarent, ex quo intelligitur, ſemidiametrum aperturae huius ſpeculi minoris eſſe debere $BQ = y$ vel ſaltem eo non multo maiorem. Quoniam igitur hic diſtantia obiecti, quae ſupra poſita eſt $= a$, noſtro caſu eſt infinita, ſi radii axi proximi in ſpeculum incidere poſſent, iis formaretur imago principalis in F ita, vt eſſet diſtantia $AF = \alpha = p$. Quia autem radii axi proximi excluduntur, nulla imago principalis formabitur. Prima ergo imago a radiis circa oram foraminis reflexis formabitur in Φ, ita, vt ſit interuallum $F\Phi = \frac{y^2}{4\alpha}$, quia hic eſt y, quod ſupra erat x et diſtantia obiecti $a = \infty$. Imago autem extrema a radiis circa oram ſpeculi Pp reflexis formetur in puncto f eritque interuallum $Ff = \frac{x^2}{4\alpha}$; quare cum ipſa imago principalis hic deſit, totum ſpatium diffuſionis hic tantum erit $\Phi f = \frac{x^2 - y^2}{4\alpha}$. Interim tamen haec puncta F, Φ, f inter ſe tam erunt propinqua, vt in calculo pro eodem haberi

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queant. Cum ergo omnes radii a ſpeculo maiore reflexi per punctum F tranſire ſint cenſendi, vt in ſpeculum QBQ incidant eius ſemidiameter BQ tantus eſſe debet, vt ſit

$$AF : AP = BF : BQ \text{ vnde fit } BQ = \tfrac{k-\alpha}{\alpha} . x$$

qui cum ipſi y debeat eſſe aequalis, habebimus

$$y = \tfrac{k-\alpha}{\alpha} . x. \text{ hincque } k = \tfrac{\alpha(y+x)}{x}.$$

Sin autem minus ſpeculum intra A et F eſſet conſtitutum; reperiretur

$$BQ = \tfrac{\alpha-k}{\alpha} x = y; \text{ hincque } k = \tfrac{\alpha(x-y)}{x},$$

quae vero expreſſio in ſuperiori contenta eſt cenſenda, propterea quod radium foraminis y tam poſitiue, quam negatiue capere licet. Cum igitur nunc primae imaginis F diſtantia a ſpeculo ſecundo ſit $k - \alpha = \frac{\alpha y}{x}$ quam ſupra vocauimus $= b$. ita, vt ſit $b = \frac{\alpha y}{x}$, ſecunda imago a ſpeculo QBQ reflexa cadet in punctum G, ita, vt ſit $BG = \beta = \frac{bq}{b-q}$; ita, vt radii a ſpeculo QBQ reflexi omnes per punctum hoc G tranſire ſint cenſendi, ſiquidem hic animum a diffuſione imaginis abſtrahimus. Nunc igitur inſuper efficiendum eſt, vt iſti radii omnes in ipſum foramen $\pi A \pi$ ingrediantur, id quod cum ſit $BQ = A\pi$ eueniet, ſi modo punctum G propius verſus A cadat, quam verſus B. ſeu debebit eſſe $\beta > \frac{1}{2} k$. Inuenimus vero

$\beta =$

$$\beta = \frac{bq}{b-q} = \frac{\alpha y q}{\alpha y - qx} \text{ et } k = \frac{\alpha(y+x)}{x},$$

ita vt nunc eſſe debeat

$$\frac{\alpha y q}{\alpha y - q x} > \frac{\alpha(y+x)}{2x} \text{ vnde oritur } q > \frac{\alpha y(x+y)}{x(3y+x)}$$

ex qua ergo formula diſtantia focalis ſpeculi minoris definiri poterit, quae ergo determinabitur per ſemidiametros foraminis et ipſius ſpeculi maioris vna cum focali diſtantia ſpeculi maioris $p = \alpha$ ſin autem ſpeculum minus conſtituatur intra F et A, iam vidimus fore $AB = k = \frac{\alpha(x-y)}{x}$ et cum nunc ſit diſtantia $b = -\frac{\alpha y}{x}$ diſtantia $BG = \beta = \frac{\alpha y q}{\alpha y + xq}$ quae vt maior ſit, quam $\frac{1}{2}k$, neceſſe eſt fiat $q > \frac{\alpha y(x-y)}{x(3y-x)}$ vnde ſi x ſit $> 3y$, debebit eſſe q negatiuum, ita, vt ſit

$$q > -\frac{\alpha y(x-y)}{x(x-3y)}$$

at ſi eſſet $x = 3y$, capi poſſet $q = \infty$, ſicque ſpeculum minus fieret planum. Quod denique ad diffuſionem imaginis ſecundae in G repraeſentatae attinet, ea iterum erit quaſi truncata ſua imagine principali, quod ſi litteris $G\Phi g$ repraeſentetur ad ſimilitudinem litterarum $F\Phi f$ totum ſpatium diffuſionis tantum erit cenſendum $= \Phi g$, cuius quantitas ex formula praecedentis problematis reperietur, ſi loco x^2 ſcribatur $x^2 - y^2$, vnde ob $a = \infty$ erit hic

$$\Phi g = \frac{\beta^2}{b^2} \frac{(x^2-y^2)}{4\alpha} + \frac{(b+\beta)(b-\beta)^2 (x^2-y^2)}{4\alpha\alpha\beta}$$

atque nunc radiorum in Φ concurrentium obliquitas ad axem erit $= \frac{b}{\alpha\beta} \cdot y$; obliquitas vero radiorum in $g = \frac{b}{\alpha\beta} \cdot x$.

Coroll.

Coroll. 1.

§. 19. Si ergo minus ſpeculum vltra locum imaginis F collocetur, eius diſtantia a primo ſpeculo debet eſſe

$$A\,B = \frac{\alpha(x+y)}{x} = \alpha + \frac{y}{x}.\,\alpha$$

ita, vt ſit $F\,B = \frac{\alpha y}{x}$ hocque ergo caſu diſtantia A B maior erit, quam diſtantia focalis ſpeculi principalis tum vero huius ſecundi ſpeculi diſtantia focalis eſſe debet $q > \frac{\alpha y(x+y)}{x(x+3y)}$.

Coroll. 2.

§. 20. Hic autem manifeſto ſupponitur, punctum G a puncto B verſus A cadere, ita, vt diſtantia β euadat poſitiua; ſi enim eſſet $q > b$, punctum G ad alteram partem ſpeculi Q B Q caderet radiique G Q producti manifeſto extra foramen praetergrederentur. Quare hic pro Q alterum limitem probe obſeruari oportet, vt ſit $q < b$. ſiue $q < \frac{\alpha y}{x}$, tum vero etiam $q > \frac{\alpha y(x+y)}{x(x+3y)}$.

Coroll. 3.

§. 21. Sin autem ſpeculum Q B Q intra focum F collocetur, oportebit eſſe diſtantiam Tab. II. Fig. 6.

$A\,B = \frac{\alpha(x-y)}{x} = \alpha - \frac{y}{x}\,\alpha.$ ita, vt ſit $F\,B = \frac{\alpha y}{x}$

tantoque interuallo prima imago poſt ſecundum ſpeculum cadat, fiatque $b = -\frac{\alpha y}{x}$; vnde deducitur di-

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ſtantia B G $= \beta = \frac{\alpha y}{\alpha y + q x}$, quae diſtantia ſemper eſt poſitiua ſeu verſus A dirigitur, niſi forte q ſit quantitas negatiua; quae cum ſuperare debeat $\frac{1}{2} k$, debet eſſe $2 xyq > \alpha y (x-y) + x (x-y) q$ deberet ergo eſſe $2 xy > x (x-y)$ ſeu $y > \frac{1}{3} x$. Quare ſi vt ſemper in praxi euenit ſit $y < \frac{1}{3} x$, huic conditioni ſatisfieri nequit; ſi ſcilicet alterum ſpeculum ſit concauum.

Coroll. 4.

§. 22. Hoc ergo caſu neceſſe eſt, vt minus ſpeculum ſit conuexum eiusque diſtantia focalis negatiua. Statuatur ergo $q = -\mathfrak{q}$, vt fiat $\beta = \frac{-b\mathfrak{q}}{b+\mathfrak{q}}$, qui valor ob $b = -\frac{\alpha y}{x}$ abit in hunc $\beta = \frac{\alpha y \mathfrak{q}}{\mathfrak{q} x - \alpha y}$, qui valor vt primo ſit poſitiuus, debet eſſe $\mathfrak{q} > \frac{\alpha y}{x}$ deinde vt fiat $2 \beta > k$ debet eſſe

$$2 xy \mathfrak{q} > x (x-y) \mathfrak{q} - \alpha y (x-y)$$

ex qua fit

$$\alpha y (x-y) > x (x - 3y) \mathfrak{q}$$

vnde pro $\mathfrak{q}$ elicitur alter limes

$$\mathfrak{q} < \frac{\alpha y (x-y)}{x (x-3y)}, \text{ altero exiſtente } \mathfrak{q} > \frac{\alpha y}{x}.$$

Coroll. 5.

§. 23. Sin vero praeter conſuetudinem foramen tantum fiat, vt ſit $3y > x$, tum ſpeculo minori concauo vti licebit dummodo eius diſtantia focalis ſit $q > \frac{\alpha y (x-y)}{x (3y-x)}$, quemadmodum ex Coroll. 3 eſt manifeſtum,

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ſeſtum, atque hoc caſu quoniam littera q nulla alia conditione reſtringitur hoc ſpeculum adeo planum fieri poterit.

Scholion.

§. 22. Haec duo ſpecula ita hic ſumus contemplati, quemadmodum in teleſcopiis Gregorianis vſurpari ſolent atque hic tantum ad obiecti punctum medium in axe tubi ſitum ſpectauimus vnde radii axi paralleli in ſpeculum principale incidant; alterum vero ſpeculum ita inſtruximus, vt omnes radios a priori reflexos recipiat eosque porro in foramen projiciat. Cum autem etiam partes obiecti extra axem ſitae viſui offerri debeant, quoniam inde radii ſub aliqua exigua obliquitate in ſpeculum incidunt, tubum, in quo haec duo ſpecula inſeruntur, aliquantillum diuergentem confici oporteret vel quod eodem redit tubum aliquanto ampliorem effici conueniet quam eſt diameter ſpeculi; deinde ob eandem rationem etiam ſpeculum minus vltra limites ipſi aſſignatos extendi deberet, vt etiam iſtos radios obliquos poſt reflexionem recipere poſſet, ſed quoniam parum intereſt, ſiue extremitates obiecti pari lumine conſpiciantur atque eius medium, ſiue minore, hac amplificatione facile eo magis carere poterimus, quod tota haec obliquitas non vltra aliquot minuta in magnis praeſertim multiplicationibus excreſcat. Longe aliter autem ſe habitura eſſet huius rei tractatio, ſi etiam ſpecula ad

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axem inſtrumenti oblique poſita in vſum vocarentur, quemadmodum in ipſo huius inuentionis principio a Newtono eſt tactum, ſed quia reflexio radiorum oblique incidentium haud exiguam gignit confuſionem, hoc argumentum hic neutiquam attingimus.

CAPVT

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CAPVT II.

DE COMPVTO CONFVSIONIS, DVM PRAETER LENTES ETIAM SPECVLA AD INSTRVMENTA DIOPTRICA CONFICIENDA ADHIBENTVR.

Problema 1.

§. 23.

Si loco primae et ſecundae lentis ſpecula vſurpentur, inuenire formulas, quae ob haec duo ſpecula in expreſſionem ſupra in Libro I. inuentam, qua ſcilicet ſemidiameter confuſionis eſt inuenta, introduci in calculum debent.

Solutio.

In primo libro §. 91. oſtendimus a duabus lentibus oriri ſpatium diffuſionis

$$Gg = \mu \beta^2 x^2 \begin{cases} \frac{\alpha^2}{b^2}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \\ + \frac{b^2}{\alpha^2}\left(\frac{1}{b} + \frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\beta}\right)^2 + \frac{\nu}{b\beta}\right) \end{cases}$$

 quae

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quae expreſſio ponendo $\alpha = A\,a$, $\beta = B\,b$, tum vero etiam $\frac{A}{1+A} = \mathfrak{A}$ et $\frac{B}{1+B} = \mathfrak{B}$, abit in hanc:

$$Gg = \frac{\mu A^2 B^2 . x^2}{a} \left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} + \frac{b}{A^4 . a} \left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}} \right) \right)$$

atque ſi hic porro, vti deinceps in tractatu de Teleſcopiis fecimus, ponamus $\frac{\alpha}{b} = \frac{A\,a}{b} = -P$, iſta expreſſio induet hanc formam

$$Gg = \frac{A^2 B^2 x^2}{a} \left(\mu \left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} \right) - \frac{\mu}{A^3 P} \left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}} \right) \right)$$

Si nunc loco duarum harum lentium duo ſubſtituantur ſpecula, ad quae litterae a, α; b, β cum x ſimiliter ſint relatae, in Probl. 3. Cap. praeced. §. 15. inuenimus fore ſpatium diffuſionis

$$Gg = \frac{\beta^2}{b^2} . \frac{(a+\alpha)(a-\alpha)^2 . x^2}{8 a^3 \alpha} + \frac{(b+\beta)(b-\beta)^2 . x^2}{8 \alpha^2 b \beta}$$

quae forma poſito $\alpha = A\,a$; $\beta = B\,b$ et $\frac{\alpha}{b} = -P$ induet hanc formam

$$Gg = \frac{A^2 B^2 . x^2}{a} \left(\frac{(1+A)(1-A)^2}{8 A^3} - \frac{(1+B)(1-B)^2}{8 A^3 . B^3 . P} \right)$$

ex qua cum ſuperiori collata cognoſcimus, ſi loco primae lentis ſpeculum ſubſtituatur, tum in computo confuſionis loco formulae

$\mu \left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} \right)$ ſcribi debere hanc $\frac{(1+A)(1-A)^2}{8 A^3}$,

ac ſi etiam loco lentis ſecundae ſpeculum ſubſtituatur, tum ſimili modo loco formulae

$\mu \left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}} \right)$ ſcribi debere hanc $\frac{(1+B)(1-B)^2}{8 B^3}$;

ac

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ac ſi circumſtantiae permitterent, vt etiam loco tertiae lentis ſpeculum ſimile ſubſtitueretur, tum in computo confuſionis loco formulae

$\mu\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$ ſcribi deberet haec formula $\frac{(1+C)(1-C)^2}{4C^3}$.

vnde ſatis ſuperque intelligitur, quomodo quantitas confuſionis aeſtimari debeat, quando loco lentium ſpecula adhibentur.

Coroll. 1.

§. 24. Quatenus autem ſpeculum obiectiuum foramine eſt pertuſum, cuius radius $= y$, eatenus in factore communi loco x^2 ſcribi oportet $x^2 - y^2$ ita, vt iam expreſſio pro ſpatio diffuſionis inuenta futura ſit

$$Gg = \frac{A^2 . B^2 (x^2 - y^2)}{a}\left(\frac{(1+A)(1-A)^2}{4A^3} - \frac{(1+B)(1-B)^2}{4A^3B^3P}\right)$$

vbi notandum eſt, formulam $x^2 - y^2$ proportionalem eſſe ſuperficiei reflectenti in primo ſpeculo, prorſus vti x^2 proportionale erat ſuperficiei refringenti lentis obiectiuae.

Coroll. 2.

§. 25. Atque haec formula $x^2 - y^2$ etiam extenditur ad omnes lentes ſequentes, quotquot binis ſpeculis inſuper adiunguntur; ita, ex gr. ſi duae lentes praeter ſpecula adhibeantur, totum ſpatium diffuſionis Ii ita exprimitur:

$Ii =$

$$Ii = \frac{A^2B^2C^2D^2(x^2-y^2)}{a}\left(\frac{(1+A)(1-A)^2}{8A^3} - \frac{(1+B)(1-B)^2}{8A^3B^3P}\right.$$
$$+ \frac{\mu}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$$
$$\left. - \frac{\mu}{A^3B^3C^3PQR}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right)\right)$$

vnde patet, quid propter ſpecula in noſtris formulis generalibus immutari debeat.

Coroll. 3.

§. 26. Cum autem noſtra ſpecula tantum ad teleſcopia accommodari queant, vbi eſt $a = \infty$ $A = 0$. et $Aa = \alpha = p$, ex formulis vinculo incluſis denominator A^3 in factorem communem transfertur ſicque pro ſpatio diffuſionis a binis ſpeculis et duabus lentibus orto habebitur haec expreſſio:

$$Ii = \frac{B^2C^2D^2(x^2-y^2)}{p}\left(\frac{1}{8} - \frac{(1+B)(1-B)^2}{8B^3P}\right.$$
$$+ \frac{\mu}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$$
$$\left. - \frac{\mu}{B^3C^3PQR}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right)\right)$$

Scholion I.

§. 27. Quoniam autem pro ſecundo ſpeculo tam littera $B = \frac{\beta}{b}$, quam $P = -\frac{\alpha}{b}$ non amplius ab arbitrio noſtro pendet, ſed earum valores iam ante ſunt definiti, videamus, quomodo iſti valores in computum ſint introducendi, atque hic duos caſus euolui conueniet, prouti minus ſpeculum ſiue vltra focum ſpeculi principalis conſtituitur, ſiue citra. Quod quo ad noſtras formas ſuccinctius exprimi poſſit, ponamus in

in genere $y = \epsilon x$ ita, vt ſit $x^2 - y^2 = (1 - \epsilon^2) x^2$, vbi ſcilicet ϵ denotat fractionem foraminis magnitudinem definientem.

I. Primo igitur quando diſtantia minoris ſpeculi Tab. II.
A B maior eſt, quam diſtantia focalis p; tum vidi- Fig. 5.
mus (§. 19.) eſſe hanc diſtantiam A B ſeu primum interuallum $= (1 + \epsilon) \alpha = (1 + \epsilon) p$, quod cum per formulas noſtras generales ſit $= A\alpha(1 - \frac{b}{p}) = p(1 - \frac{b}{p})$ erit $\frac{b}{p} = -\epsilon$. Deinde vero etiam vidimus eſſe, $b = \epsilon p$ et porro ſi diſtantia focalis minoris ſpeculi ponatur $= q$, erit $\mathfrak{B} = \frac{bq}{b-q}$ hincque $\frac{\beta}{b} = B = \frac{q}{b-q} = \frac{q}{\epsilon p - q}$. At vero pro q hos dedimus limites: $q < \epsilon p$ et $q > \frac{\epsilon(1+\epsilon)p}{1+3\epsilon}$ quibus valoribus ſubſtitutis ſpatium illud diffuſionis Ii ita exprimetur:

$$Ii = \frac{(1-\epsilon^2) \cdot C^2 D^2 q^2 x^2}{(\epsilon p - q)^2 \cdot p} \left(\frac{1}{v} + \frac{\epsilon^2 (\epsilon p - \epsilon q)^2 \cdot p}{\epsilon q^3} - \frac{\mu \cdot \epsilon (\epsilon p - q)^3}{q^3 \cdot Q} \left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{v}{\mathfrak{C}C} \right) + \frac{\mu \cdot \epsilon (\epsilon p - q)^3}{C^2 \cdot q^3 QR} \left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{v}{D\mathfrak{D}} \right) \right)$$

II. Sin autem diſtantia ſecundi ſpeculi A B mi- Tab. II.
nor fuerit, quam p, tum primo erit haec ipſa diſtan- Fig. 6.
tia $= (1 - \epsilon) p$, quae cum ſit $p(1 - \frac{b}{p})$ erit $\frac{b}{p} = \epsilon$. Deinde erit diſtantia $b = -\epsilon p$ et quia ſecundum ſpeculum debet eſſe conuexum, poſito $q = -\mathfrak{q}$ fiet $\frac{\beta}{b} = B = \frac{-\mathfrak{q}}{\mathfrak{q} - \epsilon p}$ verum pro $\mathfrak{q}$ hos dedimus limites $\mathfrak{q} > \epsilon p$ et $\mathfrak{q} < \frac{\epsilon(1-\epsilon)p}{1-3\epsilon}$ quibus valoribus ſubſtitutis ſpatium illud diffuſionis ita exprimetur:

Tom. II. Qqq $Ii =$

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$$I\,i = \frac{(1-\varepsilon^2).C^2D^2.q^2x^2}{(q-\varepsilon p)^2.p}\Big(\frac{1}{v} - \frac{\varepsilon^2(2q-\varepsilon p)^2.p}{4q^3}$$
$$- \frac{\mu.\varepsilon(q-\varepsilon p)^3}{q^3.Q}\Big(\frac{\lambda''}{\mathfrak{C}^3} + \frac{v}{C\mathfrak{C}}\Big)$$
$$+ \frac{\mu.\varepsilon.(q-\varepsilon p)^3}{q^3.C^3.QR}\Big(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{v}{D\mathfrak{D}}\Big)\Big)$$

Quodſi lens in ipſo foramine ſpeculi obiectivi conſtituatur, tum inſuper datur interuallum ſecundum, primo quippe aequale, ac primo quidem caſu erit $= (1+\varepsilon)p$. Quod cum per formulas generales ſit

$$= -\frac{AB\alpha}{P}\Big(1-\frac{1}{Q}\Big) = \frac{\varepsilon pq}{\varepsilon p - q}\Big(1-\frac{1}{Q}\Big);$$

hinc reperitur

$$\frac{1}{Q} = 1 - \frac{(\varepsilon p - q)(1+\varepsilon)}{\varepsilon q} \text{ ſeu } \frac{1}{Q} = \frac{(2\varepsilon+1)q - \varepsilon(1+\varepsilon)p}{\varepsilon q}$$

hincque $\frac{1}{pQ} = \frac{\varepsilon(1+\varepsilon)p - (2\varepsilon+1)q}{q}$

vbi notandum eſt, Q fieri non poſſe negatiuum; niſi q contineatur intra hos limites

$$q < \frac{\varepsilon(1+\varepsilon)p}{1+2\varepsilon} \text{ et } q > \frac{\varepsilon(1+\varepsilon)p}{1+3\varepsilon}.$$

Haec ſcilicet valent pro caſu priore; pro caſu vero poſteriore reperitur

$$\frac{1}{Q} = \frac{\varepsilon(1-\varepsilon)p - (1-2\varepsilon)q}{\varepsilon q} \text{ et } \frac{1}{PQ} = \frac{(2\varepsilon-1)q + \varepsilon(1-\varepsilon)p}{q}$$

vbi pariter notetur, Q fieri negatiuum, ſi q capiatur intra hos limites

$$q > \frac{\varepsilon(1-\varepsilon)p}{1-2\varepsilon} \text{ et } q < \frac{\varepsilon(1-\varepsilon)p}{1-3\varepsilon}$$

at vero Q fieri poſitiuum, ſi capiatur intra hos limites: $q < \frac{\varepsilon(1-\varepsilon)p}{1-2\varepsilon}$ et $q > \varepsilon p$.

Scho-

Scholion. 2.

§. 27. Quae hic attulimus, ad ſpatia diffuſionis, ex ſpeculis et lentibus quotcunque ortae pertinent. Concluſio vero, quae in ſuperiore libro hinc ad ſemidiametrum confuſionis ipſam determinandam eſt deducta, etiam hic quandam mutationem patitur. Quoniam enim ſemidiametrum confuſionis ex vltimae imaginis diffuſione concluſimus, notandum eſt, etiam hoc vltimum ſpatium diffuſionis ſua imagine principali fore truncatum. Quoniam enim a primo ſpeculo nulla gignitur imago principalis ob defectum radiorum axi proximorum, etiam ſequentia ſpatia diffuſionis, quotcunque fuerint lentes imagine principali deſtituentur; vnde cum horum ſpatiorum vltimum minus ſit, propter ipſam hanc mutilationem, inde etiam minor confuſio in oculo orietur, quam ob cauſſam etiam ſemidiameter confuſionis prouti eum in primo libro definiuimus minorem valorem adipiſcetur, quam inueſtigationem ſequenti problemate ſuſcipiemus.

Problema 2.

§. 28. Data vltima imagine diffuſa, quae tam per bina ſpecula, quam omnes lentes ſequentes formatur, inuenire confuſionem in ipſo oculo inde oriundam, qua ſcilicet viſio immediate afficitur.

Solutio.

Repraeſentet L λ *l* vltimum ſpatium diffuſionis Tab. III. Fig. 7. tam per ſpecula, quam omnes ſequentes lentes forma-

 tum,

tum, quippe quod eſt obiectum immediatum viſionis; vnde radii immediate in oculum ingrediuntur, in quo ſpatio punctum L denotet locum imaginis principalis, vbi radii axi proximi concurrerent, ſi ſpeculum obiectiuum eſſet integrum ob foramen autem huius ſpeculi, iſta imago principalis plane deerit et imago diffuſa demum in puncto λ incipiet, vbi radii circa oram foraminis reflexi et per omnes lentes transmiſſi concurrunt, alter vero terminus ſit in l, vbi radii ab extremitate ſpeculi obiectiui reflexi ac per lentes transmiſſi vniuntur. Quod nunc primo ad magnitudinem huius ſpatii $\lambda\, l$ attinet, ſupra vidimus, id eſſe proportionale formulae $xx - yy$, ſiue poſito $y = \varepsilon x$, huic $(1 - \varepsilon\varepsilon)\, xx$ vnde ſtatuamus hoc ſpatium $\lambda l = V(1-\varepsilon\varepsilon)xx$. Deinde radiorum in termino λ cum axe concurrentium obliquitas, quam ſupra ipſi y proportionalem eſſe vidimus, ponatur $= \mathfrak{B}y = \varepsilon\,\mathfrak{B}\,x$; obliquitas vero radiorum extremorum in puncto l concurrentium erit $= \mathfrak{B}\,x$, vbi litterae V et $\mathfrak{B}$ eosdem valores habent, quos in primo libro §. 163. aſſignauimus.

His praemiſſis quaeramus eum oculi locum, vnde haec imago diffuſa minima cum confuſione conſpiciatur. Hunc in finem concipiamus punctum quoddam medium in imagine ζ, a quo oculus ad diſtantiam ſuam iuſtam $= l$ ſit remotus, ita, vt ſit $\zeta l = O$ radiique ex hoc puncto ζ emiſſi praeciſe in puncto retinae V congregentur. Hinc ergo puncta cis et

vltra

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vltra hoc punctum ζ vel λ vel *l* verſus ſita non in ipſa retina V, ſed vel poſt eam in υ vel ante eam in *v* repraeſentabuntur radiique in his punctis ſe decuſſantes in ipſa retina circellos ſiue maiores ſiue minores referent atque nunc totum negotium huc reducitur, vt hi circelli quam minimi euadant, quia hoc modo in oculo minima confuſio producetur. Primum igitur videndum eſt, quanti huiusmodi circuli a punctis intra ζ et λ ſitis in retina oriantur et quinam eorum futurus ſit maximus, quoniam enim hi circelli partim a diſtantia a puncto ζ, partim a radiorum obliquitate pendent, quae a λ verſus ζ progrediendo continuo creſcit; facile intelligitur, ex puncto quodam medio, puta ω, maximum circellum oriri, quandoquidem tam ex ipſo puncto L, vbi obliquitas eſt nulla, quam ex puncto ζ nullus talis circellus orietur. Deinde a ζ ad *l* regrediendo continuo maiores huiusmodi circelli orirentur, ita, vt radii ex ipſo puncto *l* emiſſi ab hac parte maximum circellum gignant; ex quo manifeſtum eſt, ſi punctum ζ ita fuerit aſſumtum, vt maximi modo dicti circelli ex punctis ω et *l* orti fiant inter ſe aequales; tum confuſionem in ipſa viſione natam omnium fore minimam. Si enim punctum ζ propius ad ω moueretur; tum circellus quidem ab hac parte ortus fieret minor, alter vero ex puncto *l* ortus tanto maior euaderet; atque contrarium eueniret, ſi punctum ζ propius verſus *l* caperetur. Vt igitur nunc tam locum

puncti ζ, quam ei reſpondentis puncti ω inueſtigemus; totum ſpatium $L\,l$, etſi id noſtro caſu parte $L\,\lambda$ eſt truncatum, in computum ducamus ponamusque breuitatis gratia $L\,l = f$, eritque ex principiis ſupra expoſitis $f = V\,x^2$ et $L\,\lambda = V\,y^2 = \varepsilon^2\,V\,x^2$.; vnde fit, vti initio commemorauimus, $\lambda\,l = (1 - \varepsilon^2)\,V\,x^2$. Praeterea vero vocemus ſpatia $L\,\zeta = \zeta$, et $L\,\omega = \omega$, et quia radii ex hoc puncto ω egreſſi ſupra retina maximum circellum producere ponuntur, ad hunc inueniendum obliquitatem radiorum in puncto hoc ω noſſe oportet. Quia autem obliquitas in L eſt nulla, in l vero $= \mathfrak{B}\,x$ et in $\lambda = \varepsilon.\,\mathfrak{B}\,x$ euidens eſt, obliquitatem creſcere in ratione ſubduplicata diſtantiae a puncto L; vnde obliquitas radiorum in ω erit $= \mathfrak{B}x.\sqrt{\frac{\omega}{f}}$.

Radii igitur ex ω egreſſi concurrent poſt oculum in puncto υ, ita, vt ſit per principia ſupra ſatis ſtabilita $V\,\upsilon = \frac{uu}{ll}\,\zeta\,\omega$, denotante u profunditatem oculi O V. Radiorum autem in hoc puncto υ concurrentium obliquitas ex iisdem principiis erit

$$= \frac{l}{u}.\,\mathfrak{B}\,x\,\sqrt{\frac{\omega}{f}};$$

ex quibus duobus momentis concluditur circelli in retina depicti radius $= \frac{u}{l}.\,\zeta\,\omega.\,\mathfrak{B}\,x\,\sqrt{\frac{\omega}{f}}$ et quia eſt $\zeta\,\omega = \zeta - \omega$, erit radius iſtius circelli

$$= \frac{u}{l}\,\mathfrak{B}\,x.(\zeta - \omega)\sqrt{\frac{\omega}{f}},$$

qui ergo vt maximus euadat, ſpatium ω ita aſſumi opor-

oportet, vt fiat $(\zeta - \omega) V \omega =$ maximo, quod euenit ſumendo $\omega = \frac{1}{3}\zeta$; quocirca maximi huius circelli erit radius $= \frac{u}{l} \mathfrak{B} x . \frac{2}{3} \zeta . V \frac{\zeta}{3f}$. Nunc vero ex altera parte radii ex altero puncto l in oculum incidentes conſiderentur, qui ante retinam in puncto v colligentur, exiſtente ſpatio $V v = \frac{uu}{ll} . \zeta l = \frac{uu}{ll} (f - \zeta)$ ibique radiorum obliquitas erit $= \frac{l}{u} \mathfrak{B} x$; vnde circelli ſuper retina depicti radius erit $= \frac{u}{l} (f - \zeta) \mathfrak{B} x$ qui conſequenter radio prioris circelli inuenti aequalis ſtatui debet; ex quo obtinebitur haec aequatio $f - \zeta = \frac{2}{3} \zeta V \frac{\zeta}{3f}$ ex qua interuallum ζ definiri oportet. Sumtis autem quadratis habebimus

$$f^2 - 2f\zeta + \zeta^2 = \frac{4}{27} . \frac{\zeta^3}{f} \text{ ſiue}$$

$$f^3 - 2f^2\zeta + f\zeta^2 - \frac{4}{27}\zeta^3 = 0.$$

quam perpendenti mox patebit diviſibilem eſſe per $f - \frac{1}{3}\zeta$ diviſione autem facta prodit

$$f^2 - \frac{5}{3} f\zeta + \frac{4}{9}\zeta^2 = 0.$$

quae denuo per $f - \frac{1}{3}\zeta$ diuiſa praebet $f - \frac{4}{3}\zeta = 0$, quia vero bini priores factores hic locum habere nequeunt, quia abſurdum foret eſſe $\zeta = 3f$, vltimus factor nobis verum praebet interuallum $L\zeta = \zeta = \frac{3}{4} f$, ita, vt ſit $l\zeta = \frac{1}{4} f$ et $L\omega = \omega = \frac{1}{4} f$ his valoribus inuentis circelli minimi in oculo deſcripti radius erit $= \frac{u}{4l} f . \mathfrak{B} x$ et cum ſit $f = V x^2$, erit iſte radius $= \frac{u}{4l} V \mathfrak{B}\, x^3$ Iam vero ſi in coelo circulum conſpiceremus, cuius radius

radius apparens $= \Phi$, eius imago ſuper retina etiam eſſet circulus, cuius radius $= u\,\Phi$ hoc ergo circulo illi aequali poſito fit $\Phi = \frac{\nu \mathfrak{B} x^3}{4 l}$ et ſingula imaginis noſtrae puncta ab oculo cernentur tanquam maculae circulares, quarum ſemidiameter apparens ſit $= \frac{\nu \mathfrak{B} x^3}{4 l}$, quam expreſſionem ſupra nominauimus ſemidiametrum confuſionis.

Coroll. 1.

§. 29. In hac ſolutione aſſumſimus, punctum ω intra λ et l cadere; ſi enim termino L propius eſſet, quam punctum λ, quoniam imago tantum per ſpatium $L\,l$ eſt diffuſa, iſtud punctum ω prorſus non in computum venire poſſet, ſed maximus circellus in oculo ex hac parte ab ipſo puncto λ oriretur, atque pro hoc caſu peculiaris ſolutio requiretur, quam mox ſumus daturi.

Coroll. 2.

§. 30. Cum autem ſit $L\,\omega = \frac{1}{2} L\,f = \frac{1}{4} L\,l$, pro termino autem λ ſit $L\,\lambda = \varepsilon\,\varepsilon.\,L\,l$, punctum ω intra terminos l et λ cadet, quoties fuerit $L\,\omega > L\,\lambda$ ideoque quoties fuerit $\varepsilon < \frac{1}{2}$, quamobrem, quia in praxi ε ſemper aſſumitur $< \frac{1}{2}$, ſolutio problematis ad praxin vtique eſt accommodata.

Coroll. 3.

§. 31. Quoties igitur fuerit $\varepsilon < \frac{1}{2}$, tum certo affirmare licet ob foramen, quo ſpeculum eſt pertuſum,

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ſum, confuſionem nullo modo imminui, ſed ſemper tantam eſſe, ac ſi ſpeculum eſſet integrum, totaque ſua ſuperficie radios reflecteret, ideoque aequatio generalis ſupra inuenta pro ſemidiametro confuſionis etiam pro ſpeculis valebit, ſi modo, vt ſupra iam inuenimus, loco formularum ad ſpecula pertinentium formulae ibi aſſignatae §. 23. ſubſtituantur.

Coroll. 4.

§. 32. Atque hinc etiam cognoſcimus, ſi teleſcopium ex meris lentibus conſtet, confuſionem neutiquam diminui, etiamſi lens obiectiua circa medium obtegatur, quemadmodum nonnulli auctores ſuaſerunt, ſed optimum remedium confuſionem diminuendi certo in hoc conſtat, vt lens ocularis circa marginem obtegatur, quippe quo pacto ipſe ſemidiameter confuſionis x diminuitur, et confuſio adeo in ratione triplicata minor redditur, cum e contrario, ſi lens circa medium obtegeretur, ne minima quidem confuſionis diminutio ſit exſpectanda, niſi forte pars obtecta ſemiſſem totius lentis ſuperet, quo pacto autem claritas niminum diminueretur.

Scholion.

§. 33. Sin autem ſemidiameter foraminis $y = \epsilon . x$ ſemiſſem totius aperturae x ſuperet, ita, vt punctum ω inter L et λ cadat, problema noſtrum aliam ſolutionem poſtulat. Cum enim nunc ex parte $\zeta\lambda$ maximus

ximus circellus in oculo ab ipſo puncto λ oriatur ſitque $L\lambda = \varepsilon\varepsilon . f$ ob $Ll = f$ hincque ſpatium $\zeta\lambda = \zeta - \varepsilon\varepsilon . f$; ſpatiolum poſt oculum fiet $V\upsilon = \frac{uu}{ll}(\zeta - \varepsilon\varepsilon . f)$ ibique radiorum obliquitas $= \frac{l}{u}\varepsilon . \mathfrak{B}x$, circelli hinc ſuper retina formati erit radius $= \frac{u}{l} . \varepsilon(\zeta - \varepsilon\varepsilon . f)\mathfrak{B}x$. At ex altera parte a termino l naſcitur in retina circellus, cuius radius $= \frac{u}{l}(f - \zeta)\mathfrak{B}x$ qui duo radii ob rationes ante allegatas inter ſe aequales ſunt ſtatuendi, ex quo conſequimur $f - \zeta = \varepsilon\zeta - \varepsilon^3 f$ hincque $\zeta = \frac{f(1 + \varepsilon^3)}{1 + \varepsilon} = f(1 - \varepsilon + \varepsilon^2)$ hinc ergo erit $f - \zeta = \varepsilon(1 - \varepsilon)f$ ſicque ſemidiameter circellorum in retina erit $= \frac{u}{l} . \varepsilon(1 - \varepsilon)f . \mathfrak{B}x = \frac{u}{l} . \varepsilon(1 - \varepsilon)\sqrt{\mathfrak{B}x^3}$ conſequenter hoc caſu, quo $\varepsilon > \frac{1}{2}$, ſemidiameter confuſionis erit $= \frac{\varepsilon(1 - \varepsilon)}{l}\sqrt{\mathfrak{B}x^3}$ qui caſu praecedente, quo $\varepsilon < \frac{1}{2}$, erat $= \frac{\sqrt{\mathfrak{B}}}{4l} . x^3$; quamdiu ergo eſt $\varepsilon < \frac{1}{2}$ ſemper valet formula $\frac{\sqrt{\mathfrak{B}}}{4l}x^3$; quae etiamnum locum habet, ſi $\varepsilon = \frac{1}{2}$; verum ſtatim ac fit $\varepsilon > \frac{1}{2}$, tum demum confuſio diminui incipit, atque tandem prorſus euaneſcit, ſi fiat $\varepsilon = 1$. Quia autem claritas quoque diminuitur et tandem euaneſcit, hinc nullum plane lucrum in praxin redundare poteſt, ſiquis enim adhuc dubitet, vtrum loco lentis ſolidae, cuius radius ſit p, non adhiberi poſſet limbus vitreus paris ſuperficiei, cuius radius exterior ſit $= q$ et interior $= \varepsilon q$, ita, vt ſit $p^2 = q^2(1 - \varepsilon\varepsilon)$ atque confuſio iſtius limbi minor euadat, hoc dubium nunc facile erit reſoluere; a lente enim ſolida naſcetur confuſio vt $\frac{1}{4}p^3$;

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ex limbo autem vt $\varepsilon(1-\varepsilon)q^3$; vnde ob $p = q\surd(1-\varepsilon\varepsilon)$, erit confuſio ex lente ſolida nata ad confuſionem ex limbo oriundam, vti $(1+\varepsilon)\surd(1-\varepsilon\varepsilon) : 4\varepsilon$ quare cum ſit per hypotheſin $\varepsilon > \frac{1}{2}$ (quia altero caſu $\varepsilon < \frac{1}{2}$ ne dubium quidem exſiſtere poteſt) poſterius membrum 4ε manifeſto erit maius, quam 2, at quia ſimul $\varepsilon < 1$, erit $1+\varepsilon < 2$ ideoque multo magis $(1+\varepsilon)\surd(1-\varepsilon\varepsilon) < 2$, ex quo perſpicuum eſt, prius membrum ſemper eſſe multo minus poſteriore, ſiue confuſionem limbi multum excedere confuſionem lentis ſolidae.

Scholion 2.

§. 34. Cum autem pro vſu practico tuto ſumere queamus $\varepsilon < \frac{1}{2}$, quo caſu ſpeculum obiectiuum perforatum aeque magnam gignit confuſionem, ac ſi eſſet integrum, ſi in formula generali ſupra pro teleſcopiis exhibita, qua ſemidiameter confuſionis exprimitur, loco duarum priorum lentium noſtra ſpecula introducamus, aequatio hinc nata ſequenti modo ſe habebit

$$\frac{3}{k^3} = \frac{mx^3}{p^3}\left\{\begin{array}{l}\frac{1}{8} - \frac{(1+B)(1-B)^2}{8B^3P} + \frac{\mu}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right) \\ - \frac{\mu}{B^3C^3PQR}\left(\frac{\lambda'''}{\mathfrak{D}^3} + \frac{\nu}{D\mathfrak{D}}\right) + \text{etc.}\end{array}\right.$$

vbi notari conuenit, ſi forte lentes poſt ſpecula adhibitae ex vario vitro conficiantur; tum pro qualibet lente litteras μ et ν ex eo vitri genere ſumi debere, ex quo lens fuerit facta.

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Reliqua autem praecepta generalia pro conſtructione teleſcopiorum nullam mutationem ob ſpecula requirent, exceptis iis tantum formulis, quibus tam margo coloratus tollitur, quam omnis confuſio a diverſa radiorum refrangibilitate oriunda ad nihilum redigitur. Cum enim in has formulas induxerimus pro ſingulis lentibus litteras N, N', N'', N''' etc. quae litterae proportionales ſunt ſumtae formulis differentialibus $\frac{dn}{n-1}$; $\frac{dn'}{n'-1}$ etc. ſi loco duarum priorum lentium ſpecula ſubſtituantur, ob defectum refractionis iſtae binae litterae priores N et N' nihilo aequales ſunt cenſendae; quo obſeruato omnibus illis formulis generalibus pro ſpeculis perinde vti poterimus, atque in ſecundo libro eſt factum, dummodo quae circa diſtantias focales ſpeculorum et circa duo interualla priora in capite praecedente ſunt allata, probe obſeruentur.

Scholion 3.

§. 35. Teleſcopia autem Catadioptrica huius generis ſponte ad duo genera principalia reuocantur, ſiquidem ſupra vidimus, ſecundum ſpeculum vel vltra focum primi conſtitui poſſe, vel intra eum, atque priori caſu ſecundum ſpeculum fore concauum, altero vero conuexum. Deinde cum pro priori caſu hos limites pro ſecundi ſpeculi diſtantia focali q invenerimus

$$q < \varepsilon p \text{ et } q > \frac{\varepsilon(r+\varepsilon)p}{1+3\varepsilon}$$

exi-

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exiſtente primo interuallo $=(1+\varepsilon)p$ cui ſecundum debet eſſe aequale tum vero

$$b=\varepsilon p \text{ et } \frac{\beta}{b}=\frac{q}{b-q}=\mathfrak{B}$$

Quo hoc prius genus debite euoluamus, tres caſus conſtitui conueniet; primo ſcilicet ſumamus $q=\varepsilon p$;

$$\text{ſecundo } q=\frac{\varepsilon(1+\varepsilon)p}{1+2\varepsilon} \text{ et tertio } q=\frac{\varepsilon(1+\varepsilon)}{1+2\varepsilon}.p.$$

Pro altero vero genere ſecundum ſpeculum intra focum prioris collocabatur, ita, vt eſſet

$$b=-\varepsilon p \text{ et } \frac{\beta}{b}=\frac{q}{b-q}=\frac{-q}{\varepsilon p+q}=\mathfrak{B}$$

ibique cum diſtantia focalis q hoc caſu euadat negatiua poſito $q=-\mathfrak{q}$; hos ibidem dedimus limites

$$\mathfrak{q}>\varepsilon p \text{ et } \mathfrak{q}<\frac{\varepsilon(1-\varepsilon)}{1-2\varepsilon}.p$$

vnde iterum tres caſus euoluamus.

Primo ſcilicet ſumamus $q=-\varepsilon p$,

Secundo $q=\frac{-\varepsilon(1-\varepsilon)}{1-2\varepsilon}.p$,

Tertio $q=\frac{-\varepsilon(1-\varepsilon)}{1-2\varepsilon}.p$.

hoc autem caſu erit interuallum primum $=(1-\varepsilon)p$, cui etiam ſecundum aequale eſſe debet. Ceterum in priori genere erat $\frac{\mathfrak{r}}{p}=-\varepsilon$ ita, vt in primo ſtatim interuallo reperiatur imago realis; in altero vero genere erat $\frac{\mathfrak{r}}{p}=\varepsilon$, ita, vt in primo interuallo nulla occurrat imago realis praeterea vero, vti iam monuimus, ſumimus hic ſemper $\varepsilon<\frac{1}{2}$, vnde poſtremus adhuc caſus conſiderari merebitur, quo ſcilicet ſit $\varepsilon=\frac{1}{2}$,

quoniam tum ſecundum ſpeculum planum accipere licebit; quocirca ſecundum hos ſeptem caſus haec teleſcopia Catadioptrica ſumus pertractaturi.

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CAPVT III.

CAPVT III.

DE TELESCOPIIS CATADIOPTRICIS MINORE SPECVLO CONCAVO INSTRVCTIS.

Problema I.

§. 36.

Si ante ſpeculum principale P P foramine $\pi\pi$ pertuſum ad diſtantiam $AB = (1 + \varepsilon)p$ conſtituatur minus ſpeculum concauum Q B Q, cuius diſtantia focalis $q = \varepsilon p$, definire binas lentes C et D, ita, vt quaeuis obiecta diſtincte repraeſententur. Tab. III. Fig. 8.

Solutio.

Hic denotat p diſtantiam focalem maioris ſpeculi, cuius ſemidiameter $AP = x$ eiusque foraminis $A\pi = y = \varepsilon x$, ita, vt radius curuaturae, huius ſpeculi $= 2p$. Obiectorum igitur imago principalis ab hoc ſpeculo repraeſentabitur in F, vt ſit $AF = \alpha = p$, cuius ergo diſtantia a minore ſpeculo debet eſſe, vti ante eſt oſtenſum, $FB = \varepsilon p$ et ſemidiameter huius ſpeculi $BQ = y = \varepsilon x$. Cum igitur diſtantia focalis huius ſpeculi ſit $= q = \varepsilon p = FB$ radii hinc reflexi inter ſe fient paralleli, donec in lentem C incidant; pro

pro formulis ergo noſtris generalibus erit $\frac{1}{P} = -\varepsilon$ et $FB = b = \varepsilon p$ vnde vtique ob $P = -\frac{\alpha}{b}$ fit $P = -\frac{1}{\varepsilon}$. Deinde cum fiat $\beta = \frac{bq}{b-q} = \infty$, hincque $B = \frac{\beta}{b} = \infty$ iam quia interuallum ſecundum in genere eſt

$$= -\frac{AB.a}{P}\left(1 - \frac{1}{Q}\right) = -\frac{B p}{P}\left(1 - \frac{\varepsilon}{Q}\right)$$

hocque primo interuallo aequale eſt ponendum, fiet $Q = 1$. ſed ita tamen, vt ſit $B\left(1 - \frac{1}{Q}\right) = \frac{1+\varepsilon}{\varepsilon}$; per formulas autem generales hoc ſecundum interuallum

$= \beta + c = (1 + \varepsilon)p$; vnde ob $\beta = \infty$ fit

$$c = (1 + \varepsilon)p - \beta = -\infty \text{ ideoque}$$

$$C = \frac{\gamma}{c} = 0. \text{ et } \mathfrak{C} = 0.$$

Quare poſita lentis in foramine conſtitutae diſtantia focali $= r$, erit $r = \frac{B\mathfrak{C}}{PQ} p = -\varepsilon B \mathfrak{C}. p$; vnde cum ſit $B = \infty$ et $\mathfrak{C} = 0$, viciſſim colligitur $B\mathfrak{C} = BC = \frac{-r}{\varepsilon p}$, atque hinc pro quarta lente SDS habebimus diſtantiam focalem $s = -\frac{r}{R}$, et interuallum $CD = r\left(1 - \frac{1}{R}\right)$. Vt ergo poſtrema lens fiat conuexa, littera R debet eſſe negatiua ſiue in interuallum CD incidit imago realis in puncto H atque ex data multiplicatione m formulae generales praebent $PQR = m$; quoniam ob binas imagines reales repraeſentatio erit erecta. Hinc ergo fiet $R = \frac{m}{PQ} = -\varepsilon m$, ita, vt nunc ſit $s = \frac{r}{\varepsilon m}$ et interuallum $CD = r\left(1 + \frac{1}{\varepsilon m}\right) = r + s$, quandoquidem hic ſit ex natura rei $CH = r$ et $HD = s$. Contemplemur nunc campum apparentem et ſecun-

dum

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dum formulas noſtras generales ſecundo ſpeculo tribuamus litteram q, lenti C litteram r et lenti D litteram ſ et ſemidiameter campi apparentis erit

$$\Phi = \frac{q+r+ſ}{m-1}\xi, \text{ ſumto } \xi \text{ pro fractione } \frac{1}{4},$$

litterae autem q, r et ſ ad ſummum vnitati aequales fieri poſſunt. Poſuimus vero breuitatis gratia $\frac{q+r+ſ}{m-1} = M$, vt ſit $\Phi = M\xi$ atque formulae noſtrae generales has ſuppeditant aequationes:

$$\mathfrak{B}q = (P-1)M$$
$$\mathfrak{C}r = (PQ-1)M - q$$

quae ob valores iam inuentos $\mathfrak{B} = 1.$ et $\mathfrak{C} = 0.$ praebent ambae $q = (P-1)M = -(1+\frac{1}{k})M.$ Hinc autem inuenimus diſtantiam oculi poſt lentem D, ſcilicet $DO = O = \frac{ſs}{Mm}$ quae diſtantia cum ſit poſitiua, quandoquidem nihil impedit quominus ipſi ſ valor poſitiuus detur isque vnitati aequalis; marginem coloratum tollemus, ſi ob $N' = 0$ et $N'' = N'''$ (quandoquidem noſtrae duae lentes ex eodem vitro parantur) huic aequationi ſatisfaciamus:

$$0 = \frac{r}{PQ} + \frac{ſ}{PQR}$$

quae ergo reducitur ad hanc:

$$0 = r - \frac{ſ}{km} \text{ vnde colligitur } r = \frac{ſ}{km}$$

quare cum ſit $q = -(1+\frac{1}{k})M$ erit

$$q + r + ſ = -(1+\frac{1}{k})M + \frac{ſ}{km} + ſ = M(m-1);$$

 vnde

vnde ſequitur $M = \frac{\vartheta}{m}$; ita, vt iam ſit ſemidiameter campi apparentis $\Phi = \frac{\vartheta}{m} . \xi$. Num autem hic pro ϑ vnitas ſcribi queat, intelligemus ex lente C, cuius apertura nobis eſt praeſcripta et cuius ſemidiameter $= y = \varepsilon x$. Iam per formulas noſtras hic ſemidiameter eſſe debet $r r \xi + \frac{x}{PQ} = \frac{\vartheta r.}{\varepsilon m} \xi + \varepsilon x$, vbi ſufficit, maiori membro vti, ex quo ſequitur, eſſe debere $\frac{\vartheta r.}{\varepsilon m} \xi < \varepsilon x$, vnde ſi ſtatuamus $\vartheta = 1$ et $\xi = \frac{1}{4}$, neceſſe eſt, vt ſit $r < 4 \varepsilon^2 . m . x$; ſi igitur velimus ſumere $r > 4 \varepsilon^2 m x$; tum ϑ vnitate minus accipi debet, ex quo campus apparens in eadem ratione diminuetur. Hic autem inprimis quoque ad vltimam lentem attendi oportet, pro qua eſt $s = \frac{r}{\varepsilon m}$, ita, vt eſſe debeat $s < 4 \varepsilon x$ ſiue $s < 4 y$. vnde patet, foramen non nimis exiguum ſtatui poſſe. Totam autem confuſionem ex diuerſa radiorum refrangibilitate oriundam tolleremus ope huius aequationis:

$$0 = \frac{N''}{P^2 Q^2} . \frac{1}{r} + \frac{N'''}{P^2 Q^2 R^2} . \frac{1}{s}$$

quae abit in hanc

$$0 = N'' + \frac{N'''}{\varepsilon m},$$

quod cum nullo modo fieri poſſit, etiamſi diuerſo vitro vti vellemus, hanc confuſionem, quae ſemper eſt valde exigua, tolerari oportet. His obſeruatis cardo rei verſabitur in ſemidiametro confuſionis, quem inſenſibilem reddi conuenit ope huius aequationis:

$$\frac{1}{k^3} = \frac{m x^3}{p^3} \left(\frac{1}{4} + \frac{\varepsilon}{4} + \mu . \frac{\varepsilon^4 . p^3}{r^4} \lambda'' + \frac{\mu \varepsilon^3 p^3}{r^3 m} \lambda''' \right)$$

quae

quae aequatio abit in hanc formam:

$$p \sqrt[3]{\left(\frac{1}{k^3 m x^3} - \frac{\mu . \varepsilon^4 \lambda''}{r^3} - \frac{\mu \varepsilon^3 . \lambda'''}{m r^3}\right)} = \frac{1}{8} \sqrt[3]{(1 + \varepsilon)}$$

ex qua aequatione reperitur p: verum quantitatem x tantam aſſumi conuenit, vt inde ſufficiens claritatis gradus obtineatur. In doctrina de teleſcopiis autem pro ſufficiente claritatis gradu ſumſimus $x = \frac{m}{35}$ dig. quod autem ibi erat x ſeu $\sqrt{x^2}$, hic nobis eſt $\sqrt{(1 - \varepsilon^2) x^2}$, ita, vt hic habeamus

$$x \sqrt{(1 - \varepsilon^2)} = \frac{m}{35} \text{ dig.}$$

ſiquidem eodem claritatis gradu frui velimus, vnde foret $x = \frac{m}{50 . \sqrt{(1 - \varepsilon^2)}}$. dig. ideoque $x > \frac{m}{35}$ dig. Quia vero ſpecula non tantum radiorum reflectunt, quantum lentes transmittunt, ne hoc quidem modo tantum claritatis gradum adipiſcemur, quam in teleſcopis vulgaribus. Sin autem minori claritatis gradu contenti eſſe velimus atque ſtatuamus $x = \frac{m}{35}$ dig. ſumamusque vt ibi $k = 50$. aequatio noſtra erit

$$p . \sqrt[3]{\left(\frac{1}{m^4} - \frac{\mu . \varepsilon^4 \lambda''}{r^3} - \frac{\mu . \varepsilon^3 \lambda'''}{m r^3}\right)} = \frac{1}{8} \sqrt[3]{(1 + \varepsilon)}$$

vbi manifeſto debet eſſe $\frac{\varepsilon^4}{r^3}$ multo minus, quam prius membrum $\frac{1}{m^4}$ ſiue $r^3 > \varepsilon^4 m^4$ ideoque r multo maius quam $\varepsilon m \sqrt[3]{\varepsilon m}$ ſupra vero vidimus, eſſe debere $r < 4 \varepsilon^2 m x$; quod vt fieri poſſit debet eſſe $4 \varepsilon^2 m x$ multo maius, quam

$$\varepsilon m \sqrt[3]{\varepsilon m}, \text{ ſiue } 4 \varepsilon m > 50 . \sqrt[3]{\varepsilon m}$$

 ideo-

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ideoque $\varepsilon > \frac{4s}{m}$, quod in magnis multiplicationibus effici poſſet.

At ſi haec conditio non obſeruetur effectus in eo conſiſtet, vt non amplius ſit $\vartheta = 1$. hincque campus apparens multo minor exiſtat, quam

$$\Phi = \frac{\xi}{m} \text{ ſiue } \Phi = \frac{859}{m} \text{ min.}$$

Coroll. 1.

§. 37. Cum in teleſcopiis id ſemper inprimis ſit efficiendum, vt eorum longitudo hincque praecipue diſtantia focalis p quam minima reddatur, in aequatione vltima confuſio a lentibus oriunda tantopere diminui debet, vt prae confuſione ſpeculorum quaſi euaneſcat, quare cum in iſta formula ex primo ſpeculo naſcatur portio $\frac{1}{8}$; ex ſecundo vero $\frac{\varepsilon}{8}$ neceſſe eſt, vt portiones ſequentes ex lentibus oriundae multo fiant minores, ex quo littera r multo maior eſſe debet quam $2\varepsilon p$, ideoque r vix minus capi poterit, quam p.

Coroll. 2.

§. 38. Quodſi igitur ſtatuamus $r = p$, cum ε, vti vidimus, minus eſſe ſoleat, quam $\frac{1}{3}$, pro confuſione definienda tuto vti licebit hac aequatione

$$\frac{1}{k^3} = \frac{m x^3 (1 + \varepsilon)}{8 p^3}; \text{ vnde colligimus}$$

$$p = \frac{kx}{2} \sqrt[3]{m(1 + \varepsilon)};$$

vnde

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vnde ſi pro dato claritatis et diſtinctionis gradu capiatur

$$k\,x = m \text{ dig. erit } p = \tfrac{1}{8}\,m.\sqrt[3]{m(1+\varepsilon)}$$

quae quantitas circiter duplo minor eſt, quam in teleſcopiis dioptricis communibus, ita, vt hoc modo tota longitudo fere ad partem quartam reducatur.

Coroll. 3.

§. 39. Sumto autem $r = p$ pro campo definiendo littera δ maior accipi nequit, quam vt fiat

$$\frac{\delta p}{4\varepsilon m} = \varepsilon\,x;$$

hinc ergo pro exemplo ſpeciali, quo

$$\varepsilon = \tfrac{1}{4} \text{ et } m = 100, \text{ colligetur}$$

$$\delta = \frac{\sqrt[3]{10000}}{100} = \tfrac{1}{5} \text{ circiter},$$

ex quo patet, hoc caſu fore campum quinquies minorem, quam ſi capere liceret $\delta = 1$ ſicque in genere patet, hoc modo nimis exiguum campum obtineri.

Coroll. 4.

§. 40. Sumto autem $r = p$ pro conſtructione huiusmodi teleſcopii diſtantiae focales ſequenti modo ſe habebunt

$$p = \tfrac{1}{8}\,m\sqrt[3]{m(1+\varepsilon)};\; q = \varepsilon p;\; r = p \text{ et } s = \frac{p}{\varepsilon m};$$

tum vero interualla lentium ſeu ſpeculorum

$$AB = (1 + \epsilon)p. = BC.$$

$$CD = r + s. = p[1 + \frac{1}{\epsilon m}]$$

et diſtantia oculi $O = s = \frac{p}{\epsilon m}$

vnde patet tubum arcae, in qua ſpecula continentur, adiungendum admodum fore longum.

Scholion.

§. 41. Praeter incommoda vero, quae hic iam commemorauimus, huiusmodi teleſcopia maximo vitio laborarent propterea quod radii in lentem C incidentes inter ſe ſunt paralleli; tum enim radii peregrini qui ab obiectis vicinis directe in eandem lentem incidunt, quia etiam ſunt paralleli inter ſe, in tranſitu per lentes ſimili modo refringentur ac radii proprii, ideoque cum iis ſimul ad oculum deferentur et quoniam hi radii peregrini multo ſunt fortiores, quam proprii, ſiquidem hi duplicem refractionem iam ſunt paſſi, in oculo impreſſionem iſtorum penitus extinguent. Interim tamen quia radii peregrini ad axem magis ſunt obliqui, atque etiam in refractione maiorem obliquitatem conſeruant, ab egreſſu in oculum excludi poſſent ope exigui foraminuli, cui oculus adplicatur; hoc autem modo non ſolum claritas nimium detrimentum pateretur, ſed etiam campus inſuper reſtringeretur, quam ob cauſſam in huiusmodi teleſcopiis

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piis inprimis cauendum eſt, ne radii peregrini, qui circa minus ſpeculum praeterlabentes ab introitu in C arceri nullo modo poſſunt, cum radiis propriis ſimilem refractionem patiantur. Quod praeſtari poterit, ſi modo radii proprii in lentem C incidentes fuerint vel diuergentes vel conuergentes, vt poſt refractionem in alio foco congregentur, ac peregrini, tum enim diaphragma debito foramine in iſto foco conſtitutum facile radios peregrinos ab vlteriori progreſſu ad oculum excludet. Perſpicuum autem eſt, quo hoc remedium certius ſuccedat, illam ſiue convergentiam ſiue divergentiam ſatis notabilem eſſe debere, ſiue efficiendum eſt, vt per refractionem huius lentis C imago a radiis peregrinis formata multum diſtet ab imagine a radiis propriis formata, id quod in ſequentibus caſibus vſu veniet.

Problema 2.

§. 42. Si ante ſpeculum principale PP, foramine $\pi\,\pi$ pertuſum, ad diſtantiam $AB = [1+\varepsilon]p$ conſtituatur minus ſpeculum concauum QBQ, cuius diſtantia focalis $q = \frac{\varepsilon(1+\varepsilon)}{1+2\varepsilon}.p$, definire binas lentes C et D, ita, vt quaeuis obiecta diſtincte repraeſententur. Tab. III. Fig. 8.

Solutio.

Hic ergo, vt ante, eſt diſtantia $AF = a = p$ et $FB = b = \varepsilon p$, hincque $\frac{1}{P} = -\varepsilon$ ob $AB = [1+\varepsilon]p$.

Quia

Quia vero hic eſt

$$q = \frac{\varepsilon(1+\varepsilon)}{1+2\varepsilon} \cdot p, \text{ fiet } \frac{\beta}{b} = \frac{q}{b-q} = \frac{1+\varepsilon}{\varepsilon} = \mathrm{B};$$

ita, vt iam ſit $\beta = [1+\varepsilon]p$, quae diſtantia ipſi ſecundo interuallo B C eſt aequalis ſicque ſecunda imago in ipſam lentem C incidet, vnde fiet $c = 0$; vnde cum poſuerimus

$$\frac{\beta}{c} = -\mathrm{Q}, \text{ fiet hic } \mathrm{Q} = -\infty$$

tum vero pro tertia imagine erit

$$\gamma = \frac{cr}{c-r} = 0, \text{ ita, vt ſit } \mathrm{C} = -1 \text{ et } \mathfrak{C} = \infty.$$

Quare cum ſit

$$r = \frac{\mathrm{B}\mathfrak{C}}{\mathrm{PQ}} \cdot p = -\frac{(1+\varepsilon)\mathfrak{C}}{\mathrm{Q}} \cdot p$$

viciſſim adparet, fore $\frac{\mathfrak{C}}{\mathrm{Q}} = -\frac{\varepsilon r}{(1+\varepsilon)p}$.

His inuentis diſtantiae focales erunt

$$\mathrm{P} = p;\; q = \frac{\varepsilon(1+\varepsilon)}{1+2\varepsilon} \cdot p;\; r = r; \text{ et}$$

$$s = \frac{\mathrm{B}}{\mathrm{PQR}} \cdot p = \frac{1+\varepsilon}{\varepsilon m} \cdot p \text{ ob } \mathrm{PQR} = m.$$

Interualla vero ita erunt expreſſa

$$\mathrm{AB} = [1+\varepsilon]p = \mathrm{BC}.$$

$$\mathrm{CD} = \frac{1+\varepsilon}{\varepsilon m} \cdot p = s,$$

vti rei natura poſtulat, quandoquidem vltima imago in ipſa lente C manet conſtituta. Ceterum patet, hic duas occurrere imagines reales; alteram in F, alteram in C, ideoque imagines ſitu erecto repraeſentari et recte nos aſſumſiſſe $\mathrm{PQR} = m$. Pro

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Pro campo diiudicando erit

$$M = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s}}{m - 1} \text{ vnde fit } \Phi = M \xi;$$

tum vero eſſe debet

$$\mathfrak{B}\mathfrak{q} = (P - 1) M. \text{ hinc}$$

$$\mathfrak{q} = -\frac{(1 + 2\varepsilon)}{\varepsilon} M \text{ et}$$

$$\mathfrak{C}\mathfrak{r} = (PQ - 1) M - \mathfrak{q} \text{ hinc}$$

$$\mathfrak{r} = \left(\frac{PQ}{\mathfrak{C}} - \frac{1}{\mathfrak{C}}\right) M - \frac{\mathfrak{q}}{\mathfrak{C}}.$$

Quia vero

$$\mathfrak{C} = \infty \text{ et } \frac{\mathfrak{C}}{Q} = \frac{-r}{(1 + \varepsilon) p} \text{ erit}$$

$$\mathfrak{r} = \frac{PQM}{\mathfrak{C}} = \frac{(1 + \varepsilon) p}{\varepsilon r}. M$$

hinc ergo fit

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} = \left(\frac{(1 + \varepsilon) p}{\varepsilon r} - \frac{(1 + 2\varepsilon)}{\varepsilon}\right) M + \mathfrak{s} = (m - 1) M;$$

vnde reperitur

$$M = \frac{\varepsilon r . \mathfrak{s}}{m \varepsilon r - (1 + \varepsilon)(p - r)}$$

circa $\mathfrak{s}$ autem nihil adhuc definitur, ſed cum lentis C ſemidiameter aperturae reuera ſit $= \varepsilon x$, per formulas autem noſtras eſſe debeat

$$= \frac{1}{4} \mathfrak{r} r = \frac{(1 + \varepsilon) p . M}{4 \varepsilon}$$

ſiue ipſo campo introducto hic ſemidiameter erit $= \frac{(1 + \varepsilon) p \Phi}{\varepsilon}$, qui cum excedere nequeat εx, hoc non eſt verendum, niſi eſſet $\Phi = \frac{\varepsilon^2 x}{(1 + \varepsilon) p}$ vel maius. Iam

vt margo coloratus euaneſcat, debet eſſe

$$0 = \frac{\mathfrak{r}}{PQ} + \frac{\vartheta}{PQR}; \text{ ideoque eſſe deberet } \frac{\vartheta}{m} = 0,$$

vnde patet hoc modo marginem coloratum euitari non poſſe; ſed tamen eum fore minimum et vix ſenſibilem ob denominatores P Q et P Q R maximos.

Sumta porro littera ϑ, vti circumſtantiae permittunt, pro loco oculi habebimus $O = \frac{\vartheta s}{Mm}$. Denique conditio confuſionis tollendae praebet hanc aequationem

$$\frac{1}{k^3} = \frac{mx^3}{p^3}\left(\frac{1}{\nu} + \frac{(1+2\varepsilon)\varepsilon}{8(1+\varepsilon)^2}\right)$$

ſequentibus partibus ſponte euaneſcentibus, ita, vt ſtatui poſſit

$$p = \tfrac{1}{4} m. \sqrt[3]{m\left(\frac{1+\varepsilon(1+2\varepsilon)}{(1+\varepsilon)^3}\right)} \text{ dig.}$$

Coroll. I.

§. 43. Cum lentis in C poſitae ſemidiameter aperturae eſſe debeat $= \frac{1}{4}\mathfrak{r}\, r$, is vero reuera ſit $= \varepsilon x$; hinc colligitur $\mathfrak{r} = \frac{4\varepsilon x}{r}$. Verum ante inuenimus $\mathfrak{r} = \frac{(1+\varepsilon)p}{\varepsilon r}.M$ his ergo duobus valoribus aequatis prodit $4\varepsilon^2 x = (1+\varepsilon)p.M$; vnde ſi eſſet $\mathfrak{r} = 1$ foret $r = 4\varepsilon x$; tum vero $r = \frac{M(1+\varepsilon)p}{\varepsilon}$; quia vero eſt

$$M = \frac{\varepsilon r.\vartheta}{m\varepsilon r - (1+\varepsilon)(p-r)}$$

habebimus nunc ſubſtituto pro r illo valore

$$M = \frac{4\varepsilon^2\vartheta x}{4m\varepsilon^2 x - (1+\varepsilon)(p-4\varepsilon x)}.$$

qui

qui valor in illa aeqnatione ſubſtitutus dabit

$$\vartheta = \frac{4m\varepsilon^2 x - (1+\varepsilon)(p - 4\varepsilon x)}{(1+\varepsilon)p}.$$

Coroll. 2.

§. 44. Quia autem ϑ vnitate maius eſſe nequit, hoc valore vnitati aequali poſito prodibit

$$4m\varepsilon^2 x = 2(1+\varepsilon)p - 4\varepsilon(1+\varepsilon)x$$

hincque

$$m = \frac{(1+\varepsilon)p - 2\varepsilon(1+\varepsilon)x}{2\varepsilon^2 x}$$

quae aequatio ſubſiſtere nequit, niſi multiplicatio m aliquot millia excedat, quod in praxi nunquam locum habere poteſt.

Scholion.

§. 45. Huiusmodi vero teleſcopia duplici laborant defectu; primo enim quia lens C in ipſo imaginis loco conſtituitur, niſi lens ex puriſſimo vitro ſit confecta repraeſentatio vehementer erit inquinata, vti iam ſaepius obſeruauimus; deinde etiam haud exiguum vitium in eo conſiſtit, quod marginem coloratum non licuit ad nihilum reducere, quam ob cauſſam haec teleſcopia ſuperfluum foret vberius proſequi, ſed potius eiusmodi caſum euoluamus, in quo ſecunda imago poſt lentem C cadat ſimulque margo coloratus feliciter tolli queat, quare cum pro hoc praeſtando habeatur aequatio $0 = \frac{\varepsilon}{PQ} + \frac{\vartheta}{PQR}$, neceſſe

eſt,

eſt, vt fieri queat $\mathfrak{r} + \frac{\mathfrak{s}}{R} = 0$, quod commodiſſime fieri poterit, ſi fuerit $R = -1$, quia enim tum erit $\mathfrak{r} = \mathfrak{s}$, maximum campum adipiſci poterimus, ſi ſumere liceat $\mathfrak{r} = \mathfrak{s} = 1$ tum enim fiet $M = \frac{q+1}{m-1}$ et quamuis q ſit fractio negatiua, tamen campus hinc orietur ſatis magnus; vt vero fiat R numerus negatiuus, ſecunda imago in interuallum C D cadere debet, ita, vt Q maneat quantitas poſitiua et quia multiplicatio dat $m = PQR$ ob $P = -\frac{1}{\varepsilon}$, ſi ſumamus $R = -1$ neceſſe eſt fiat $Q = \varepsilon m$, vnde cum ſit

$$Q = -\frac{\beta}{c}, \text{ erit } c = -\frac{\beta}{\varepsilon m}$$

at vero ſecundum interuallum $BC = \beta + c$ quod cum primo $(1+\varepsilon)p$ aequale eſſe debeat, elicimus

$$\beta = (1+\varepsilon)p - c = \frac{\varepsilon(1+\varepsilon)mp}{\varepsilon m - 1}.$$

Cum vero ſit

$$\beta = \frac{bq}{b-q} \text{ et } b = \varepsilon p \text{ ſiue etiam } \frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$$

hinc erit

$$\frac{1}{q} = \frac{1+2\varepsilon}{\varepsilon(1+\varepsilon)p} - \frac{1}{\varepsilon(1+\varepsilon)mp} \text{ tum vero erit}$$

$$B = \frac{\beta}{b} = \frac{(1+\varepsilon)m}{\varepsilon m - 1} \text{ hincque}$$

$$\mathfrak{B} = \frac{(1+\varepsilon)m}{(1+2\varepsilon)m - 1} \text{ vnde fit } q = \mathfrak{B}b.$$

Porro vero cum ſit $r = \mathfrak{C}c$, erit

$$\mathfrak{C} = \frac{r}{c} = \frac{-(\varepsilon m - 1)r}{(1+\varepsilon)p} \text{ hincque}$$

$$C = \frac{-(\varepsilon m - 1)r}{(1+\varepsilon)p + (\varepsilon m - 1)r}.$$

Pro

Pro interuallo autem C D, quod eſt $\gamma + d = \gamma + s$, quia eſt

$$\gamma = \frac{cr}{c-r} = \frac{(1+\varepsilon)pr}{(1+\varepsilon)p + (\varepsilon m - 1)r},$$

quia vero etiam eſſe debet $R = -\frac{\gamma}{d} = -1$ hinc erit $s = \gamma$ ſicque interuallum $CD = 2\gamma = 2.s.$ Quia autem porro eſt

$$M = \frac{q+2}{m-1}, \text{ ſumto ſcilicet } \mathfrak{r} = \mathfrak{s} = 1, \text{ erit}$$

$$q = \frac{-((1+2\varepsilon)m-1)}{\varepsilon m}. M \text{ hincque}$$

$$q + 2 = \frac{-((1+2\varepsilon)m-1)M + 2\varepsilon m}{\varepsilon m} = M(m-1);$$

vnde ſequitur

$$M = \frac{2\varepsilon m}{\varepsilon m^2 + (1+\varepsilon)m - 1};$$

ex quo viciſſim concludimus

$$q = \frac{-2((1+2\varepsilon)m-1)}{\varepsilon m^2 + (1+\varepsilon)m - 1}.$$

Praeterea vero adhuc habetur haec aequatio

$$\mathfrak{C} = (PQ - 1)M - q$$

quae abit in hanc

$$\mathfrak{C} = -(m+1)M - q$$

ſeu ſubſtitutis valoribus

$$\frac{-(\varepsilon m - 1)r}{(1+\varepsilon)p} = \frac{-2\varepsilon m(m+1) + 2(1+2\varepsilon)m - 2}{\varepsilon m^2 + (1+\varepsilon)m - 1}$$
$$= \frac{-2\varepsilon m^2 + 2(1+\varepsilon)m - 2}{\varepsilon m^2 + (1+\varepsilon)m - 1}$$

vnde concludimus fore

$$r = \frac{2(\varepsilon m^2 - (1+\varepsilon)m + 1)(1+\varepsilon)p}{(\varepsilon m - 1)(\varepsilon m^2 + (1+\varepsilon)m - 1)}$$

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$$r = \frac{2(m-1)(1+\varepsilon)}{\varepsilon m^2 + (1+\varepsilon)m - 1} \cdot p$$

hinc cum ſit $\frac{1}{\gamma} = \frac{1}{r} - \frac{1}{c}$, reperitur

$$\gamma = \frac{2(m-1)(1+\varepsilon)}{3\varepsilon m^2 - (1+\varepsilon)m + 1} p = s.$$

vnde porro concluditur diſtantia oculi

$$O = \frac{\theta s}{Mm} = \frac{\varepsilon m^2 + (1+\varepsilon)m - 1}{2\varepsilon m^2} \cdot s$$

$$= \frac{1}{2} s \left(1 + \frac{1+\varepsilon}{\varepsilon m} - \frac{1}{\varepsilon m^2}\right) = \frac{1}{2} s \text{ proxime.}$$

quod autem ad campum apparentem attinet, quoniam ſumſimus $\mathfrak{r} = \theta = 1$, diſpiciendum eſt, num etiam ponere liceat $\xi = \frac{1}{4}$. Hoc autem patebit ex lente C, cuius ſemidiameter aperturae $= \xi r$ excedere nequit εx, poſito igitur $\xi r = \varepsilon x$ colligitur

$$\xi = \frac{\varepsilon x(\varepsilon m^2 + (1+\varepsilon)m - 1)}{2(m-1)(1+\varepsilon)p}$$

qui valor ſi fuerit minor quam $\frac{1}{4}$, eo erit vtendum, ita, vt tum ſit $\Phi = M\xi$; ſin autem ille valor prodeat maior, quam $\frac{1}{4}$ nihilominus ſumi debet $\xi = \frac{1}{4}$. Si tanquam exemplum ſumatur

$$\varepsilon = \tfrac{1}{4},\ m = 100,\ x = \tfrac{5}{2} \text{ dig. et } p = 25. \text{ dig.};$$

reuera prodit $\xi = \frac{1}{4}$, ita, vt haec poſitio $\xi = \frac{1}{4}$ parum a praxi diſcrepare videatur; vnde operae pretium erit has determinationes coniunctim ob oculos ponere.

Exemplum Teleſcopii Catadioptrici.

§ 46. Ex modo allatis prima elementa huius Teleſcopii ita ſe habebunt:

$a =$

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$$a = \infty;\ b = \varepsilon p;\ c = \frac{-(1+\varepsilon)}{\varepsilon m - 1} p;\ d = \gamma$$

$$\alpha = p;\ \beta = \frac{\varepsilon(1+\varepsilon) m p}{\varepsilon m - 1};\ \gamma = \frac{2(m-1)(1+\varepsilon)}{3\varepsilon m^2 - (1+\varepsilon) m + 1} \cdot p;\ \delta = \infty.$$

Ex quibus deducuntur ſequentes valores.

$$\mathrm{B} = \frac{(1+\varepsilon) m}{\varepsilon m - 1};\ \mathrm{C} = \frac{-2(m-1)(\varepsilon m - 1)}{3\varepsilon m^2 - (1+\varepsilon) m + 1};$$

$$\mathfrak{B} = \frac{(1+\varepsilon) m}{(1+2\varepsilon) m - 1};\ \mathfrak{C} = \frac{-2(m-1)(\varepsilon m - 1)}{\varepsilon m^2 + (1+\varepsilon) m - 1};$$

$$\mathrm{P} = -\frac{\alpha}{b} = -\frac{1}{\varepsilon};\ \mathrm{Q} = -\frac{\beta}{c} = \varepsilon m;\ \mathrm{R} = -\frac{\gamma}{\delta} = -1.$$

Ex his vero colliguntur diſtantiae focales

$$p = p;\ q = \mathfrak{B} b = \frac{\varepsilon(1+\varepsilon) m}{(1+2\varepsilon) m - 1} \cdot p;$$

$$r = \mathfrak{C} c. = \frac{2(m-1)(1+\varepsilon)}{\varepsilon m^2 + (1+\varepsilon) m - 1} \cdot p \text{ et } s = d = \gamma.$$

et pro earum aperturis

$$\mathfrak{q} = \frac{-2((1+2\varepsilon) m - 1)}{\varepsilon m^2 + (1+\varepsilon) m - 1};\ \mathfrak{r} = 1;\ \mathfrak{s} = 1.$$

hincque

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} = \frac{2\varepsilon m (m-1)}{\varepsilon m^2 + (1+\varepsilon) m - 1} \text{ ideoque}$$

$$\mathrm{M} = \frac{2\varepsilon m}{\varepsilon m^2 + (1+\varepsilon) m - 1}$$

ex quo elicitur ſemidiameter campi apparentis

$\Phi = \mathrm{M}\xi$; ac ſi liceat ſumere $\xi = \frac{1}{4}$; fiet

$$\Phi = \frac{1718, \varepsilon m}{\varepsilon m^2 + (1+\varepsilon) m - 1}. \text{ minut.}$$

at pro loco oculi inuenimus

$$O = \frac{1}{3} s \left(1 + \frac{1+\varepsilon}{6m} - \frac{1}{\varepsilon m^2}\right)$$

Super-

Supereſt igitur, vt ex conditione confuſionis definiatur diſtantia focalis p, quae reperitur

$$p = k\,x.\sqrt[3]{m.\left(\tfrac{1}{8} + \tfrac{\varepsilon.(m+1)^2((1+2\varepsilon)m-1)}{8.(1+\varepsilon)^3 m^3}\right.} $$
$$+ \tfrac{\mu.(\varepsilon m^2+(1+\varepsilon)m-1)^3}{8m^4(m-1)^3(1+\varepsilon)^3}.(\lambda''+\nu\mathfrak{C}.(1-\mathfrak{C}))$$
$$\left. + \tfrac{\mu.(3\varepsilon m^2-(1+\varepsilon)m+1)^3.\lambda'''}{8m^4(m-1)^3(1+\varepsilon)^3}\right)$$

vbi ſi tantam claritatem deſideremus, qualem ſupra teleſcopiis tribuimus ſumi debet $x = \frac{m}{50}$ dig. et pro gradu diſtinctionis $k = 50$, vt ſit $k\,x = m$.

Sin autem minori claritatis gradu contenti eſſe velimus, fortaſſe ſufficiet ponere $x = \frac{m}{100}$ dig. vel adeo $x = \frac{m}{200}$ dig.

Conſtructio huiusmodi Teleſcopii pro multiplicatione $m = 100$. ſumto $\varepsilon = \frac{1}{4}$.

§. 47. Pro maiori ergo ſpeculo, cuius ſemidiameter ſit $= x$, foraminis ſemidiameter erit $= \frac{1}{4}x$. eius vero diſtantia focalis in genere ponatur $= p$; ex qua ſequentes diſtantiae focales ita definientur

$q = \frac{125}{596}p = 0,2097.\,p;$

$r = \frac{5\cdot 99}{5248}p = 0,0943.\,p.$

$s = \frac{5\cdot 99}{14752}.\,p = 0,03355.\,p.$

Interualla autem ſequenti modo definientur

1°. $AB = \frac{5}{4}p = 1,25\,p;$

2°. $BC = \frac{5}{4}p = 1,25.\,p;$

3°. C

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3°. $CD = 2.\ s = 0,0671.\ p$;

4°. $O = 0,5248.\ s$.

Praeterea vti ſpeculi maioris ſemidiameter aperturae eſt $= x$, ita minoris erit $= \frac{1}{4} x$. cui etiam aequatur apertura lentis C; lentis vero ocularis D ſemidiameter aperturae poterit ſumi $= \frac{1}{4}.\ s$. vnde campi apparentis ſemidiameter erit circiter $\Phi = 16,368$. minut. qui campus locum habet, niſi ſit $\frac{1}{4} x < \frac{1}{4} r$ ſeu $x < r$. hoc enim ſi euenerit, vt ſit $x < r$, tum campus in eadem ratione diminuetur, atque in eadem ratione aperturam lentis D diminui conueniet. At vero pro definienda diſtantia focali p habetur iſta aequatio

$$p = \frac{1}{2} k x \sqrt[5]{\left\{ \begin{matrix} 100 + 19,45 + 0,0095\ \mu.\ (\lambda'' - 5\nu) \\ + 0,211.\ \mu.\ \lambda''' \end{matrix} \right)}$$

vbi partes ex binis lentibus oriundae vix ad dimidium accedant, tota haec quantitas radicalis certe non ad 5 exſurget, ita, vt tuto ſumi poſſit $p = \frac{5}{2}.\ k x$; ſupra autem notauimus eſſe circiter $k = 50$.

Scholion.

§. 48. Quodſi hic ſtatuamus $k = 50$ et $x = 2$ dig. diſtantia focalis ſpeculi obiectiui ex hac formula prodit $p = 250$. dig. ideoque maius viginti pedibus, quod merito maxime mirum videbitur, cum talia teleſcopia circumferantur, in quibus p non ſuperat 24. dig. atque x adeo duobus digitis maior reperitur, et quae

nihilominus centies multiplicant; cuius ergo phaenomeni cauſſam ſcrutari oportet; primo autem manifeſtum eſt, eam non in hoc eſſe ſitam, quod numerum k nimis magnum aſſumſimus; etſi enim pro microſcopiis contenti eſſe ſoleamus valore $k = 20$, tamen fateri debemus, confuſionem tum ſatis eſſe ſenſibilem, qualem tamen in his teleſcopiis non deprehendimus, et quamuis praeterea ſumeremus $k = 20$, tamen adhuc prodiret $p = 100$. dig. Euidens ergo eſt, cauſſam neceſſario in eo ſitam eſſe debere, quod poſt ſignum radicale cubicum binae priores partes ad ſpecula relatae non ſolum multo ſint minores, quam hic aſſumſimus ſed adeo nihilo aequales poni debeant. Interim tamen certum eſt, ſi haec ſpecula haberent figuram ſphaericam, vti in calculo noſtro aſſumſimus, partes inde in confuſionem influentes minores non fore, quam hic ſunt definitae; ex quo tuto concludere poſſumus, in his inſtrumentis ſpecula non ad figuram ſphaericam eſſe elaborata ſed iis ab artifice figuram parabolicam eſſe inductam, in quo Cel. Schort gloriatur, ſe modum inueniſſe ſpecula ad figuram parabolicam elaborandi, cui inuento ſine dubio exiguus valor litterae p tribui debet; quodſi enim poſt ſignum radicale binas priores partes omittamus; totus valor huius formulae radicalis ſumto $\lambda''' = 1$. ob $\mu = \frac{9}{13}$ circiter reducetur infra $\frac{2}{7}$; ſumto autem hoc valore ſequitur fore $p = 30$. dig. prorſus fere, vti experientia teſtatur; facile enim licet k aſſumere minus, quam

50; tum vero etiam aliae conſtructiones proferri poſſunt, in quibus haec duo membra poſteriora adhuc minores ſortirentur coefficientes. Quodſi ergo ambo noſtra ſpecula figuram habuerint parabolicam ſumereque liceat $p = 30$ dig., exiſtente $x = 2$ dig. erit $r = 2,829$ dig. eiusque aperturae ſemidiameter, quem ſcilicet foramen ſuppeditat $= \epsilon x = \frac{1}{2}$ dig. vnde vtique ſumi non licebit $\xi = \frac{1}{4}$, ſed tantum $\xi = \frac{3}{17}$ et campus ſupra inuentus diminui debet in ratione $\frac{1}{4} : \frac{3}{17}$ ſiue $17 : 12$ ſiue ſuo triente propemodum, ita, vt adhuc ſit eius ſemidiameter $\Phi = 11$. minut. Quodſi autem diſtantia focalis p maior aſſumi debeat, tum pro ξ adhuc minor valor reperietur.

Scholion 2.

§. 49. Teleſcopia autem vulgaria huius generis non mediocriter diſcrepant a menſuris ſupra deſcriptis; vnde operae pretium erit, menſuras talis teleſcopii, quod pro excellenti habetur, accuratius examinare. Erat autem ſpeculi maioris diſtantia focalis duorum pedum ſeu $p = 24$. dig. ſemidiameter eius $x = 2\frac{1}{2}$ dig. foraminis vero ſemidiameter $y = \frac{1}{2}$ dig. vnde ſequitur fractio $\epsilon = \frac{1}{5}$. Verum minus ſpeculum a maiore diſtabat interuallo $AB = 27\frac{1}{3}$ dig. vnde cum ſit $AF = p = 24$ dig. ſequitur diſtantia $FB = b = 3\frac{1}{3}$ dig. Quare cum poſuerimus $b = \epsilon p$; hinc non amplius fiet $\epsilon = \frac{1}{5}$, ſed tantum $\epsilon = \frac{5}{36}$; ita, vt in praxi recepta minus ſpeculum propius collocetur, Tab. II. Fig. 8.

 quam

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quam ratio foraminis poſtulat. Verum rationes non deſunt, a regula ſupra ſtabilita recedendi. Supra enim hoc ſpeculum minus, quod etiam in praxi foramini aequabatur, ita conſtituimus, vt omnes radios axi parallelos, qui a maiore ſpeculo reflectuntur, non ſolum reciperet, ſed etiam ab iis quaſi impleretur. Cum autem ob campum apparentem etiam radii ad axem obliqui a maiori ſpeculo reflectantur, quorum plures in noſtra conſtructione minus ſpeculum praetergrederentur, vtique conſultum erit, iſtud ſpeculum aliquanto propius admouere, vt etiam hos radios recipere queat. Quamobrem conueniet litterae ε duplicem valorem tribui, alterum ex ratione foraminis petitum, alterum vero ex loco minoris ſpeculi, quos ne inter ſe confuudamus, in poſterum ſtatuamus $y = \delta x$ at vero $b = \varepsilon . p$; ita, vt hoc caſu futurum ſit $\delta = \frac{1}{5}$ et $\varepsilon = \frac{5}{36}$. Neque vero hinc in noſtras formulas alia mutatio inferetur, niſi vt in locis, vbi formula εx ſeu y occurrit, eius loco ſcribamus δx, quod quidem tantum, vbi de quantitate foraminis et minoris ſpeculi ſermo eſt, occurrit; in reliquis vero omnibus formulis, vbi ε cum littera p coniungitur, nulla fit mutatio, ita, vt noſtrae formulae generales etiam hic valeant. Verum vt ad iſtud teleſcopium revertamur, diſtantia focalis ſpeculi minoris erat $q = 3$ dig. vnde concluditur diſtantia $BG = \beta = \frac{bq}{b-q} = 30$. hincque $CG = 2\frac{2}{3}$. Hic autem probe notandum eſt, ſi vel leuiſſima mutatio in loco minoris ſpeculi fiat,

tum

tum in hoc interuallo CG inſignem mutationem oriri; ſi enim loco $3\frac{1}{3}$ ſumatur $FB = b = 3\frac{3}{8}$, vt fit $BC = 27\frac{3}{8}$, reperietur $BG = \beta = 27$ hincque $CG = -\frac{3}{8}$. Quam ob cauſſam etiam minus ſpeculum ita conſtitui ſolet, vt eius locus ope cochleae tantillum immutari poſſit. In iſto autem exemplo ſpeculum minus ita eſt conſtituendum, vt inde prodeat $CG = 1\frac{1}{3}$ dig. Vnde viciſſim verus valor ipſius b definiri poterit; quia enim fit $BG = \frac{3b}{b-3}$, ob $CB = 24 + b$ erit $CG = \frac{3b}{b-3} - b - 24$; quae diſtantia vt fiat $= \frac{4}{3}$ dig. elicietur

$$b = \frac{\sqrt{1525} - 29}{3} = 3,35041.$$

qui valor aſſumtum $3\frac{1}{3}$ tantum ſuperat particula $\frac{1}{60}$ ita, vt in reliquo calculo ſumi poſſit $b = 3\frac{1}{3}$. Pergamus nunc in noſtro examine et quia lentis in C diſtantia focalis erat $= 4$ dig. $= r$, ob $c = -\frac{4}{3}$ dig. fiet $CH = \gamma = 1$ dig. Deinde vero erat interuallum $CD = 3$ dig. et lentis ocularis D diſtantia focalis $s = 2$ dig. ſicque prodibit diſtantia $HD = d = 2$ dig. ideoque $d = s$, vti natura teleſcopii poſtulat. Quocirca ſingula huius teleſcopii elementa ita ſe habebunt

$$a = 24;\ b = 3,35041.;\ c = -1,33333;\ d = 2;$$
$$\beta = 28,68374;\ \gamma = 1.$$

et diſtantiae focales

$$p = 24;\ q = 3;\ r = 4, \text{ et } s = 2. \text{ dig.}$$

 inter-

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interualla vero

$$AB = BC = 27,35041.; \quad CD = 3. \text{ dig.}$$

Hinc vero reliquae noſtrae litterae inuenientur

$$B = \frac{\beta}{b} = 8,5613; \quad \mathfrak{B} = 0,89541.$$

$$C = \frac{\gamma}{c} = -0,75.; \quad \mathfrak{C} = -3. \text{ et}$$

$$\epsilon = \frac{b}{p} = 0,13960. = \frac{1}{7,1633}$$

ac denique

$$P = -\frac{\alpha}{b} = -7,1633.$$

$$Q = -\frac{\beta}{c} = 21,51281.$$

$$R = -\frac{\gamma}{d} = -\frac{1}{2}.$$

His inuentis valoribus proprietates huius Teleſcopii ſequenti modo definiri poterunt: quod

1°. ad multiplicationem attinet, quia eſt $m = PQR$, erit $m = 77,05$.

2°. vt nunc etiam campum apparentem definiamus, primo ex apertura lentis C, cuius ſemidiameter eſt $y = \frac{1}{2}$ dig. ſumto $\xi = \frac{1}{4}$, erit $\frac{1}{4}\mathfrak{r}r = \frac{1}{2}$ dig. ideoque $\mathfrak{r} = \frac{1}{2}$ dig. tum vero eſt

$$\mathfrak{q} = \frac{(P-1)M}{\mathfrak{B}} = -9,1168.\,M.$$

ſimilique modo

$$\mathfrak{C}\mathfrak{r} = (PQ-1)M - \mathfrak{q} = -145,98.\,M$$

hincque $\mathfrak{r} = 48,66.\,M.$

Cum

Cum igitur ante eſſet $r = \frac{1}{4}$ dig. hinc concluditur

$$M = \frac{r}{97,72} = \frac{q + r + s}{76,05}$$

vnde elicitur

$$s = \frac{8517}{9772} - 0,5 = 0,3751.$$

qui valor cum vnitate ſit minor veritati erit conſentaneus; ſi enim vnitate maior prodiiſſet, tum litterae r valorem ſemiſſe minorem tribuere debuiſſemus. Quocirca ſemidiameter campi apparentis erit

$$\Phi = M\xi = \frac{1}{4}M = 859.\ M.\ \text{min.} = 8'\ 50''.$$

ſiue diameter campi erit $= 17'\ 40''$.

3°. Videamus, an per hoc teleſcopium etiam margo coloratus deſtruatur, quae conditio cum poſtulet

$$0 = \frac{r}{PQ} + \frac{s}{PQR} \text{ ſiue } r = 2s,$$

quod cum non multum a veritate diſcrepet, margo vtique debebit eſſe inſenſibilis; interim tamen perfectius margo coloratus tolleretur, ſi prodiiſſet exacte $r = 2s$; id quod quidem leuiſſima mutatione fieri poſſet. Tandem autem reſtabit, vt etiam inueſtigemus, quam exacte aequatio ſemidiametrum confuſionis complectens hic impleatur, ſiue cum hic iam cognoſcamus litteras m; x; p; B; C; vna cum μ, ν et λ ex indole vitri et figura lentium, definiemus inde litteram k, quam nouimus vix infra 50 admitti poſſe

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poſſe. Sumamus autem primo ambo ſpecula ad figuram ſphaericam eſſe elaborata, quoniam facile erit facto calculo duos terminos priores rejicere, quando nouerimus haec ſpecula eſſe parabolica. Ex forma autem generali ſupra § 34. data patet fore

$$\frac{1}{k} = 0,222.\sqrt[3]{\Big(1 + \frac{\varepsilon(1+\mathfrak{B})(1-\mathfrak{B})^2}{\mathfrak{B}^3}}$$
$$- \frac{4\mu}{m.B^3\mathfrak{C}^3}(\lambda'' + \nu.\mathfrak{C}.(1-\mathfrak{C}))$$
$$- \frac{8\mu}{m.B^3C^3}.\lambda'''\Big)$$

ob $x = \frac{3}{5}$; $p = 24$, et $m = 77,05$.

Deinde cum ſit $\varepsilon = 0,1396$,

$$B = 8,5613,\ \mathfrak{B} = 0,89541.$$
$$\mathfrak{C} = -3, \text{ et } C = -\tfrac{5}{4},$$

ſinguli hi termini ita in numeris euoluentur.

$$\frac{1}{k} = 0,222.\sqrt[3]{\big(1 + 0,1216 + 0,000003.\mu.(\lambda'' - 12.\nu)}$$
$$+ 0,00039.\mu.\lambda'''\big)$$

hinc ergo colligimus, ſi primum ſpeculum eſſet ſphaericum, certe proditurum eſſe

$$\frac{1}{k} > 0,222; \text{ hoc eſt } \frac{1}{k} > \frac{2}{9} \text{ ideoque } k < \frac{9}{2},$$

vnde certe confuſio enormis naſceretur; quod cum neutiquam fieri debet, neceſſe eſt, vt primum ſpeculum ſit parabolicum vel proxime ſaltem, vt primus terminus euaneſcat. Si porro ſpeculum minus eſſet ſphaericum, prodiret adhuc $\frac{1}{k} > 0,111$ ſeu $k < 9$. vnde confu-

confuſio adhuc intolerabilis naſceretur, ex quo concludimus etiam a ſecundo ſpeculo nullam confuſionem naſci. Reiectis ergo binis prioribus terminis habebitur

$$\frac{1}{k} = 0,222.\sqrt[3]{(0,000003.\mu.(\lambda'' - 12.\nu) + 0,00039.\mu.\lambda''')}$$

vbi ſtatim patet ſolum poſtremum membrum in computum venire, vnde ergo cum ſumi poſſit $\mu\lambda''' = 1.$ prodit

$$\frac{1}{k} = 0,222.\ 0,073. = \frac{2}{9}.\frac{1}{13} \text{ ſiue } k = 59.$$

qui valor iam tantus eſt, vt nulla confuſio ſit metuenda atque hinc iam multo magis intelligimus, ſummam ſollertiam ad huiusmodi teleſcopia conficienda requiri, quae ſi ab artifice exſpectari poteſt, nullum eſt dubium, quin ſpecies teleſcopiorum a nobis ante expoſita his, quae paſſim reperiuntur, longe ſit anteferenda. In §pho igitur ſuperiori 46. vt eum ad modo examinatum teleſcopium accommodemus, ſumi poterit ε quatenus ad p refertur $= \frac{1}{7}$, quatenus autem ad x refertur $= \frac{1}{8}$ vt fiat $y = \frac{1}{3}x$; vnde pro quauis multiplicatione huiusmodi teleſcopia formari poterunt, quae certe multo maiorem campum patefacient, ſimulque marginem coloratum perfectius tollent. Verum ſi ſpeculum minus fiat conuexum, multo maiora commoda inde ſperare licebit, vti in ſequente Capite oſtendemus. Caſum enim, qui hic

 adhuc

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adhuc deſiderari poſſet, quo imago realis in interuallum B C caderet, ne quidem attingemus, quoniam tam campum nimis paruum produceret, quam vitio marginis colorati vehementer laboraret. Cum enim tum eſſet $R > o$, aequatio pro margine tollendo $o = r + \frac{s}{R}$ ſubſiſtere non poſſet, niſi r foret negatiuum et quia q etiam eſt negatiuum, campus fere ad nihilum redigeretur.

CAPVT

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CAPVT IV.

DE
TELESCOPIIS CATADIOPTRICIS MINORE SPECVLO CONVEXO INSTRVCTIS.

Problema I.

§. 50.

Conſtructionem huiusmodi teleſcopiorum deſcribere, quibus obiecta ſitu inuerſo repraeſententur, ſeu vbi vnica imago realis occurrat.

Solutio.

Cum in hoc genere diſtantia amborum ſpeculorum ſit $AB = (1 - \epsilon)p$, ideoque $b = -\epsilon . p$ ob $\alpha = p$ erit $P = -\frac{\alpha}{b} = +\frac{1}{\epsilon}$, vbi ϵ deſignat fractionem aliquanto minorem, quam ratio foraminis ad ſpeculum maius $\frac{y}{x}$ deſignat, ita, vt poſito $y = \delta x$ ſit $\epsilon < \delta$, ob rationem ante allegatam §. 49. qua ſcilicet obtinetur, vt etiam radii obliqui a minore ſpeculo excipiantur. Interim tamen ſemidiameter aperturae minoris ſpeculi maneat $= \delta x = y$, ita, vt hoc ſpeculum foramini aequetur, vti initio aſſumſimus. Nunc ſtatim conſideremus aequationem, qua margo colo-

coloratus deſtruitur, quae ſi praeter ſpecula duae lentes adhibeantur, reducitur ad hanc formam: $o = \mathfrak{r} + \frac{\mathfrak{s}}{R}$, vnde vt ambae litterae $\mathfrak{r}$ et $\mathfrak{s}$ valores poſitiuos habere queant, vti ratio campi poſtulat, conueniet litterae R valorem tribui negatiuum et quidem vnitate non minorem, vt ſumto $\mathfrak{s} = 1$ prior lens C, cuius apertura iam per foramen determinatur, campum non reſtringat. Ponamus igitur $R = -i$ et cum ex data multiplicatione m ob repraeſentationem inuerſam ſit $PQR = -m$ fiet hinc $PQ = \frac{m}{i}$ et $Q = \frac{\varepsilon m}{i}$. Eſt vero $Q = -\frac{\beta}{c}$ et quia eſt

$$\beta + c = BC = AB = (1 - \varepsilon)p;$$

hinc colligimus

$$c = -\frac{i(1-\varepsilon)p}{\varepsilon m - i} \text{ et } \beta = \frac{m.\varepsilon(1-\varepsilon)p}{\varepsilon m - i}$$

quare cum in genere ſit $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$ erit

$$\frac{1}{q} = -\frac{(1-2\varepsilon)m - i}{m\varepsilon(1-\varepsilon)p} \text{ hincque } q = -\frac{m\varepsilon(1-\varepsilon)p}{(1-2\varepsilon)m + i}.$$

Porro ex valoribus b et β colligimus

$$B = \frac{\beta}{b} = \frac{-m(1-\varepsilon)}{\varepsilon m - i} \text{ et } \mathfrak{B} = \frac{+m(1-\varepsilon)}{(1-2\varepsilon)m + i}.$$

Deinde cum ſit $C = \frac{\gamma}{c}$ et $\mathfrak{C}c = r$ hinc inuenimus

$$\mathfrak{C} = \frac{r}{c} = -\frac{(\varepsilon m - i)r}{i(1-\varepsilon)p} \text{ ideoque}$$

$$C = \frac{-(\varepsilon m - i)r}{i(1-\varepsilon)p + (\varepsilon m - i)r} = \frac{\gamma}{c}$$

ex quo porro colligitur

$$\gamma = \frac{i(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}.$$

Deni-

Denique cum ſit $R = -\frac{\gamma}{d} = -\frac{\gamma}{s}$ ob $s = d$, erit

$$s = -\frac{\gamma}{R} = \frac{\gamma}{i} = \frac{(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}$$

hincque tertium interuallum

$$CD = \gamma + s = \frac{(1+i)(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r};$$

Nunc autem aperturae praebent has aequationes

1°. $\mathfrak{B}\,\mathfrak{q} = (P - 1)\,M$, vnde fit

$$\mathfrak{q} = \frac{((1-2\varepsilon)m + i)\,M}{\varepsilon m};$$

2°. $\mathfrak{C}\,\mathfrak{r} = (PQ - 1)\,M - \mathfrak{q} = \frac{\varepsilon m^2 - (1-\varepsilon)im - i^2}{\varepsilon i m}\,M$

ſeu $\mathfrak{C}\,\mathfrak{r} = \frac{(m+i)(\varepsilon m - i)}{\varepsilon i m}.\,M$

vnde elicitur $\mathfrak{r} = -\frac{(m+i)(1-\varepsilon)p}{\varepsilon m r}.\,M$

vnde cum ſit $\mathfrak{s} = i\,\mathfrak{r}$ ideoque $\mathfrak{r} + \mathfrak{s} = (1+i)\,\mathfrak{r}$, erit

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} = \frac{(1-2\varepsilon)mr + ir - (1+i)(m+i)(1-\varepsilon)p}{\varepsilon m r}.\,M$$

$= M\,(m - 1)$, ſicque facta diuiſione per M inueniemus

$$r = \frac{-(m+i)(1+i)(1-\varepsilon)p}{\varepsilon m^2 - (1-\varepsilon)m - i}$$

qui valor cum ſit negatiuus, ex eo etiam prodibit interuallum CD negatiuum, vnde patet hunc valorem in praxi locum habere non poſſe.

Verum cum ſaepenumero problemata duas pluresue ſolutiones admittant, idem etiam hic vſu venit, hocque problema praeter ſolutionem hic inuentam in-

 ſuper

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ſuper aliam complectitur, quam per diuiſionem ex calculo expulimus. Quod quod facilius appareat, calculum ita inſtituamus; cum primo ſit

$$\mathfrak{q} = \frac{(1-2\varepsilon)m+i}{\varepsilon m}.\,\mathrm{M} \text{ deinde } \mathfrak{s} = i\,\mathfrak{r}, \text{ erit}$$

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s} = \frac{(1-2\varepsilon)m+i}{\varepsilon m}\mathrm{M}+(i+1)\mathfrak{r} = \mathrm{M}(m-1)$$

vnde colligitur

$$\mathrm{M} = \frac{\varepsilon m(1+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i} \text{ ideoque}$$

$$\mathfrak{q} = \frac{(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i}$$

altera vero aequatio dabit

$$\mathfrak{C}\,\mathfrak{r} = \frac{(m-i)\varepsilon m(1+i)\mathfrak{r}-i(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon i m^2-(1-\varepsilon)i m-i^2}.$$

vnde fit

$$\mathfrak{C} = \frac{(1+i)(m+i)(\varepsilon m-i)}{i(\varepsilon m^2-(1-\varepsilon)m-i)}.$$

ſupra vero iam inuenimus

$$\mathfrak{C} = \frac{-(\varepsilon m-i)r}{i(1-\varepsilon)p}$$

vnde patet aequalitatem horum duorum valorum duplici modo obtineri poſſe 1°. ſcilicet, ſi fuerit $i = \varepsilon m$, quo quippe vterque valor euaneſcit; 2°. autem, quo facta diuiſione per

$$\varepsilon m - i \text{ fit } \frac{(1+i)(m+i)}{\varepsilon m^2-(1-\varepsilon)m-i} = \frac{-r}{(1-\varepsilon)p}$$

haecque eſt ſolutio incongrua ante inuenta. Statuamus igitur nunc $i = \varepsilon m$ fietque $\mathfrak{C} = 0$, littera vero r hinc

r hinc plane non determinatur, et noſtra ſolutio ſequenti modo ſe habebit:

$$\alpha = p;\ b = -\varepsilon p.;\ c = -\infty;\ d = \frac{r}{\varepsilon m};$$
$$\beta = \infty;\quad \gamma = r;$$

vbi notetur, fore $\beta + c = (1 - \varepsilon)p$.

Hinc porro erit

$$\mathrm{B} = \infty;\ \mathfrak{B} = 1.\ \mathrm{C} = 0;\ \mathfrak{C} = 0.$$

tum vero

$$\mathrm{P} = \frac{1}{\varepsilon};\ \mathrm{Q} = 1;\ \mathrm{R} = -\varepsilon m;$$

ita, vt ſit $\mathrm{PQR} = -m$.

Quia vero $\mathrm{B} = \infty$ et $\mathrm{C} = \mathfrak{C} = 0$, productum in ſe manet indefinitum; verum cum ſit

$$r = \frac{\mathrm{B}\mathfrak{C}}{\mathrm{PQ}}.p = \varepsilon \mathrm{B}\mathfrak{C}.p,$$ hinc viciſſim erit $\mathrm{B}\mathfrak{C} = \frac{r}{\varepsilon p}$.

Praeterea vero erunt diſtantiae focales

$$q = -\varepsilon p;\ \text{et}\ s = d = \frac{r}{\varepsilon m}$$

atque interualla

$$\mathrm{AB} = \mathrm{BC} = (1 - \varepsilon)p\ \text{et}\ \mathrm{CD} = r\left(1 + \frac{1}{\varepsilon m}\right)$$

Denique cum ſit

$$\mathfrak{q} = \frac{(1+\varepsilon m)(1-\varepsilon)\mathfrak{x}}{\varepsilon m - 1}\ \text{et}\ \mathfrak{y} = \varepsilon m \mathfrak{x}\ \text{erit}$$
$$\mathrm{M} = \frac{\varepsilon(\varepsilon m + 1)\mathfrak{x}}{\varepsilon m - 1} = \frac{(\varepsilon m + 1)\mathfrak{y}}{m(\varepsilon m - 1)}$$

ideoque ſemidiameter campi apparentis

$$\Phi = \frac{1}{4}.\frac{(\varepsilon m + 1)\mathfrak{y}}{m(\varepsilon m - 1)} = 859.\frac{(\varepsilon m + 1)\mathfrak{y}}{m(\varepsilon m - 1)}\ \text{minut.}$$

vbi

vbi ſumere licebit $\theta = 1$, ſi modo lens ocularis vtrinque fiat aeque conuexa. Oculi vero poſt hanc lentem diſtantia reperitur

$$O = \frac{\theta s}{M m} = \frac{\varepsilon m - 1}{\varepsilon m + 1} \cdot s.$$

Quia autem lentis C ſemidiameter aperturae maior eſſe nequit, quam $y = \delta x$, ponamus $\frac{1}{4} r\, r = \delta x$ ſiue $\frac{\theta r}{4 \varepsilon m} = \delta x$; vnde, ſumto $\theta = 1$, definitur $r = 4 \delta \varepsilon m x$ hincque $s = 4 \delta x$. Verum etiam ad aperturam minoris ſpeculi eſt attendendum, cuius ſemidiameter reuera eſt $= \delta x$ et qui ob campum eſſe deberet $= \frac{1}{4} q q$; quam ob cauſſam neceſſe eſt ſit

$$\frac{(1-\varepsilon)(\varepsilon m + 1)\theta p}{4 m (\varepsilon m - 1)} < \delta x \text{ ideoque } \theta < \frac{4 m (\varepsilon m - 1) \delta x}{(1-\varepsilon)(\varepsilon m + 1) p}.$$

Tuto igitur ſumere licebit $\theta = 1$, ſi modo fuerit

$$4 m (\varepsilon m - 1) \delta x > (1 - \varepsilon)(\varepsilon m + 1) p.$$

Contra vero θ vnitate minus accipi deberet. Tantum igitur ſupereſt, vt ex formula ſemidiametri confuſionis definiamus diſtantiam focalem ſpeculi principalis p, quae ita reperitur expreſſa

$$p = k x \sqrt[3]{m \left(\frac{1-\varepsilon}{\varepsilon} + \mu . \frac{\varepsilon^4 p^3}{r^3} . \lambda'' + \mu . \frac{\varepsilon^3 p^3}{m r^3} \lambda''' \right)}$$

ſiquidem ambobus ſpeculis figura ſphaerica inducatur, at ſi ambo habeant figuram parabolicam, debebit eſſe

$$r = k \varepsilon x \sqrt[3]{\mu m \left(\varepsilon \lambda'' + \frac{\lambda'''}{m} \right)}$$

ita, vt iam aliter non definiatur, niſi ex quantitate ſpeculi, cum ſine dubio ſemper eſſe debeat p multo maius

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maius, quam x. Quia vero iam ante definiuimus, $r = 4\,\delta\,\varepsilon\, m\, x$, habebitur nunc

$$4\,\delta\, m = k\,\sqrt[3]{\mu}\; m\left(\varepsilon\,\lambda'' + \frac{\lambda'''}{m}\right)$$

Cum nunc ſit proxime $\mu = 1$. ſumique poſſit $\lambda'' = 1$. et λ''' binario ſit minus, k vero infra 50 capi non debeat, valorem ipſius ε aeſtimare poterimus; tantus enim eſſe debet, vt numerus $\frac{4\,\delta\, m}{\sqrt[3]{(\varepsilon\, m + 2)}}$ non minor prodeat, quam 50; vnde patet pro ε ſumi debere fractionem valde paruam, ſi enim eſſet $\delta = \frac{1}{2}$ et $m = 100$, colligitur circiter $\varepsilon = \frac{1}{50}$.

Exemplum.

§. 51. Ponamus $m = 100$, $x = 2$ dig. $y = \frac{1}{2}$ dig. ideoque $\delta = \frac{1}{4}$ et vt $\frac{4\,\delta\, m}{\sqrt[3]{(\varepsilon\, m + s)}}$ ſatis magnum obtineat valorem, ſumamus $\varepsilon = \frac{1}{55}$ ſic enim prodit $k = \frac{100}{\sqrt[3]{7}}$ ſeu $k > 50$ hinc ergo erit $r = 10$. dig. et $s = 2$ dig. Deinde cum pro ſpeculo minore debeat eſſe

$$8000 > 57 \cdot p. \text{ erit } p < \frac{1000}{57}.$$

Vnde tuto ſumi poterit $p = 25$. dig. ſicque erit $q = -\frac{5}{4}$ dig. et interuallum $AB = BC = 23\frac{3}{4}$ dig. et $CD = 12$. dig. Oculi vero diſtantia $O = \frac{4}{3}$ dig. at campi apparentis ſemidiameter $\Phi = 12' \, 53''$, vbi probe notandum, hic ambo ſpecula aſſumi perfecte parabolica.

Scholion.

§. 52. Quamuis autem haec conſtructio perfecte ſuccedat, tamen tale teleſcopium tam inſigni vitio erit praeditum, vt omni vſu deſtituatur; cum enim radii a minore ſpeculo reflexi iterum fiant inter ſe paralleli, radii peregrini circa hoc ſpeculum tranſeuntes et in lentem C incidentes cum illis refractionem communem patientur, ſimulque cum iis in oculum deferentur, ita, vt verum obiectum cum vicinis prorſus permixtum viſioni repraeſentetur neque vllo modo ſeparari poterunt. Cum igitur huius vitii cauſſa in eo ſit ſita, quod radii a minore ſpeculo reflexi fiant paralleli ſeu interuallum $\beta = \infty$, ne hoc fiat, diligenter erit cauendum, quod fiet, ſi diſtantia β minor fuerit interuallo B C, ita, vt in hoc interuallum imago realis incidat litteraque Q negatiuum obtineat valorem. Praeterea vero quia etiam R negatiuum valorem habere debet ob marginem coloratum, duae iam habebuntur imagines reales et obiecta ſitu erecto cernentur. Neque vero duabus tantum lentibus adhibendis ſcopo noſtro ſatisfacere poterimus, ſed tertiam inſuper lentem in ſubſidium vocari oportebit, quae commodiſſime ita inſtrui poterit, vt aperturam quam minimam requirat, ſiquidem hoc modo ſegregatio radiorum peregrinorum feliciſſime ſuccedet, quemadmodum in ſequente problemate oſtendemus.

Pro-

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Problema 2.

§. 52. Huiusmodi teleſcopium cum ſpeculo minore conuexo et tribus lentibus vitreis conſtruere, quod obiecta ſitu erecto diſtincte repraeſentet.

Solutio.

Maneat, vt ante, $y = \delta x$ et interuallum ſpeculorum $AB = (1 - \epsilon)p = BC$. vt ſit $b = -\epsilon p$. Iam cum debeat eſſe $\beta < (1 - \epsilon)p$ et tamen ſuperare debeat eius ſemiſſem $\frac{1}{2}(1 - \epsilon)p$, ſtatuamus $\beta = \zeta(1 - \epsilon)p$, ita, vt ζ inter limites 1 et $\frac{1}{2}$ contineatur, hinc ergo fiet Tab. III. Fig. 9.

$$q = \frac{\zeta\epsilon(1-\epsilon)}{\epsilon - \zeta(1-\epsilon)} \cdot p = \frac{-\zeta\epsilon(1-\epsilon)}{\zeta - \epsilon(\zeta+1)} \cdot p$$

Tum vero erit

$$B = \frac{\beta}{b} = \frac{-\zeta(1-\epsilon)}{\epsilon} \text{ et } \mathfrak{B} = \frac{\zeta(1-\epsilon)}{\zeta - \epsilon(\zeta+1)}.$$

Porro vero erit $c = (1 - \epsilon)(1 - \zeta)p$ ſicque habebimus

$$P = \frac{1}{\epsilon};\ Q = \frac{-\beta}{c} = \frac{-\zeta}{1-\zeta}.$$

Statuatur igitur praeterea $R = -k$ fiatque

$$PQRS = m = \frac{\zeta k}{\epsilon(1-\zeta)} \cdot S,$$

vnde reliquae diſtantiae focales erunt

$$r = (1 - \epsilon)(1 - \zeta)\mathfrak{C}.\, p;$$

$$s = \frac{(1-\epsilon)(1-\zeta)C\mathfrak{D}}{k} \cdot p \text{ et}$$

$$t = \frac{-\zeta(1-\epsilon).CD}{\epsilon m} \cdot p.$$

 reli-

reliquaque interualla

$$CD = (1-\varepsilon)(1-\zeta)(1+\tfrac{1}{k})\,C.p$$
$$D\dot{E} = (1-\varepsilon)(1-\zeta)(1-\tfrac{1}{S})\,CD.p.$$

vnde intelligimus eſſe debere $C > 0$ ideoque $\mathfrak{C} < 1$; et $(1-\frac{1}{S})D > 0$. Vt vero fiat $t > 0$, debet eſſe $D < 0$ ideoque $S < 1$. Conſideretur nunc aequatio pro margine colorato tollendo, quae eſt

$$0 = \mathfrak{r} + \frac{\mathfrak{s}}{K} + \frac{\mathfrak{t}}{RS} \text{ ſiue } \mathfrak{r} = \frac{\mathfrak{s}}{k} + \frac{\mathfrak{t}}{kS};$$

vt iam ſecunda lens nulla apertura indigeat, ſtatuatur

$$\mathfrak{s} = 0. \text{ eritque } \mathfrak{r} = \frac{\mathfrak{t}}{kS}$$

aequationes autem pro litteris $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ poſito

$$M = \frac{q+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{m-1} = \frac{q+(1+kS)\mathfrak{t}}{m-1}, \text{ ſunt}$$

1°. $\mathfrak{B}q = \frac{1-\varepsilon}{\varepsilon}.M$

2°. $\mathfrak{C}\mathfrak{r} = \frac{-((1-\varepsilon)\zeta+\varepsilon)}{\varepsilon(1-\zeta)}M - q$

3°. $0 = \frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}.M - q - \mathfrak{r}.$

Ex prima autem habetur

$$q = \frac{\zeta-\varepsilon(\zeta+1)}{\varepsilon\zeta}.M$$

Ex tertia autem fit

$$q = \frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}M - \mathfrak{r}$$

qui duo valores inter ſe aequati dant

$$M = \frac{\varepsilon\zeta(1-\zeta)\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon} \text{ hincque}$$
$$q = \frac{(1-\zeta)(\zeta-\varepsilon(1+\zeta))\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon}.$$

Tum

Tum vero ob $M = \frac{q + \varepsilon(1 + kS)}{m - 1}$ reperietur etiam

$$M = \frac{kS\varepsilon}{m - 1 - \frac{\zeta(\varepsilon + k) + \varepsilon}{\varepsilon(1 - \zeta)}}$$

ex quorum valorum aequalitate ob $m = \frac{\zeta k}{\varepsilon(1 - \zeta)}$. S reperitur tandem

$$\zeta(\zeta kS - \varepsilon(1 - \zeta) - \zeta(\varepsilon + k) + \varepsilon) =$$
$$kS(\zeta^2(1 + k) - \zeta(1 + \varepsilon) + \varepsilon) \text{ ſeu}$$
$$\zeta^2 = S(\zeta(1 + \varepsilon) - k\zeta^2 - \varepsilon)$$

vnde concludimus eſſe debere

$$\zeta(1 + \varepsilon) > k\zeta^2 + \varepsilon \text{ ſiue } k < \frac{\zeta(1 + \varepsilon) - \varepsilon}{\zeta^2}.$$

Praeterea vero vt ex ſecunda aequatione pro $\mathfrak{C}$ prodeat valor poſitiuus, neceſſe eſt, vt ſit $q < 0$. ideoque etiam $\mathfrak{B} < 0$. vnde ſpeculum minus foret concauum; verum vt fiat $\mathfrak{B} < 0$, debet eſſe $\zeta < \varepsilon(\zeta + 1)$ ſeu $\varepsilon > \frac{\zeta}{\zeta + 1}$. Hoc vero non ſufficit, ſed inſuper neceſſe eſt, vt ſit

$$-q > \frac{(1 - \varepsilon)\zeta + \varepsilon}{\varepsilon(1 - \zeta)} \cdot M \text{ ſeu } \frac{-\zeta + \varepsilon(\zeta + 1)}{\zeta} > \frac{(1 - \varepsilon)\zeta + \varepsilon}{(1 - \zeta)}$$

vnde ſequitur $\varepsilon > \frac{\zeta}{1 - \zeta}$, quod cum nullo modo fieri queat, quia ζ intra limites 1 et $\frac{1}{2}$ continetur et ε vnitate minus eſſe debet, nunc demum intelligimus, hunc caſum locum habere non poſſe.

Alia Solutio.

§. 53. Quoniam igitur hoc incommodum inde naſcitur, quod ſumſimus R negatiuum, conſideremus

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alterum caſum, quo S fit negatiuum, manente R poſitiuo, et quoniam Q poſitum eſt negatiuum, ponamus $Q = -i$ et $S = -k$, vt ſit

$$PQRS = \frac{iRk}{\epsilon} = m;$$

calculus autem commodior euadet, ſi littera i retineatur, et cum ſit $i = \frac{\beta}{c}$ et $\beta + c = (1 - \epsilon)p$, euidens eſt, capi debere $i > 1$, eritque

$$\beta = \frac{i(1-\epsilon)}{1+i}p \text{ et } c = \frac{1-\epsilon}{1+i}p. \text{ vnde fit}$$

$$B = +\frac{\beta}{b} = -\frac{i(1-\epsilon)}{\epsilon(1+i)} \text{ et}$$

$$\mathfrak{B} = +\frac{i(1-\epsilon)}{i(1-2\epsilon)-\epsilon}; \text{ hincque } q = -\frac{\epsilon\epsilon(1-\epsilon)}{i(1-2\epsilon)-\epsilon}\cdot p.$$

Reliquae vero diſtantiae focales erunt

$$r = +\frac{(1-\epsilon)\cdot\mathfrak{C}}{(1+i)}\cdot p; \quad s = -\frac{(1-\epsilon)\cdot C\mathfrak{D}}{(1+i)R}\cdot p \text{ et}$$

$$t = -\frac{(1-\epsilon)CD}{(1+i)Rk}\cdot p,$$

et duo reliqua interualla erunt

$$CD = +\frac{(1-\epsilon)C}{1+i}\left(1 - \frac{\epsilon}{R}\right)p;$$

$$DE = -\frac{(1-\epsilon)C.D}{(1+i)R}\left(1 + \frac{\epsilon}{k}\right)p.$$

Vt igitur fiat $t > 0$, debet eſſe CD negatiuum, quo ipſo etiam vltimum interuallum fit poſitiuum. Vt vero et penultimum fiat poſitiuum, debet eſſe $C\left(1 - \frac{\epsilon}{R}\right)$ poſitiuum. Conditio porro marginis colorati ſumto $s = 0$. praebet $r = \frac{t}{Rk}$ ſiue $t = Rk.r$. et cum ſit

$$M = \frac{q+r+t}{m-1} = \frac{q+(1+Rk)r}{m-1}$$

ſatis-

ſatisfieri oportet his tribus aequationibus

$$1^\circ.\ \mathfrak{B}\,\mathfrak{q} = \frac{i-\epsilon}{\epsilon}.\,M$$

$$2^\circ.\ \mathfrak{C}\,\mathfrak{r} = -\frac{(i+\epsilon)}{\epsilon}\,M - \mathfrak{q}$$

$$3^\circ.\ 0 = +\frac{(iR+\epsilon)}{\epsilon}\,M + \mathfrak{q} + \mathfrak{r}.$$

Ex tertia ergo fit

$$\mathfrak{q} + \mathfrak{r} = -\frac{(iR+\epsilon)}{\epsilon}\,M;\ \text{hincque}$$

$$\mathfrak{q} + \mathfrak{r}(1 + Rk) = -\frac{(iR+\epsilon)}{\epsilon}\,M + Rk\mathfrak{r} = M(m-1)$$

vnde colligitur

$$M = \frac{R\,k.\,\mathfrak{r}}{m + \frac{iR}{\epsilon}},\ \text{ſimulque}$$

$$\mathfrak{q} = -\frac{Rk(iR+\epsilon)}{m\epsilon + iR}\,\mathfrak{r} - \mathfrak{r} = -\frac{(kiR^2 + R(i+k\epsilon) + m\epsilon)}{m\epsilon + iR}.\,\mathfrak{r}.$$

ex quo valor ipſius $\mathfrak{q}$ prodit negatiuus, qui cum ex prima forma prodeat poſitiuus, ſiquidem eſt $\mathfrak{B} > 0$, patet, etiam hanc ſolutionem locum habere non poſſe, ſiquidem ſecundum ſpeculum eſt conuexum, vti aſſumſimus.

Tertia Solutio.

§. 54. Pro repraeſentatione igitur erecta vnicus tantum caſus ſupereſt, quo ſumto Q poſitiuo ambae litterae R et S negatiuos obtinent valores. Statuamus igitur $Q = +i$; $R = -k$ et $S = -k'$, vt ſit

$$PQRS = m = \frac{ikk'}{\epsilon}\ \text{hincque}\ k' = \frac{\epsilon m}{ik}.$$

Porro

Porro erit

$$\mathfrak{C} = \frac{i(1-\varepsilon)}{i-1} . p;\; c = -\frac{(1-\varepsilon)}{i-1} . p;\; \text{vnde ſit}$$

$$B = \frac{-i(1-\varepsilon)}{\varepsilon(i-1)} \text{ et } \mathfrak{B} = \frac{i(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}$$

quare diſtantiae focales ſequenti modo ſe habebunt:

$$q = \frac{-\varepsilon i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon} . p;\; r = \frac{-(1-\varepsilon)\mathfrak{C}}{i-1} . p;$$

$$s = \frac{-(1-\varepsilon)C\mathfrak{D}}{(i-1)k} . p. \text{ et } t = \frac{-(1-\varepsilon)CD}{(i-1)kk'} . p.$$

$$= \frac{-i(1-\varepsilon).CD}{\varepsilon(i-1)m} . p.$$

Interualla vero lentium erunt

$$CD = \frac{-(1-\varepsilon)C}{i-1}\left(1+\frac{1}{k}\right)p.$$

$$DE = \frac{-(1-\varepsilon)CD}{(i-1)k}\left(1+\frac{1}{k'}\right)p.$$

vnde intelligimus, eſſe debere $C < 0$ et $D > 0$. ideoque $\mathfrak{D} < 1$. et $\mathfrak{D} > 0$. Nunc autem conditio marginis colorati dabit $0 = r + \frac{t}{kk'}$, vnde patet, eſſe debere $r < 0$. ſeu ob lentem C campum diminui. Ponamus ergo hic $r = -\omega$, vt fiat $t = \omega . kk' = \frac{\varepsilon m}{i}\omega$. quandoquidem etiam hic aſſumſimus $s = 0$; pro campo ergo apparente erit

$$M = \frac{iq + \omega(\varepsilon m - i)}{i(m-1)}$$

cui ſequentes tres aequationes ſunt adiungendae:

1°. $\mathfrak{B}q = \frac{1-\varepsilon}{\varepsilon} . M.$

2°. $-\mathfrak{C}\omega = \frac{i-\varepsilon}{\varepsilon} M - q.$

3°. $0 = -\frac{(ik+\varepsilon)}{\varepsilon} M - q + \omega.$

Ex

Ex hac vltima ergo concludimus

$$q = \omega - \frac{(ik+\varepsilon)}{\varepsilon} \cdot M.$$

addatur vtrinque $\omega\left(\frac{\varepsilon m}{i} - 1\right)$, eritque

$$q + \omega\left(\frac{\varepsilon m}{i} - 1\right) = \frac{\varepsilon m}{i}\,\omega - \frac{(ik+\varepsilon)}{\varepsilon} M = M(m-1)$$

ex quo colligitur

$$M = \frac{\varepsilon^2 m \omega}{i(m\varepsilon + ik)};$$ vnde viciſſim

$$q = \frac{\varepsilon m(i - ik - \varepsilon) + i^2 k}{i(\varepsilon m + ik)} \cdot \omega.$$

Ex prima vero aequatione fit

$$q = \frac{(\varepsilon i(1 - 2\varepsilon) + \varepsilon^2)\, m\omega}{i^2(m\varepsilon + ik)}$$

quorum valorum aequalitas ſuppeditat hanc aequationem

$$\varepsilon i m(i - ik - \varepsilon) + i^3 k = \varepsilon m(i(1 - 2\varepsilon) + \varepsilon)$$

ſeu

$$\varepsilon m(i^2(1 - k) - i(1 - \varepsilon) - \varepsilon) + i^3 k = 0.$$

ex qua aequatione inuenimus

$$k = \frac{\varepsilon m(i^2 - i(1-\varepsilon) - \varepsilon)}{i^2(\varepsilon m - i)} \text{ ſeu } k = \frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m - i)}$$

qui valor debet eſſe poſitiuus; quem in finem ſumi debet $i > 1$ et $i < \varepsilon m$. Iam ſubſtituto valore ipſius k reperitur

$$M = \frac{\varepsilon(\varepsilon m - i)\omega}{\varepsilon i m - i(1-\varepsilon) - \varepsilon}.$$

Ex ſecunda denique aequatione colligimus

$$\mathfrak{C} = -\frac{k(\varepsilon m - i)}{\varepsilon m + ik}$$

Secunda vero aequatio dat

$$\mathfrak{C} = -\frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m+ik)}$$

qui valor ergo eſt negatiuus, ideoque et C < o, vti ſupra iam requirebatur. Litterae autem $\mathfrak{D}$ et D arbitrio noſtro manent permiſſae, dummodo D poſitiue capiatur; quod tandem ad ipſam quantitatem p attinet, eam ex confuſione definiri conuenit ope formulae notae, vbi inprimis diſpiciendum erit, vtrum ſpeculis figura ſphaerica inducta ſit an parabolica.

Coroll. 1.

§. 55. Si ergo littera t in calculum introducatur, quam licebit vnitati aequalem ſumere, pro campo apparente habebimus

$$M = \frac{i(\varepsilon m - i)}{m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)} \cdot t$$

quippe qui valor per 859 min. multiplicatus dat ſemidiametrum campi Φ. Vidimus autem, litteram i intra limites 1 et εm capi debere.

Coroll. 2.

§. 56. Si caperetur $i = 1$, foret $\mathfrak{C} = \infty$ et radii a ſpeculo minore reflexi fierent inter ſe paralleli, vnde vitium ſupra memoratum oriretur, quod ſcilicet radii peregrini ita cum propriis permiſcerentur, vt nullo modo ſeparari poſſent, qui caſus cum ſit ſollicite euitandus, litteram i vnitate multo maiorem accipi conueniet, neque tamen alteri limiti εm aequalis

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lis aſſumi poteſt, quia alioquin campus prorſus euaneſceret.

Coroll. 3.

§. 57. Calculum inſtituenti facile patebit, maximum in hac expreſſione M locum non habere et eius valorem eo magis diminutum iri, quo maior littera i accipiatur. Quare cum eſſe debeat $i > 1$, ſi ſumamus $i = 2$, erit

$$M = \frac{2(\varepsilon m - 1)t}{m(2\varepsilon m + \varepsilon - 2)}$$

ſicque pro magnis multiplicationibus $M = \frac{1}{m} \cdot t$ qui valor etiam prodit, ſi capiatur $i = 3$ vel 4 etc. dummodo i ſit multo minus, quam εm, qui campus ſimplex cenſeri ſolet. Sin autem medium inter limites ſumendo capiatur

$$i = \frac{\varepsilon m + 1}{2} \text{ fiet } M = \frac{(\varepsilon m + 1)t}{2m(\varepsilon m + \varepsilon + 1)}$$

et pro magnis multiplicationibus campus ad dimidium redigetur.

Coroll. 4.

§. 58. Idem etiam patet ex primitiuo valore ipſius M, qui eſt

$$M = \frac{q + r + t}{m - 1}, \text{ pro quo } r = -\omega = -\frac{it}{\varepsilon m}.$$

Etſi autem q addi debet, tamen ex ſuperioribus patet, eſſe $q < \omega$; erat enim ex tertia aequatione

$$q = \omega - \frac{(ik + t)}{\varepsilon} M.$$

 Scho-

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Scholion.

§. 59. Circa campum autem inprimis eſt inquirendum, an loco t ſcribere liceat vnitatem, quod iudicium ex prima lente C eſt petendum, cuius ſemidiameter aperturae reuera eſt $= \delta x$ ob campum autem eſſe debet $= \frac{1}{4} \mathfrak{r} r$. Cum igitur ſit $\mathfrak{r} = -\frac{it}{\varepsilon m}$ et

$$r = \frac{-(1-\varepsilon)\mathfrak{C}}{i-1} \cdot p \text{ ſeu } r = \frac{\varepsilon m(1-\varepsilon)(i+\varepsilon)}{i^2(\varepsilon m + ik)} \cdot p.$$

Iam ſupra autem inuenimus eſſe

$$\varepsilon m + ik = \frac{\varepsilon m(i(\varepsilon m + \varepsilon - 1) - \varepsilon)}{i(\varepsilon m - 1)} = \frac{\varepsilon m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}{i(\varepsilon m - i)}$$

Quocirca erit

$$r = \frac{(\varepsilon m - i)(1-\varepsilon)(i+\varepsilon)}{i(\varepsilon i m - i(1-\varepsilon) - \varepsilon)} p$$

vnde, niſi fuerit

$$\frac{(\varepsilon m - i)(1-\varepsilon)(i+\varepsilon)}{\varepsilon m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)} \cdot p > 4 \delta x$$

tum ſumere licebit $t = 1$. Contra vero t tanto minus vnitate capi debebit, vbi notaſſe iuuabit, eſſe $\delta > \varepsilon$. Quoniam autem hae formulae nimis ſunt complicatae, quam vt in genere omnia momenta pro conſtructione teleſcopii commode exprimi queant: ſtatuamus $i = \frac{1}{2}(\varepsilon m + 1)$ vt interuallum CD minus euadat, etſi campus ad ſemiſſem redigitur; deinde enim videbimus, quomodo campus amplificari poſſit. Poſito autem

$$i = \frac{\varepsilon m + 1}{2} \text{ erit } k = \frac{2\varepsilon m(\varepsilon m + 2\varepsilon + 1)}{(\varepsilon m + 1)^2},$$

qui valor abit in $k = 2$ pro magnis multiplicationibus;

Dein-

Deinde vero

$$\mathfrak{C} = \frac{-(\varepsilon m + 2\varepsilon + 1)(\varepsilon m - 1)^2}{2(\varepsilon m + 1)((\varepsilon m + 1)(\varepsilon m + \varepsilon - 1) - 2\varepsilon)}$$

vnde C reperitur.

Scholion 2.

§. 60. Quia vero valor $i = \frac{\varepsilon m + 1}{2}$ merito nimis magnus videri poteſt, pro i potius medium geometricum ſumamus ſitque $i = \sqrt{\varepsilon m}$ ac primo pro campo apparente fiet

$$M = \frac{\varepsilon}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} . t:$$

Deinde vero habebimus $k = \frac{\varepsilon + \sqrt{\varepsilon m}}{\sqrt{\varepsilon m}}$ hincque

$$B = \frac{-(1-\varepsilon)\sqrt{\varepsilon m}}{\varepsilon(\sqrt{\varepsilon m} - 1)} \text{ et } \mathfrak{B} = \frac{(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m} + \varepsilon}.$$

$$\mathfrak{C} = \frac{-(\varepsilon + \sqrt{\varepsilon m})(\sqrt{\varepsilon m} - 1)}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} \text{ et } C = \frac{-(\varepsilon + \sqrt{\varepsilon m})(\sqrt{\varepsilon m} - 1)}{2\varepsilon m + \varepsilon\sqrt{\varepsilon m}}.$$

Ex his ſi ponamus $D = \vartheta$, vt ſit $\mathfrak{D} = \frac{\theta}{1+\theta}$ reperientur diſtantiae focales:

$$p = p;\; q = \frac{-\varepsilon(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m} + \varepsilon} . p,$$

$$r = \frac{(1-\varepsilon)(\varepsilon + \sqrt{\varepsilon m})}{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon} . p.$$

$$s = \frac{\theta}{1+\theta} . \frac{(1-\varepsilon)}{2\sqrt{\varepsilon m} + \varepsilon} . p.$$

$$t = \frac{\theta(1-\varepsilon)(\varepsilon + \sqrt{\varepsilon m})}{\varepsilon m(\varepsilon + 2\sqrt{\varepsilon m})} . p.$$

Interualla vero lentium erunt

$$AB = BC = (1 - \varepsilon)\, p.$$

$$CD = \frac{(\;-\varepsilon)(\varepsilon + 2\sqrt{\varepsilon m})}{2\varepsilon m + \varepsilon\sqrt{\varepsilon m}} . p$$

$$DE = \frac{\theta(1-\varepsilon)(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon)}{\varepsilon m(\varepsilon + 2\sqrt{\varepsilon m})} . p.$$

Pro loco autem oculi erit

$$O = \frac{t.t}{Mm} = \frac{\epsilon m + \sqrt{\epsilon m} + \epsilon}{\epsilon m} . t. = t\,(1 + \frac{1}{\sqrt{\epsilon m}} + \frac{1}{m}).$$

Pro aperturis autem inuenimus

$$q = \frac{(1 - 2\epsilon)\sqrt{\epsilon m} + \epsilon}{(\epsilon m + \sqrt{\epsilon m} + \epsilon)\sqrt{\epsilon m}} . t$$

$$r = -\frac{1}{\sqrt{\epsilon m}}; \text{ et } \mathfrak{s} = 0.$$

Licebit autem ſumere $t = 1$, niſi prodeat

$$\frac{(1 - \epsilon)(\epsilon + \sqrt{\epsilon m})}{(\epsilon m + \sqrt{\epsilon m} + \epsilon)\sqrt{\epsilon m}} . p > 4\,\delta\,x.$$

Lenti autem in D, pro qua eſt $\mathfrak{s} = 0$, apertura tribui debet, cuius ſemidiameter ſit $= \frac{x}{PQR} = \frac{\epsilon x}{\epsilon + \sqrt{\epsilon m}}$ ita, vt huius lentis apertura ſit tam exigua, vt ad radios peregrinos arcendos apprime ſit accommodata. Interim tamen quia campus apparens hic nimis eſt exiguus, vtique operae erit pretium, huic generi teleſcopiorum maiorem campum procurare, quod in ſequente problemate praeſtabimus.

Problema 3.

§. 61. Teleſcopiorum generi in problemate praecedente deſcripto nouum gradum perfectionis addere, dum eius campus apparens amplificatur.

Solutio.

Fit hoc additione nouae lentis, ita, vt nunc teleſcopium ex duobus ſpeculis et quatuor lentibus componatur. Maneat autem, vt ante,

$$P = \frac{1}{\epsilon};\ Q = i;\ R = -k \text{ et } S = -k';$$

quibus

quibus accedente littera T ſit, $\frac{ikk'T}{\epsilon} = m$ deinde ſit etiam, vt ante,

$$B = \frac{-i(1-\epsilon)}{\epsilon(i-1)} \text{ hincque } \mathfrak{B} = \frac{i(1-\epsilon)}{i(1-2\epsilon)+\epsilon};$$

ex quibus diſtantiae focales ita formabuntur;

$$q = -\frac{\mathfrak{B}}{P}p = -\epsilon\mathfrak{B}p;\ r = \frac{B\mathfrak{C}}{PQ}p = \frac{\epsilon B\mathfrak{C}}{i}p;$$

$$s = \frac{\epsilon . BC\mathfrak{D}.}{ik}p;\ t = \frac{\epsilon . BCD . \mathfrak{E}}{ikk'}.p; \text{ et}$$

$$u = -\frac{\epsilon BCDE}{ikk'T}.p = -\frac{BCDE}{m}.p;$$

et interualla

$$AB = BC = (1-\epsilon)p;\ CD = \frac{\epsilon BC}{i}\left(1+\frac{1}{k}\right)p;$$

$$DE = +\frac{\epsilon BCD}{ik}.\left(1+\frac{1}{k'}\right)p., \text{ et}$$

$$EF = \frac{\epsilon BCDE}{ikk'}\left(1-\frac{1}{T}\right)p.$$

vbi cum ſit $B < 0$, debet eſſe $C < 0$; deinde $D > 0$. Porro vt fiat u poſitiuum, debet eſſe $E < 0$ hincque ob vltimum interuallum $T < 1$. Nunc ſtatuatur etiam $r = -\omega$; $\vartheta = 0$; et vt campus maximus euadat, $u = t$, vt ſit $M = \frac{q-\omega+2t}{m-1}$. Vt vero margo coloratus euaneſcat, debet eſſe

$$\omega = \frac{t}{kk'} + \frac{u}{kk'T} = \frac{t}{kk'}\left(1+\frac{1}{T}\right)$$

et quia debet eſſe $T < 1$, ſumatur ſtatim $T = \frac{1}{2}$ vt ſit $m = \frac{ikk'}{2\epsilon}$; hincque $kk' = \frac{2\epsilon m}{i}$ tum igitur erit $\omega = \frac{3i}{2\epsilon m}.t$, ac viciſſim $t = \frac{2\epsilon m\omega}{3i}$; vnde ſit

$$M = \frac{q+\omega\left(\frac{4\epsilon m}{3i}-1\right)}{m-1.}$$

Nunc

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Nunc autem conſiderari oportet ſequentes quatuor aequationes:

$$\text{I}^{\circ}.\ \mathfrak{B}\,q = \frac{1-\varepsilon}{\varepsilon}.\,M$$

$$\text{II}^{\circ}.\ -\mathfrak{C}\,\omega = \frac{i-\varepsilon}{\varepsilon}\,M - q$$

$$\text{III}^{\circ}.\ 0 = -\left(\frac{ik+\varepsilon}{\varepsilon}\right)M - q + \omega$$

$$\text{IV}^{\circ}.\ \mathfrak{E}\,t = \frac{ikk'-\varepsilon}{\varepsilon}\,M - q + \omega.$$

Ex tertia igitur habemus

$$q - \omega = -\left(\frac{ik+\varepsilon}{\varepsilon}\right)M$$

addatur vtrinque $\frac{4\varepsilon m\omega}{3i}$, ac prodibit

$$M(m-1) = \frac{4\varepsilon m\omega}{3i} - \left(\frac{ik+\varepsilon}{\varepsilon}\right)M$$

vnde inuenitur

$$M = \frac{4\,\varepsilon\,m\,\omega}{m+\frac{ik}{\varepsilon}} = \frac{4\varepsilon^2 m\omega}{3i(m\varepsilon+ik)}$$

ſeu ſubſtituto valore ipſius ω

$$M = \frac{2\varepsilon}{m\varepsilon+ik}.\,t;$$

atque inſuper ex eadem aequatione erit

$$q = \frac{3i(m\varepsilon+ik)-4\varepsilon m(ik+\varepsilon)}{3i(m\varepsilon+ik)}.\,\omega$$

at vero prima aequatio dat

$$q = \frac{4(1-\varepsilon)\varepsilon.m\omega}{3i(m\varepsilon+ik)\mathfrak{B}},$$

quorum valorum aequalitas praebet

$$3i(m\varepsilon+ik) - 4\varepsilon m(ik+\varepsilon)$$
$$= \frac{4\varepsilon(1-\varepsilon)m}{\mathfrak{B}} = \frac{4\varepsilon m(i(1-2\varepsilon)+\varepsilon)}{i}$$

vnde

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vnde fit

$$ik(4\varepsilon m-3i)=\varepsilon m(3i-4\varepsilon)-\frac{4\varepsilon m(i(1-2\varepsilon)+\varepsilon)}{i}$$

$$=\frac{\varepsilon m}{i}(3i^2-4i(1-\varepsilon)-4\varepsilon)\ \text{ſeu}$$

$$ik=\frac{\varepsilon m(3i^2-4i(1-\varepsilon)-4\varepsilon)}{i(4\varepsilon m-3i)}$$

qui valor, vt ſit poſitiuus, debet eſſe $i<\frac{4}{3}\varepsilon m$ ſimulque

$$i>\tfrac{2}{3}(1-\varepsilon+\sqrt{1+\varepsilon+\varepsilon^2})$$

Hinc autem valore ipſius k definito ſecunda aequatio dabit

$$\mathfrak{C}=\frac{-4\varepsilon m(i^2-i(1-\varepsilon)-\varepsilon)}{3i^2(m\varepsilon+ik)}=\frac{-4\varepsilon m(i-1)(i+\varepsilon)}{3i^2(m\varepsilon+ik)}$$

ſiue ex altero valore ipſius q

$$\mathfrak{C}=\frac{-\varepsilon m(1+4k)+3ik}{3(m\varepsilon+ik)}$$

erit ergo ob $\mathfrak{C}<0$ etiam $C<0$ vti requiritur, ex quorum valorum aequalitate idem valor pro k, qui ante, prodit. Notetur autem hic eſſe

$$\varepsilon m+ik=\frac{4\varepsilon m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}{i(4\varepsilon m-3i)}$$

vnde fit

$$M=\frac{i(4\varepsilon m-3i).f.}{2m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}.$$

Deinde vero littera D arbitrio noſtro permittitur, dummodo ſumatur poſitiua.

Quarta denique aequatio nobis praebet valorem litterae

$$\mathfrak{E} = \frac{4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon}{i(m\varepsilon + ik)}$$

quare vt E prodeat negatiuum, oportet eſſe $\mathfrak{E} > 1$ ſiue

$$4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon > i(\varepsilon m + ik)$$

et valore ipſius $\varepsilon m + ik$ ſubſtituto

$$(4\varepsilon m - 3i)(4\varepsilon i m - 2i(1-\varepsilon) - 2\varepsilon)$$
$$> 4\varepsilon m(\varepsilon i m - i(1-\varepsilon) - \varepsilon)$$

quod vt fiat neceſſe eſt ſit

$$6.\varepsilon^2 i m^2 - 2\varepsilon m(3i^2 + i(1-\varepsilon) + \varepsilon)$$
$$+ 3i^2(1-\varepsilon) + 3\varepsilon i > 0.$$

quod ſponte euenit, cum certo ſit $i < \varepsilon m$.

Tandem pro loco oculi habebimus

$$O = \frac{tu}{M.m} = \frac{2(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}{i(4\varepsilon m - 3i)}.u \text{ ſiue}$$
$$O = \tfrac{1}{2}u\left(1 + \frac{3i^2 - 2i(1-\varepsilon) - 4\varepsilon}{i(4\varepsilon m - 3i)}\right)$$

Supereſt porro, vt diiudicemus, an pro t vnitas accipi queat, quod licebit, ſi fuerit

$$r < \frac{4\delta x}{\omega} \text{ ſeu } r < \frac{8\delta\varepsilon m x}{3i}.$$

Contra vero accipi debet $t = \frac{8\delta\varepsilon m x}{3ir}$ quo caſu campus in eadem ratione diminuetur, in qua t ab vnitate deficit. Quod autem ad quantitatem p attinet, ea

ea ex aequatione nota definiri debet, ſpeculorum ratione habita, vtrum ſint ſphaerica an parabolica.

Corollarium.

§. 62. Quia lens in D, quam minimo foraminulo pertundi ſufficit, a lente C diſtat interuallo

$$CD = \frac{\varepsilon . BC}{i}\left(1 + \frac{1}{k}\right)p$$

radii autem peregrini in lentem C incidentes poſt eam colliguntur ad diſtantiam $r = \frac{\varepsilon . B\mathfrak{C}}{i} p$; vt hi radii excludantur, neceſſe eſt, vt hae duae diſtantiae a ſe inuicem diſcrepent, ſeu notabilis differentia eſſe debet inter has quantitates $C\left(1 + \frac{1}{k}\right)$ et $\mathfrak{C}$, hoc eſt inter $1 + \frac{1}{k}$ et $1 - \mathfrak{C}$ ſeu inter $\frac{1}{k}$ et $-\mathfrak{C}$. Eſt vero

$$\frac{1}{k} = \frac{i^2(4\varepsilon m - 3i)}{\varepsilon m(3i^2 - 4i(1-\varepsilon) - 4\varepsilon)} \text{ et}$$

$$-\mathfrak{C} = \frac{(4\varepsilon m - 3i)(i-1)(i+\varepsilon)}{3i(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}$$

quare cum ratio inter has quantitates debeat eſſe admodum inaequalis, haec fractio

$$\frac{3i^3(\varepsilon i m - i(1-\varepsilon) - \varepsilon)}{\varepsilon m(i-1)(i+\varepsilon)(3i^2 - 4i(1-\varepsilon) - 4\varepsilon)}$$

plurimum ab vnitate diſcrepare debet; at differentia inter numeratorem et denominatorem ſatis eſt magna, vt aequalitas non ſit metuenda.

Coroll. 2.

§. 63. Quodſi autem ſumamus $i = 2$, fractio illa ab vnitate diuerſa euadet $= \frac{6(2\varepsilon m + \varepsilon - 2)}{\varepsilon m(1+\varepsilon)(2+\varepsilon)}$, quae

vtique ſatis ab vnitate diſcrepat, vt tranſitus radiorum peregrinorum neutiquam ſit metuendus. Campi autem ratio maxime exigit, vt ipſi i tam paruum valorem tribuamus, quam circumſtantiae permittunt. Ceterum multo magis ille tranſitus euitabitur, ſi capiatur $i > 2$.

Exemplum I.

§. 64. Pro multiplicatione $m = 50$. Ponamus hic $\delta = \frac{1}{4}$; $\varepsilon = \frac{1}{5}$, et quia haec multiplicatio poſtulat $x = 1$. dig. erit $y = \frac{1}{4}$ dig. Deinde ſtatuamus

$i = 3$. erit

$(i + \varepsilon)(i - 1) = 6,4$

$3i^2 - 4i(1 - \varepsilon) - 4\varepsilon = 16,6.$

$\varepsilon m = 10;$

$4\varepsilon m - 3i = 31.$

$B = -6$; $\mathfrak{B} = \frac{6}{5}$; $k = \frac{166}{93} = 1,785.$

$\varepsilon m + ik = 15,355;$

$\mathfrak{C} = -0,6175$; $C = -0,3817.$

$\mathfrak{E} = 2,4921$; $E = -1,6702$

vnde elementa primitiua ſequenti modo definientur, ponendo ϑ loco D, vt ſit $\mathfrak{D} = \frac{\vartheta}{1 + \vartheta}$.

$a = p$; $\beta = 1,2.p$; $\gamma = 0,1526.p$;

$b = -\frac{1}{5}p = -0,2.p$; $c = -0,4\,p$; $d = 0,0855.p$;

$\delta = 0,$

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$\delta = 0,0855.\,\vartheta.\,p.;\ e = 0,0229.\,\vartheta\,p.$

$\varepsilon = -\,0,0382.\,\vartheta\,p;\ f = 0,0764.\,\vartheta.\,p.$

ex quibus interualla colliguntur

$AB = BC = 0,\,8.\,p;\ CD = 0,\,2381.\,p;$

$DE = 0,\,1084.\,\vartheta\,p;\ EF = 0,\,0382.\,\vartheta\,p;$

ſicque tubus foramini ſpeculi annectendus erit circiter $= \frac{5}{7}\,p.$

Diſtantiae vero focales erunt

$q = \mathfrak{B}\,b = -\,0,\,24.\,p;$

$r = \mathfrak{C}\,c = 0,\,247.\,p;$

$s = \mathfrak{D}\,d = 0,\,0855.\,\frac{\theta}{1+\theta}.\,p;$

$t = \mathfrak{E}\,e = 0,\,0571.\,\vartheta.\,p;$

$u = f = 0,\,0764.\,\vartheta.\,p.$

Praeterea pro hoc caſu habebimus

$M = \frac{93}{2745}\,t = 0,\,0339.\,t$
(8, 5307323.)

Tum vero $\mathfrak{q} = 0,\,113.\,t$

$\mathfrak{r} = -\,\omega = -\,0,\,45.\,t.$

Nunc igitur videamus, an pro t ſumi poſſit vnitas, nec ne? quem in finem conſideremus valorem

$\mathfrak{r}\,r = 4\,\delta\,x;$ ſeu $0,\,111.\,p.\,t = 1$ dig.

vnde fit $t = \frac{1}{0,111.\,p} = \frac{9}{p}$ vnde apparet, ſi p fuerit

nouem digitorum vel minus, tum ſumi poſſe $t = 1$. ſin autem fuerit $p > 9$ dig. tum ſumi debet $t = \frac{9}{p}$ et campus tanto fiet minor. Circa locum oculi vero notandum eſt, eſſe $O = \frac{1}{3} u \left(1 + \frac{16,6}{97} \right) = 0,58. u$. Nunc vero reſtat praecipua inueſtigatio diſtantiae focalis p, quae ex menſura confuſionis colligitur

$$p = k x \sqrt[5]{50 \left(0,125 - 0,0283 \right.}$$
$$+ 0,00131. \mu (\lambda + \nu. \mathfrak{C}. 1 - \mathfrak{C}.)$$
$$+ 0,0031. \mu \left(\frac{(1+\theta)^3 \lambda'}{\theta^3} + \frac{\nu.(1+\theta)}{\theta^2} \right)$$
$$+ \frac{0,00005.\mu}{\theta^3} (\lambda'' + \nu. \mathfrak{E}. 1 - \mathfrak{E})$$
$$\left. + \frac{0,00036.\mu}{\theta^3}. \lambda''' \right)$$

Circa hanc expreſſionem vero ſequentia obſeruemus:

I°. Si ſpeculum principale ſit parabolicum; primum membrum poſt ſignum radicale 0,125 omitti debet; ac ſi etiam minus ſpeculum eſſet parabolicum; tum quoque ſecundum terminum omittere liceret. Conſultius autem videtur ſolum primum ſpeculum parabolicum efficere; alteri vero figuram ſphaericam perfectam inducere, tum enim ſequentia membra ita inſtrui, ſiue litterae λ, λ', λ'' cum littera ϑ ita aſſumi poterunt, vt iſta membra a ſecundo, quod eſt negatiuum, perfecte tollantur; ſicque tota confuſio ad nihilum redigatur. Quod ſi ſucceſſerit, ſufficiet litteram

p ex

p ex ſola apertura definire, ſumendo ſcilicet $p = 4x$ vel $6x$ vel $7x$, prouti viſum fuerit. Hoc ergo caſu ob $x = 1$ dig. diſtantia focalis p tuto minor, quam 9 dig. accipi poterit.

II°. Cum igitur ſumi poſſit $p < 9$. dig. ponere licebit $t = 1$. et campi apparentis ſemidiameter erit $= 859$. M. minut. $= 29$. minut. Tum autem binas poſtremas lentes vtrinque aeque conuexas confici oportet, vnde ſi lentes ex vitro communi pro quo eſt $n = 1,55$ parentur, erit $\lambda''' = 1 + (\frac{\sigma - \rho}{2\tau})^2 = 1,6299$. At $\lambda'' = 1 + 0,6299.(1 - 2\mathfrak{C}) = 10,9991$.

III°. Quia adeo capere liceret $p = 4$ dig. ne diſtantia focalis vltimae lentis fiat nimis parua, ſufficiet ſtatuere $\vartheta = 1$. atque hinc erit vltimum membrum noſtrae formulae $= 0,00055$. Pro penultimo membro erit

$$\nu.\mathfrak{C}.1 - \mathfrak{C} = -0,8649;\text{ ideoque}$$

$$\lambda'' + \nu.\mathfrak{C}\,1 - \mathfrak{C} = 10,1342,$$

ac propterea totum membrum $= 0,00047$. Quocirca ambo poſtrema membra iunctim ſumta dabunt $0,00102$.

IV°. Pro prima autem lente erit

$$\nu.\mathfrak{C}.1 - \mathfrak{C} = -0,2323;$$

vnde

vnde totum membrum inde natum fiet

$$= 0,00123.\lambda - 0,00028.$$

Pro ſecunda autem lente erit

$$\frac{(1+\theta)^3}{\theta^3}\lambda' + \frac{\nu.(1+\theta)}{\theta^2} = 8.\lambda' + 2\nu$$

hincque totum membrum erit

$$= 0,0232.\lambda' + 0,00135.$$

V°. His ergo inuentis litteras λ et λ' ita definiri oportet, vt fiat

$$0,0283 = 0,00123.\lambda + 0,0232.\lambda' + 0,00209$$

ſiue

$$0,0262 = 0,00123.\lambda + 0,0232.\lambda'$$

vbi notandum litteras λ et λ' vnitate minores eſſe non poſſe ſtatuamus ergo $\lambda' = 1$; et eſſe debebit $0,0030 = 0,00123\,\lambda$; hincque $\lambda = \frac{0,00300}{0,00123} = \frac{300}{123} = 2,44$.

Hinc igitur conſequimur ſequentem conſtructionem:

Teleſcopium Catadioptricum pro multiplicatione $m = 50$.

§. 65. Ex iis, quae modo euoluimus, obtinemus ſequentes determinationes:

I°. Pro ſpeculo principali, quod exactiſſime ſecundum figuram parabolicam elaborar debet, diſtan-

diſtantia focalis accipi poſſet $p = 4$ dig. Interim tamen litteram p quaſi indeterminatam in calculo retineamus.

Semidiameter aperturae huius ſpeculi $x = 1$ dig. et ſemidiameter foraminis $y = \delta x = \frac{1}{4}$ dig. Ante hoc ſpeculum ad interuallum $= 0,8 . p$ conſtituatur ſpeculum Secundum Q B Q.

II°. Pro quo debet eſſe diſtantia focalis $q = -0,24 . p$, ita, vt hoc ſpeculum debeat eſſe conuexum et ad figuram ſphaericam exacte elaboratum. Eius aperturae ſemidiameter $= \frac{1}{4}$ dig. Poſt hoc ſpeculum in ipſo foramine ſpeculi maioris ad diſtantiam $BC = \frac{4}{5} . p = 0,8 . p$. conſtituitur.

III°. Lens prima, ex vitro communi $n = 1,55$ paranda, cuius diſtantia focalis ſit $r = 0,247 . p$ capiendo

$$\text{rad. fac.} \begin{cases} \text{ant.} = \dfrac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) + \tau\sqrt{(\lambda - 1)}} = \dfrac{r}{1,4283} = 0,1729 . p \\ \text{poſter.} = \dfrac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{\lambda - 1}} = \dfrac{r}{0,3896} = 0,6339 . p \end{cases}$$

Semidiameter aperturae $= \frac{1}{4}$ dig; vt foraminis, et interuallum vsque ad lentem ſecundam

$= 0,2381 . p = CD.$

IV°. Pro ſecunda lente S D S, cuius diſtantia focalis $s = 0,0427 . p$ ob $\mathfrak{D} = \frac{1}{2}$ et $\lambda' = 1$ capiatur

$$\text{rad. fac.}\begin{cases}\text{anter.} = \dfrac{s}{\sigma - \frac{1}{2}(\sigma - \varrho)} = \dfrac{s}{[illegible]} = 0{,}04697\,p. \\ \text{poſter.} = \dfrac{s}{\varrho + \frac{1}{2}(\sigma - \varrho)} = \dfrac{s}{[illegible]} = 0{,}04697\,p\end{cases}$$

Eius aperturae ſemidiameter:

$$= \frac{x}{PQR} = \frac{1}{27{,}771} = 0,\ 037.\ \text{dig.}$$

et interuallum ad tertiam lentem

$DE = 0,\ 1084.\ p.$

V°. Pro tertia lente, cuius diſtantia focalis $t = 0{,}0571.\ p$, capiatur radius vtriusque faciei $= 0,\ 0628.\ p.$ eius aperturae ſemidiam. $= \frac{1}{4}\ t = 0,\ 0142.\ p.$ et interuallum ad quartam lentem $= 0,\ 0382.\ p.$

VI°. Pro quarta lente, cuius diſtantia focalis $u = 0,\ 0764.\ p$, capiatur radius vtriusque faciei $= 0,\ 0840.\ p.$ eius aperturae ſemidiam. $= \frac{1}{4}\ u = 0,\ 0191.\ p.$ et interuallum ad oculum

$= 0,\ 58.\ u = 0,\ 0445.\ p.$

VII°. Tubi ergo anterioris ambo ſpecula continentis longitudo aliquanto maior eſt, quam 0, 8 p. Tubi vero poſterioris lentes continentis longitudo erit $= 0,\ 4292.\ p.$ ſicque totius inſtrumenti

menti longitudo erit circiter $= 1,4292.\,p$. ita, vt ſumto $p = 5$. dig.; haec longitudo futura ſit 7. dig.

VIII° Campi autem apparentis ſemidiameter iam ſupra indicatus eſt $= 29$. minut., qui pro multiplicatione $m = 50$ ſatis eſt notabilis.

IX°. Diaphragmatis ſiue ſeptis in locis imaginum realium collocandis hic plane non erit opus, cum ſecunda lens tam exiguam habeat aperturam, quae radios peregrinos omnes excludat. Interim tamen ſi in loco primae imaginis realis, quae poſt primam lentem cadit ad interuallum $\gamma = 0,1526.\,p$ collocetur diaphragma, eius foraminis ſemidiameter ſumi debet $= 0,127.\,p$ hoc vero diaphragmate vix erit opus, cum radiorum peregrinorum in lentem primam incidentium imago cadat poſt hanc lentem ad diſtantiam $r = 0,247\ p$, dum ea radiorum propriorum cadit ad diſtantiam $\gamma = 0,1526.\,p$, quod diſcrimen ſatis eſt notabile.

X°. Si quis metuat, ne a tam exiguo ſpeculo, cuius ſemidiameter eſt $= 1$ dig. quodque adeo foramine eſt pertuſum, nimis exigua luminis copia ad oculum transmittatur, is tantum menſuram digitorum pro lubitu augeat nihil

 enim

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enim impedit, quominus menſura digiti adeo duplicetur. Hoc enim modo claritas ad lubitum augeri poterit neque tamen inſtrumenti longitudo, quae per ſe eſt parua, ob hanc cauſſam enormis euadet.

Exemplum II.

Pro multiplicatione $m = 100$.

§. 66. Statuamus hic $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$ vt ſit $\varepsilon m = 20$. Tum vero ſumamus $i = 4$, quo tubus breuior euadat, atque habebimus

$$P = \frac{1}{\varepsilon} = 5;\ Q = i = 4;$$

$$R = -k = -\frac{43}{68} = -0,63235 \text{ ob}$$

$$3i^2 - 4i(1-\varepsilon) - 4\varepsilon = 34\tfrac{2}{5} \text{ et } 4\varepsilon m - 3i = 68$$

porro

$$S = -k' = -\frac{680}{43} = -15,814 \text{ et } T = \frac{1}{2} = 0,5.$$

Vnde fit

$$PQ = 20;\ PQR = -12,647;$$

$$PQRS = 200 \text{ et } PQRST = 100.$$

Reliquae vero litterae reperientur

$$\mathfrak{B} = \frac{16}{13} = 1,231.$$

$$B = -\frac{16}{3} = -5,333.$$

$$\mathfrak{C} = -\frac{17 \cdot 21}{383} = -0,93211$$
$$(9,9694694)$$

C =

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$$C = -\tfrac{357}{745} = -0,4824$$
$$(9,6834398)$$

et

$$\mathfrak{D} = \tfrac{\vartheta}{1+\vartheta};\ \mathfrak{E} = \tfrac{17.785}{8.3.383} = 3,4755$$
$$(0,5410119)$$
$$D = \vartheta;\ E = -\tfrac{3,4755}{2,4755} = -1,4039$$
$$(0,1473490)$$

Vnde colligimus

$\log. B\mathfrak{E} = 0,6964410;\ \log. BC\mathfrak{E} = 0,9514233;$
$\log. BC = 0,4104114;\ \log BCE = 0,5577604(-)$

His praemiſſis elementa noſtra erunt

$\alpha = p;\ b = -\frac{\alpha}{P} = -\frac{1}{5}.\alpha = -0,2.p.$

$\beta = Bb = 1,0666.p;\ c = -0,2666.p.$

$\gamma = Cc = 0,1286.p;\ d = 0,20344.p.$

$\delta = Dd = 0,20344.\vartheta.p;\ e = 0,01286.\vartheta.p.$

$\varepsilon = Ee = -0,01806.\vartheta.p;\ f = 0,03612.\vartheta.p.$

vnde ſtatim obtinemus interualla

$AB = 0,8.p;\ BC = 0,8.p;\ CD = 0,3320.p.$
$DE = 0,2163.\vartheta.p;\ EF = 0,01806.\vartheta.p.$

Diſtantiae vero focales ita ſe habebunt:

$q = \mathfrak{B}b = -0,246.p;\ r = \mathfrak{C}c = 0,2485.p.$

$s = 0,2034.\frac{\vartheta}{1+\vartheta}.p;\ t = \mathfrak{E}e = 0,0447.\vartheta.p.$

$u = f = 0,0361.\vartheta.p.$

Praeterea vero erit $\omega = 0,3.t = -\mathfrak{r}$ vnde aequatio $\mathfrak{r}\,r = 4\,\delta\,x$ abit in hanc: $0,06455\,t.p = x$; quare ſi ſumatur $x = 2$ dig.; hinc fiet $t = \frac{2}{0,06455.p}$. Dummodo igitur fuerit $p < 30$ dig. capere licebit $t = 1$. binasque vltimas lentes vtrinque aeque conuexas fieri oportet. Verum ſi etiam hic liceat totam confuſionem ad nihilum redigere, ob $x = 2$. dig. ſumi adeo poſſet $p = 8$. dig. etiamſi praeſtet ipſi p maiorem valorem tribuere; vnde patet tuto aſſumi poſſe $\vartheta = 1$.

Praeterea vero pro campo apparente habebitur $M = \frac{28}{1515}.t$; quare ſi capi poterit $t = 1$. ſemidiameter campi apparentis erit $\Phi = \frac{859.28}{1515}$. min. $= 15\frac{7}{8}$ min. et pro loco oculi habebimus

$$O = 0,563.u. = 0,02037.p.$$

Denique vt tota confuſio euaneſcat, primum ſpeculum perfecte parabolicum confici neceſſe eſt, atque tum eſſe debebit

$$\frac{1(1+B)(1-B)^2}{8B^3} = \frac{\mu}{B^3\mathfrak{C}^3PQ}(\lambda + \nu.\mathfrak{C}.1-\mathfrak{C})$$
$$- \frac{\mu}{B^3C^3PQR}(8.\lambda' + 2\nu)$$
$$+ \frac{\mu}{B^3C^3\mathfrak{E}^3PQRS}(\lambda'' + \nu.\mathfrak{E}.1-\mathfrak{E})$$
$$- \frac{\mu}{B^3C^3.E^3.m}\lambda'''.$$

vbi vt ante ſi refractio vitri ſit

$$n = 1,55.\ \text{erit}\ \lambda''' = 1,6299\ \text{et}$$
$$\lambda'' = 1 + 0,6299.(1-2\mathfrak{E})^2 = 23,308.$$

vnde

vnde aequatio noſtra praebebit

$$\begin{aligned} 0,02864 = 0,000382.\lambda &- 0,00016 \\ + 0,034843.\lambda' &+ 0,00200 \\ &- 0,00001 \\ &+ 0,00015 \\ &+ 0,00032 \end{aligned}$$

ſiue $0,02634 = 0,000382.\lambda + 0,03484.\lambda'$

quae aequalitas quia λ et λ' vnitate minores eſſe nequeunt, ſubſiſtere non poteſt. Quamobrem coacti ſumus ipſi ϑ maiorem valorem tribuere; ſit ergo $\vartheta = 2$, et noſtra aequatio fiet

$$\begin{aligned} 0,02809 = 0,000382.\lambda &- 0,00016 \\ + 0,01143.\lambda' &+ 0,00075 \\ &- 0,00001 \\ &+ 0,00002 \\ &+ 0,00004 \end{aligned}$$

ſiue $0,02744 = 0,000382.\lambda + 0,01143.\lambda'$

Ne hinc valor ipſius λ prodeat nimis magnus, ſumamus $\lambda' = 2$ eritque $0,00458 = 0,000382.\lambda$, hincque $\lambda = \frac{4580}{382} = 12$. Sin autem ſumſiſſemus $\lambda' = 2\frac{1}{3}$ obtinuiſſemus $\lambda = \frac{770}{382} = 2$.

Vtamur ergo his poſtremis valoribus $\lambda = 2$; et $\lambda' = 2\frac{1}{3}$, exiſtente $\vartheta = 2$; hincque $\mathfrak{D} = \frac{2}{3}$; vnde colligitur ſequens

Con-

Conſtructio Teleſcopii Catadioptrici pro $m = 100$.

§. 67. Haec ergo conſtructio conſtabit ſequentibus determinationibus.

I°. Primum ſpeculum perfecte ſecundum figuram parabolicam elaboretur, cuius diſtantia focalis ſit $= p$, quam ad minimum 8 dig. ſtatui oportet; eius aperturae ſemid. $= x = 2$. dig. foraminis autem ſemidiam. $= \frac{1}{2}$ dig. et diſtantia a ſpeculo minore $AB = 0,8.\,p$.

II°. Minus ſpeculum figuram ſphaericam habeto, cuius diſtantia focalis ſit $q = -0,246.\,p$; et ſemidiamet. aperturae $= \frac{1}{2}$ dig. indeque diſtantia ad primam lentem $BC = 0,8.\,p$.

III°. Pro prima lente, cuius diſtantia focalis

$$r = 0,2485.\,p, \text{ numeri vero}$$

$$\mathfrak{C} = -0,9321 \text{ et } \lambda = 2,$$

capiatur radius faciei

$$\text{anter.} = \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{\lambda - 1}} = \frac{r}{2,5666 - 0,9051} = 0,1205.\,p.$$

$$\text{poſter.} = \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) + \tau\sqrt{\lambda - 1}} = \frac{r}{-0,1485 + 0,9051} = -1,0210.\,p.$$

Semidiam. apert. foramini aequalis $= \frac{1}{2}$ dig.

et diſtantia ad lentem ſecund. $CD = 0,3320.\,p$.

IV°. Pro

IV°. Pro ſecunda lente, cuius diſtantia focalis $s = 0,1356.p$ et numeri $\mathfrak{D} = \frac{2}{3}$ et $\lambda' = 2,3333.$ capiatur radius faciei

anter. $= \frac{s}{\sigma - \mathfrak{D}(\sigma - \varrho) \pm \tau\sqrt{(\lambda' - 1)}} = \frac{s}{1,7147} = 0,0791.p.$

poſter. $= \frac{s}{\varrho + \mathfrak{D}(\sigma - \varrho) \mp \tau\sqrt{(\lambda' - 1)}} = \frac{s}{0,1034} = 1,3114.p.$

Eius aperturae ſemidiam. $= \frac{x}{PUK} = 0,16.$ dig. et diſtantia a lente tertia $DE = 0,4326.p.$

V°. Pro lente tertia, cuius diſt. focal. $t = 0,0894.p.$ capiatur radius vtriusque faciei $= 0,0983.p.$ eius aperturae ſemid. $= \frac{1}{4}t = 0,0246.p.$ et diſtantia ad lentem quartam $EF = 0,03612.p.$

VI°. Pro lente quarta, cuius diſt. foc. $u = 0,0722.p.$ capiatur radius vtriusque faciei $= 0,0794.p.$ eius aperturae ſemidiam. $= \frac{1}{4}u = 0,0198.p.$ et diſtantia oculi $O = 0,563.u = 0,0204.p.$

VII°. Longitudo ergo tubi prioris aliquanto maior erit, quam $0,8.p.$ tubi autem affixi longitudo $= 0,8211.p$; hincque totius inſtrumenti circiter $= 1,6211.p.$

VIII°. Campi apparentis ſemidiameter $= 15\frac{1}{4}$ min. et quae ſupra obſeruauimus praeterea, etiam hic locum habent.

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Exempl. III.

Pro multiplicatione $m = 150$.

§. 68. Maneant, vt ante, $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$ vt ſit $\varepsilon m = 30$. ſumatur autem $i = 5$ et vt claritate ſufficiente fruamur, ſit $x = 3$. dig. vt ſit $y = \frac{3}{4}$ dig. et hinc colligimus

$$P = 5;\ Q = 5;\ R = -k = -0,6652.$$

$$S = -k' = -1,8040; \text{ et } T = \tfrac{1}{2}. \text{ hinc}$$

$$PQ = 25;\ PQR = -16,63;$$

$$PQRS = 300 \text{ et } PQRST = 150.$$

inde vero reliquae litterae reperientur:

$$\mathfrak{B} = \tfrac{5}{4} = 1,25;\ B = -5;$$

$$\mathfrak{C} = -\tfrac{33,28}{33,326} = -0,9986$$
$$(9,9994001)$$

$$C = -\tfrac{0,9986}{1,9986} = -0,49966.$$
$$(9,6986742)$$

$$\mathfrak{D} = \tfrac{\theta}{1+\theta};\ D = \vartheta;$$

$$\mathfrak{E} = \tfrac{118,32}{33,326} = 3,5504.$$
$$(0,5502750)$$

$$E = -\tfrac{3,5504}{2,5504} = -1,3921.$$
$$(0,1436667)$$

vnde colligimus

$$\log. B\mathfrak{C} = 0,6983701;$$

log.

$\log. BC = 0,3976442$;

$\log. BC\mathfrak{C} = 0,9479192$;

$\log. BCE = 0,5413109\,(-)$

His praemiſſis elementa noſtra erunt

$\alpha = p$; $b = -0,2.p$; $\beta = p$; $c = -0,2.p$.

$\gamma = 0,099932.p$; $d = 0,15023.p$.

$\delta = 0,15023.\vartheta.p$; $e = 0,00833.\vartheta.p$.

$\varepsilon = -0,01159.\vartheta.p$; $f = +0,02318.\vartheta.p$.

vnde colligimus interualla

$AB = 0,8.p = BC$; $CD = 0,25016.p$.

$DE = 0,15856.\vartheta.p$; $EF = 0,01159.\vartheta.p$.

Diſtantiae vero focales ita ſe habebunt:

$q = -0,25.p.$; $r = 0,19972.p$;

$s = 0,15023.\frac{\vartheta}{1+\vartheta}.p$; $t = 0,02956.\vartheta.p$; et

$u = 0,02318.\vartheta.p$.

Porro eſt $\omega = \frac{1}{4}x = -\mathfrak{r}$; vnde aequatio $\mathfrak{r}r = 4\delta x$ dabit

$$x = \frac{12}{0,19972.p} = \frac{60}{p}$$

proxime dum ergo p ſit < 60, tuto ſumere licebit $x = 1$. et quia tum erit $M = \frac{2}{166,550}$; hincque ſemidiameter campi $\Phi = 10\frac{1}{3}$ min. et pro loco oculi

$O = 0,555.u. = 0,01285.\vartheta.p$.

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Denique ſi primum ſpeculum conficiatur parabolicum, omnis confuſio tolletur huic aequationi ſatisfaciendo

$$0,0288 = 0,00030144.\lambda - 0,00013994 + 0,0036177.\frac{\lambda'(1+\vartheta)^3}{\vartheta^3} + 0,00084146.\frac{1+\vartheta}{\vartheta^2} + \frac{0,0001058}{\vartheta^3} + \frac{0,00024176}{\vartheta^3}$$

ſiue

$$0,0289399 = 0,00030144.\lambda + 0,0036177.\frac{(1+\vartheta)^3}{\vartheta^3}\lambda' + 0,00084146.\frac{1+\vartheta}{\vartheta^2} + \frac{0,0003410}{\vartheta^3}$$

Hic patet ſtatim, ſumi non poſſe $\vartheta = 1$. tentetur ergo poſitio $\vartheta = \frac{3}{2}$ eritque

$$0,0289399 = 0,00030144.\lambda + 0,0167487.\lambda' + 0,00093495 + 0,0001013.$$

ſiue

$$0,0279037 = 0,00030144.\lambda + 0,0167487.\lambda'$$

quare ſi hic ſtatuatur $\lambda' = 1$; fiet

$$\lambda = \frac{0,0111550}{0,00030144} = \frac{11155}{301} = 37.$$

ſin autem ſumamus $\lambda = 1$. fiet

$$\lambda' = \frac{0,0276023}{0,0167487} = \frac{276023}{167487} = 1,648.$$

Sin autem λ ſtatueretur 2 vel 3, valor ipſius λ' vix inde mutaretur vnde pro vſu practico praeſtare videtur, ſi ipſi λ' certus quidam valor tribuatur quia

quia enim tum ob leuiſſimos errores λ multum variare poteſt, plures lentes pro variis valoribus λ parari poterunt; ex quibus aptiſſimam experientia declarabit. Statuamus ergo $\lambda' = \frac{5}{2}$ ac reperietur

$$\lambda = \frac{0,2027807}{0,0253014} = \frac{27807}{3014} = 9.$$

vnde in praxi ternae lentes parari poterunt ex valoribus $\lambda = 8; = 9; = 10.$

Poſito ergo $\vartheta = \frac{5}{2}$; vt ſit $\mathfrak{D} = \frac{5}{3}$ ſumatur $\lambda' = \frac{5}{2}$ et $\lambda = 9$. vnde colligitur ſequens:

Conſtructio Teleſcopii Catadioptrici pro $m = 150$.

§. 69. Haec conſtructio ſequentibus determinationibus continetur:

I°. Speculum obiectiuum accuratiſſime ſecundum figuram parabolicam elaboretur; cuius diſtantia focalis minor non ſit duodecim digitis; quam hic littera p deſignemus. Eius aperturae ſemidiameter vero ſit $x = 3$. dig. foraminis vero ſemidiameter $= \frac{1}{4}$ dig. et diſtantia ad ſpeculum minus $AB = 0,8\ p.$

II°. Speculum minus exactiſſime ad figuram ſphaericam elaboretur, cuius diſtantia focalis ſit $q = -0,25 . p.$ quippe quod eſt conuexum. Eius aperturae ſemidiameter $= \frac{1}{4}$ dig. et diſtantia ad primam lentem $BC = 0,8 . p.$

III°. Pro prima lente, cuius diſtantia focalis eſt $r = 0,19972 \cdot p$. numerique $\mathfrak{C} = -0,9986$ et $\lambda = 9$. capiatur radius faciei

anter. $= \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \pm \tau \sqrt{8}.} = \frac{r}{0,5021} = 0,39777 \cdot p.$

poſter. $= \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \mp \tau . \sqrt{8}.} = \frac{r}{1,3160} = 0,15176. p.$

Sin autem ſumeretur $\lambda = 10$, prodiret radius faciei

anter. $= \frac{r}{0,3468} = 0,57589 \cdot p.$

poſter. $= \frac{r}{1,4747} = 0,13543 \cdot p.$

vnde concludimus in genere ſumi poſſe radium faciei

anter. $= (0,39777 \mp 0,17812.\ \omega)\, p.$

poſter. $= (0,15176 \pm 0,01633.\ \omega)\, p.$

vbi ω per experientiam definiri conueniet.

Huius autem lentis ſemidiameter aperturae $= \frac{1}{4}$ dig et diſtantia ad lentem ſecundam

$CD = 0,25016. p.$

IV°. Pro ſecunda lente, cuius diſtantia focalis $s = 0,090138. p.$ et numeri $\mathfrak{D} = \frac{1}{5}$ et $\lambda' = 1,5$ capiatur radius faciei

anter. $= \frac{s}{\sigma - \mathfrak{D}(\sigma - \varrho) \pm \tau \sqrt{0,5}} = \frac{s}{0,1251} = 0,72046. p.$

poſter. $= \frac{s}{\varrho + \mathfrak{D}(\sigma - \varrho) \mp \tau \sqrt{0,5}} = -\frac{s}{0,0312} = -2,8864. p.$

Eius

Eius aperturae ſemidiameter

$= \frac{x}{PQR} = \frac{2}{11}$ dig. $= 0,18$. dig.

et diſtantia ad lentem tertiam

$DE = 0,23784.p.$

V°. Pro tertia lente, cuius diſtantia focalis

$t = 0,04434.p.$

ſumatur radius faciei vtriusque $= 0,048774.p.$

eius aperturae ſemidiameter $= 0,01108.p.$

et diſtantia ad lentem quartam $EF = 0,01738.p.$

VI°. Pro lente quarta, cuius diſtantia focalis

$u = 0,03477.p.$

capiatur radius vtriusque faciei $= 0,03824.p.$

eius aperturae ſemidiameter $= \frac{1}{4}u = 0,00869.p.$

et diſtantia ad oculum

$O = 0,555.u = 0,01927.p.$

VII°. Longitudo ergo tubi prioris ſpecula continentis aliquantum ſuperabit $0,8.p$; poſterioris vero erit $= 0,52465.p.$ ita, vt totius inſtrumenti longitudo ſit circiter $= 1,32465.p.$ Tum vero ſemidiameter campi apparentis erit $= 10\frac{1}{3}$ minut.

Scholion.

§. 70. Remedium in ſubſidium praxeos, quod hic pro prima lente attulimus, etiam facile ad exempla prae-

praecedentia accommodatur. Ponamus enim pro hac lente inuentos eſſe radios facierum f et g, et nunc quaeſtio eo redit, quomodo hos radios variari oporteat, vt diſtantia focalis maneat eadem. Ponatur prior $= f + x$; poſterior $= g - y$, et neceſſe eſt, vt fiat $\frac{fg}{f+g} = \frac{(f+x)(g-y)}{f+g+x-y}$ vnde ſumto x pro lubitu ſiue negatiue ſiue poſitiue capi debebit $y = \frac{g^2 . x}{f^2 + (f+g)x}$; quare cum x et y ſint ſatis parua erit $y = \frac{g^2 x}{f^2}$, ſiue $x : y = f^2 : g^2$, ita, vt poſito $x = f^2 \omega$ futurum ſit $y = g^2 . \omega$. Pro lente ergo prima, cuius radii ſupra inuenti ſint f et g, alias ſucceſſiue ſubſtitui conueniet, quarum radii ſint $f \pm f^2 . \omega$ et $g \mp g^2 . \omega$. Deinde hic etiam notaſſe iuuabit, pro lente prima minorem aperturam ſufficere poſſe, quam hic aſſignauimus foramini aequalem. Sufficiet enim apertura, cuius ſemidiameter $= \frac{1}{4} r\, r. = \frac{1}{16} . r = 0{,}01248 . p.$ vnde ſi $p = 12$ dig. iſte ſemidiameter foret $= 0{,}1497$. dig. $= \frac{1}{7}$ dig. circiter; ac ſi adeo eſſet $p = 20$. dig.; foret iſte ſemidiameter $= \frac{1}{4}$ dig. ex quo concludimus, ſufficere, ſi huic lenti apertura tribuatur, cuius ſemidiameter ſit $\frac{1}{4}$ dig. quo pacto ingentem copiam radiorum peregrinorum ab introitu arcebimus, ſicque reliqui eo felicius a ſecunda lente excludentur; etſi eius apertura non tam eſt exigua, vt in praecedentibus exemplis, cuius rei ratio eſt, quod litteram i in multo minore ratione auximus, quam multiplicationem m; quam ob cauſſam in ſequente exemplo litterae i multo maiorem valo-

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valorem tribuemus, quia inde nihil aliud eſt metuendum, niſi exigua diminutio campi.

Exemplum 4.
pro multiplicatione $m = 200$.

§. 71. Manentibus litteris $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$, capiatur $i = 10$ et vt ſufficiens claritatis gradus obtineatur, ſumamus $x = 5$. dig. vt ſit ſemidiameter foraminis $= \delta x = \frac{5}{4}$ dig. et $\varepsilon m = 40$. Hinc ergo colliguntur valores

$$P = 5;\ Q = 10;\ R = -k = -0,8221;$$

$$S = -k' = -9,7312 \text{ et } T = \tfrac{1}{2};\ \text{hincque}$$

$$PQ = 50;\ PQR = -41,105;$$

$$PQRS = 400 \text{ et } PQRST = 200.$$

reliquae vero litterae ita determinabuntur

$$\mathfrak{B} = \tfrac{40}{31} = 1,2903;\quad B = -\tfrac{40}{9} = -4,4444.$$

$$\mathfrak{C} = -1,0153;\qquad C = -0,50381.$$

$$(0,0066052)(-);\qquad (9,7022655)(-)$$

$$\mathfrak{E} = 3,2841;\qquad E = -1,4377.$$

$$(0,5164093)\qquad (0,1576942)$$

vnde colliguntur ſequentes logarithmi

$\log. B\mathfrak{C} = 0,6544183;\ l.\ BC = 0,3500786.$

$\log. BC\mathfrak{E} = 0,8664879;\ l.\ BCE = 0,5077728 -$

 hinc

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hinc elementa ſequenti modo definientur:

$\alpha = p$; $b = -0,2.p$; $\beta = 0,8889.p$;

$c = -0,0889.p$; $-\gamma = 0,04478$;

$d = 0,054473.p$; $\delta = 0,054473.\vartheta.p$;

$e = 0,005598.\vartheta.p$; $\varepsilon = -0,007798.\vartheta.p$;

et $f = 0,015596.\vartheta.p$.

ex quibus colliguntur interualla

$AB = 0,8.p = BC$; $CD = 0,09925.p$;

$DE = 0,060071.\vartheta.p$; $EF = 0,007798.\vartheta.p$.

Diſtantiae vero focales

$q = -0,2581.p$; $r = 0,09025.p$;

$s = 0,05447\frac{\theta}{1+\theta}.p$; $t = 0,01838.\vartheta.p$;

et $u = 0,015596.\vartheta.p$.

Porro eſt $\omega = -\mathfrak{r} = \frac{3}{4}t$; vnde aequatio $\mathfrak{r}r = 4\delta x$ dabit $t = \frac{160}{p}$. dig.; vnde patet, dummodo p minor ſit, quam 160. dig. tuto ſumi poſſe $t = 1$; at ſi liceat confuſionem ad nihilum redigere, adeo ſumere licebit $p = 20$. dig. tum autem fiet $M = \frac{1}{150}$; vnde ſemidiameter campi erit $\frac{859}{125}$ min. $= 7\frac{1}{8}$ min. Praeterea vero pro loco oculi habebitur $O = 0,6.u$. Tantum igitur ſupereſt, vt confuſionem ad nihilum redigamus, quod fiet hac aequatione:

$$0,029074 = 0,00020418 . \lambda - 0,0000972 .$$
$$+ 0,0020329 . \frac{(\theta+1)^5}{\theta^3} \lambda'$$
$$+ 0,00047286 . \frac{1+\theta}{\theta^2} .$$
$$+ \frac{0,000111}{\theta^3} .$$
$$+ \frac{0,00000116}{\theta^3} .$$

ſiue

$$0,029171 = 0,00020418 . \lambda$$
$$+ 0,0020329 . \frac{(1+\theta)^5}{\theta^3} . \lambda'$$
$$+ 0,0004729 . \frac{1+\theta}{\theta^2} .$$
$$+ \frac{0,0001122}{\theta^3} .$$

vbi iam nihil obſtat, quominus ſtatuatur $\vartheta = 1$. hincque habebimus

$$0,028113 = 0,0002042 . \lambda + 0,016264 . \lambda' .$$

Ne igitur hinc valor ipſius λ prodeat nimis magnus, commode ſtatui poterit $\lambda' = 1\frac{1}{2}$, atque reperietur $\lambda = \frac{3717}{204} = 18$. proxime. Commodius vero erit ſumere $\lambda' = 1\frac{2}{3}$; vnde fiet $\lambda = \frac{1006}{204} = 5$. Retineamus igitur valores $\vartheta = 1$; $\lambda' = 1\frac{2}{3}$, vt fiat $\lambda = 5$, cui adiungere poterimus valores finitimos $\lambda = 4$ et $\lambda = 6$. quo praxi melius conſulatur; atque hinc colligetur ſequens

Conſtructio Teleſcopii Catadioptrici pro multiplicatione $m = 200$.

§. 72. Statuamus hic, vt hactenus, diſtantiam focalem ſpeculi principalis $= p$, quum, vt vidimus, mino-

minorem quam 20 dig. aſſumi non conuenit. Praeſtabit autem eam haud mediocriter maiorem aſſumere.

I°. Speculum igitur primum adcuratiſſime forma parabolica elaboretur, cuius diſtantia focalis ſit $= p$;
Eius aperturae ſemidiameter $x = 5$. dig.
et ſemidiameter foraminis $y = 1\frac{1}{4}$ dig.
Diſtantia vero ad ſpeculum minus $AB = 0,8.p$.

II°. Pro ſecundo ſpeculo minore conuexo eius figura accuratiſſime ſphaerice elaboretur, vt ſit eius diſtantia focalis $q = -0,2581.p$.
Eius aperturae ſemidiameter $= 1\frac{1}{4}$ dig.
et diſtantia ad primam lentem in foramine $= BC = 0,8.p$.

III°. Pro lente prima, cuius diſtantia focalis $r = 0,09025.p$. et numeri $\mathfrak{C} = -1,0153$. et $\lambda = 5$, capiatur radius faciei

anter. $= \frac{r}{\sigma - \mathfrak{C}(\sigma - \varrho) \mp \tau\sqrt{4}} = \frac{r}{3,0361 \mp 1,8102}$

poſter. $= \frac{r}{\varrho + \mathfrak{C}(\sigma - \varrho) \pm \tau\sqrt{4}} = \frac{r}{-1,2610 \mp 1,8102}$

hinc radius faciei

anter. $= 0,070734.p$.
poſter. $= 0,16645.p$.

Sin

Sin autem ſumeremus $\lambda = 4$, prodiret radius faciei

anter. $= \frac{r}{3{,}0861 + 1{,}5677} = 0{,}05944.\,p.$

poſter. $= \frac{r}{-1{,}2680 \pm 1{,}5677} = 0{,}30113.\,p.$

At ſi ſumeretur $\lambda = 6.$ foret radius faciei

anter. $= \frac{r}{3{,}0861 + 2{,}0239} = 0{,}08496.\,p.$

poſter. $= \frac{r}{-1{,}2680 \pm 2{,}0239} = 0{,}11940.\,p.$

ex quibus caſibus deducimus in ſubſidium praxcos ſequentes concluſiones:

Prior: Si $\lambda = 5 - \omega$, denotante ω fractionem arbitrariam, erit radius faciei

anter. $= (0{,}07073 - 0{,}01129.\,\omega)\,p$

poſter. $= (0{,}16645 + 0{,}13468.\,\omega).\,p.$

Poſter: Sin autem $\lambda = 5 + \omega$, erit radius faciei

anter. $= (0{,}07073 + 0{,}01423.\,\omega)\,p.$

poſter. $= (0{,}16645 - 0{,}04705.\,\omega).\,p.$

Eius aperturae ſemidiameter $= 1\frac{1}{4}$ dig.

et diſtantia ad lentem ſecundam

$CD = 0{,}09925.\,p.$

IV°. Pro ſecunda lente, cuius diſtantia focalis eſt $s = 0{,}02723.\,p.$ et numeri $\mathfrak{D} = \frac{1}{2}$ et

 $\lambda' =$

$\lambda' = 1,6667$. capiatur radius faciei

anter. $= \frac{s}{\frac{1}{2}(\sigma + \varrho) + \tau \sqrt{0,6667}} = \frac{s}{0,5090 + 0,7350}$

poſter. $= \frac{s}{\frac{1}{2}(\sigma + \varrho) \pm \tau \sqrt{0,6667}} = \frac{s}{0,5050 \pm 0,7350}$

ſeu anter. $= 0,01652.p.$

poſter. $= 0,16018.p.$

eius aperturae ſemidiam. $= \frac{x}{FQR} = \frac{1}{3}$ dig.

et diſtantia a lente tertia $DE = 0,06007.p.$

V°. Pro lente tertia, cuius diſtantia focalis

$t = 0,01838.p.$

capiatur radius faciei vtriusque $= 0,02022.p.$

Eius apert. ſemidiam. $= \frac{1}{4}t = 0,00459.p.$

et diſtantia a lente quarta $EF = 0,007798.p.$

VI°. Pro lente quarta, cuius diſtantia focalis

$u = 0,015596.p.$

capiatur radius faciei vtriusque $= 0,01715.p$

Eius aperturae ſemid. $= \frac{1}{4}u = 0,0039.p.$

et diſtantia ad oculum $= 0,6.u = 0,00936.p.$

VII°. Hinc ergo longitudo tubi prioris erit quaſi $= p$, quia maior eſſe debet, quam $\frac{9}{7}.p.$ poſterioris vero lentes continentis $= 0,17648.p.$

ita,

ita, vt tota longitudo futura ſit circiter

$= 1,\ 17648 . p.$

Campi vero apparentis ſemidiameter erit

$= 7\frac{1}{8}$ minut.

VIII°. Si pro lente prima tantum ad claritatem ſpectemus, eius aperturae ſemidiameter deberet eſſe $= \frac{x}{PQ} = \frac{1}{13}$ dig. ſin autem ad campum ſpectemus, hic ſemidiameter eſſe debet

$= \frac{1}{4}\, \mathfrak{r}\, r = \frac{3}{32} . r = 0,\ 00846 . p.$

qui, ſi adeo eſſet $p = 40$ dig. fieret

$0,\ 3384$ dig. $= \frac{1}{3}$ dig.

Quare cum ſemidiameter foraminis $= 1\frac{1}{4}$ dig. tuto oram huius lentis obtegere licebit, donec eius aperturae ſemidiameter fiat $= \frac{1}{3}$ dig. quo pacto radii peregrini iam maximam partem excludentur.

IX°. Cum igitur ne opus quidem ſit tantam magnitudinem primae lenti tribuere, ipſum foramen maioris ſpeculi multo minus ſtatuere licebit, quam $1\frac{1}{4}$ dig. hocque modo dum ipſum hoc ſpeculum maiorem ſuperficiem adipiſcetur, etiam claritatis gradus augebitur, neque vero ideo neceſſe erit, et minoris ſpeculi magnitudinem imminuere, cum ſufficiens radio-

diorum copia in ſpeculum cadere poſſit. Radii peregrini colliguntur poſt lentem C in diſtantia $r = 0,09025.p$. radii vero proprii in diſtantia $\gamma = 0,0448.p$.

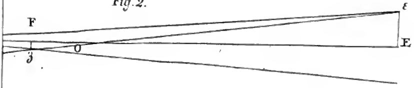
X°. Cum deinde prima imago realis poſt lentem primam cadat ad diſtantiam $\gamma = 0,0448.p$. radii autem peregrini in hanc lentem incidentes ſuam imaginem forment ad diſtantiam $r = 0,09025.p$; quae cum illa plus quam duplo ſit maior, neutiquam metuendum erit, ne radii peregrini ad oculum vsque propagentur.

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Fig. 1.



Fig. 2.



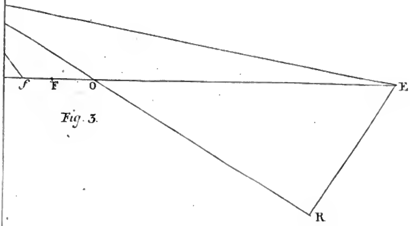
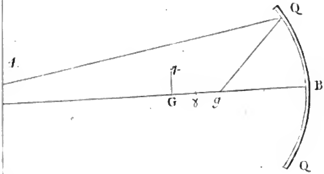


Fig. 3.

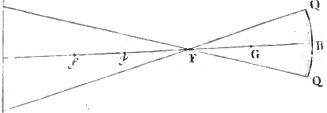
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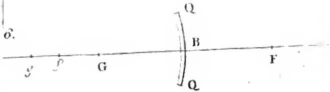
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5.



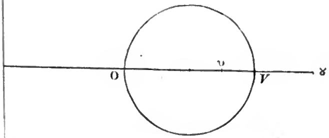
6.

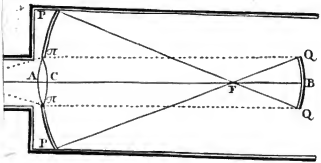


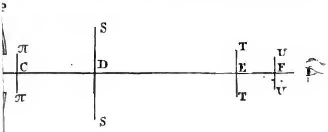
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